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ON SOME GENERALIZATIONS OF LC–SPACES

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Abstract. The aim of this paper is to extend the notion of LC-spaces, i.e. spaces whose Lindelöf subsets are closed. We will consider four weaker forms of this concept and investigate their relationships with LC-spaces as well as among themselves. Accordingly, we continue the study of LC-spaces and related spaces.

1. Introduction

Lindelöf spaces have always played a highly expressive role in topology. They were introduced by Alexandroff and Urysohn back in 1929 and their name is due to Lindelöf's proof in 1903 that from any collection of open sets covering an euclidean space one can extract a countable subcollection covering the space.

Special classes of Lindelöf spaces such as hereditarily Lindelöf spaces and maximal Lindelöf spaces have had considerable impact on General Topology. A class of spaces that occurs in the study of maximal Lindelöf spaces [**3**] is the notion of LC-spaces, — a concept having much in common with P-spaces.

A topological space whose Lindelöf subsets are closed is called an LC-space by Mukherji and Sarkar [**16**] and by Gauld, Mrsevic, Reilly and Vamanamurthy [**8**]. LC-spaces are also known as L-closed spaces ([**10**], [**11**], [**12**], [**13**]). They generalize KC-spaces (= compact subsets are closed) [**21**] and Hausdorff P-spaces (= F_σ-sets are closed) [**15**]. Every LC-space is a cid-space (= countable subsets are closed and discrete) [**6**] and so T_1 and anticompact (= compact subsets are finite). Note that cid-spaces have been called weak LC-spaces by Mukherji and Sarkar [**16**].

In recent years there has been a significant interest in LC-spaces. Examples of LC-spaces that are not P-spaces can be found in [**12**], [**13**], [**15**] and [**16**]. By making use of an example of Kunen of a rigid Tychonoff Lindelöf P-space, Henrikson and Woods [**12**] provided an example of a Lindelöf Tychonoff cid-space that is not an LC-space. Generalizations of some results from [**11**] can be found in a paper by Ganster and Jankovic [**6**] where several examples are given as well.

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Grant and Reilly [**9**] considered the situation when an LC-space is discrete and when a Hausdorff LC-space is countably compact. Very recently, Dontchev and Ganster [**5**] showed that the product of two LC-spaces need not be an LC-space, and Ganster, Kanibir and Reilly [**7**] pointed out that a locally LC-space need not be an LC-space.

In this paper we consider LC-spaces from a more generalized point of view. We introduce some new classes of spaces that contain properly the class of LC-spaces. More precisely, we consider a modified version of C-spaces (= compact sets have compact closures) as the spaces in which every Lindelöf subset has Lindelöf closure. We also investigate spaces where all Lindelöf sets are F_σ-sets, spaces whose Lindelöf F_σ-sets are closed, and the spaces where between each Lindelöf set and its closure there lies a Lindelöf F_σ-set. In the last section we consider (weakly) locally Lindelöf spaces and their relationships to generalized LC-spaces.

Our terminology is standard. The closure and the interior of a subset A of a space (X, τ) are denoted by cl A and int A, respectively (or $\mathrm{cl}_\tau A$ and $\mathrm{int}_\tau A$ if there is a possibility of confusion). The set of all positive integers is denoted by ω.

2. Generalized LC-spaces

Definition 1. A topological space (X, τ) is called an LC-space ([**16**], [**8**]) if every Lindelöf subset of X is closed.

Note that LC-spaces are also known under the name L-closed [**11**]. We will now introduce the following four generalizations of LC-spaces.

Definition 2. A topological space (X, τ) is called

(1) an L_1-space if every Lindelöf F_σ-set is closed,
(2) an L_2-space if $\mathrm{cl}\, L$ is Lindelöf whenever $L \subseteq X$ is Lindelöf,
(3) an L_3-space if every Lindelöf subset L is an F_σ-set,
(4) an L_4-space if, whenever $L \subseteq X$ is Lindelöf, then there is a Lindelöf F_σ-set F with $L \subseteq F \subseteq \mathrm{cl}\, L$.

Our first result summarizes some immediate consequences of Definition 2.

Theorem 2.1. *(i) If (X, τ) is an LC-space then (X, τ) is an L_i-space, $i = 1$, 2, 3, 4.*

(ii) (X, τ) is an LC-space if and only if it is an L_1-space and an L_3-space.

(iii) Every Lindelöf space is an L_2-space, and every L_2-space having a dense Lindelöf subset is Lindelöf.

(iv) Every space which is L_1 and L_4 is an L_2-space.

(v) Every L_2-space is an L_4-space, and every L_3-space is an L_4-space.

(vi) Every L_3-space is T_1, and every T_1 L_1-space is cid.

(vii) The property L_3 is hereditary, and the properties L_1, L_2 and L_4 are hereditary on F_σ-sets.

(viii) Every P-space is an L_1-space.

In 1979, Bankston [**2**] introduced the so-called anti-operator on a topological space. A space (X, τ) is said to be anti-Lindelöf if each Lindelöf subset of X is countable. Recall also that (X, τ) is called a Q-set space if each subset of (X, τ) is an F_σ-set. The proof of the following result is straightforward and hence omitted.

Proposition 2.2. *(i) Every T_1 anti-Lindelöf space is an L_3-space. Hence every T_1, anti-Lindelöf L_1-space is an LC-space.*

(ii) Every Q-set space is an L_3-space, hence so is every strongly σ-discrete and every regular submaximal space with countable Souslin number [**1**].

Note that, although Q-set spaces are L_3-spaces they need not be LC-spaces as the set of all integers with the cofinite topology shows. Our next result provides a condition under which L_3-spaces are Q-set spaces.

Proposition 2.3. *Every hereditarily Lindelöf L_3-space is a Q-set space.*

We now provide some examples to show that among the L_i-spaces there are no more implications than those listed in Theorem 2.1.

Example 2.4. Let R be the set of reals and let τ be the usual topology. Then (R, τ) is a hereditarily Lindelöf L_2-space and thus an L_4-space, but neither an LC-space nor an L_1-space nor an L_3-space.

Example 2.5. Let R be the set of reals and let τ be the rational sequence topology on R (see [**19**, Example 65]). Then (R, τ) is a separable non-Lindelöf space where each point has a countable neighbourhood. Hence (R, τ) is anti-Lindelöf and so an L_3-space and also an L_4-space. Clearly (R, τ) is neither an L_2-space nor an L_1-space.

Example 2.6 (see [**20**]). Let X be the set of reals and let τ be the density topology on X. It is consistent with the axioms of set theory that the only hereditarily Lindelöf subspaces are the countable ones [**20**]. Since the density topology is perfect, all Lindelöf subsets are hereditarily Lindelöf and hence countable. Since (X, τ) is a cid-space it is thus an LC-space. Additionally we note that (X, τ) is neither Lindelöf nor separable.

Our next task is to show that there exists a Hausdorff L_1-space that is not an LC-space. For this we need some preparation. Let R be the set of reals and let τ be the usual topology on R. $B \subseteq R$ is called a Bernstein set if B and $R - B$ intersect every uncountable closed subset of (R, τ). $L \subseteq R$ is said to be a Lusin set if L is uncountable and the intersection of L with any meager subset of (R, τ) is at most countable. For the convenience of the reader let us mention some basic facts about these notions (see e.g. [**18**]).

Proposition 2.7. *(i) There exists a Bernstein subset of (R, τ), and if B is a Bernstein set then $R - B$ is also a Bernstein set.*

(ii) If B is a Bernstein set then B is a dense Baire subspace. In particular, for any open set U, $U \cap B$ is uncountable.

(iii) Under the continuum hypothesis (CH), every subset of R of 2^{nd} category contains a Lusin set.

Now let $R = B_1 \cup B_2$ be the disjoint union of two Bernstein sets B_1 and B_2, and let $\{G_n : n \in \omega\}$ be a countable base for (R, τ). For each $n \in \omega$ choose a Lusin set $L_n \subseteq B_1 \cap G_n$ and let $L = \cup\{L_n; n \in \omega\}$. Then L is also a Luzin set such that for any nonempty open set U in (R, τ), $U \cap L$ is uncountable. Let $X = L \cup B_2$. We will define a new topology σ on X in the following way: a basic neighbourhood of $x \in L$ is a set W_x containing x and having the form $W_x = (U \cap L) - C$ where U is open in (R, τ) and C is countable. A basic neighbourhood of $x \in B_2$ has the form $\{x\} \cup ((V \cap L) - C)$ where V is an open set in (R, τ) containing x and C is countable. Note that every countable subset of (X, σ) is closed. Since the Countable Complement Extension Topology τ^* on R is hereditarily Lindelöf (see [**19**, Example 63]) and $\tau^*|_L = \sigma|_L$, L is a Lindelöf subspace of (X, σ) which is not closed. Hence (X, σ) is not an LC-space.

Let A be closed and Lindelöf in (X, σ). Then $A \cap B_2$ is countable since B_2 is closed and discrete in (X, σ). Suppose that A is uncountable. Then $A \cap L$ is not meager in (R, τ). If $(A \cap L)^*$ denotes the set of condensation points of $A \cap L$ then $(A \cap L)^*$ is closed in (R, τ), $(A \cap L) - (A \cap L)^*$ is countable and so $(A \cap L) \cap (A \cap L)^*$ is not nowhere dense in (R, τ), i.e. there exists a nonempty open set W in (R, τ) such that $W \subseteq (A \cap L)^*$. Let $x \in W \cap B_2$ and suppose that $x \notin \mathrm{cl}_\sigma A$. Then there exists an open set V in (R, τ) containing x and a countable set C such that $V \subseteq W$ and $(\{x\} \cup (V \cap L) - C) \cap (A \cap L)$ is empty, i.e. $V \cap A \cap L$ is countable, which is a contradiction to $x \in (A \cap L)^*$. So we have $W \cap B_2 \subseteq \mathrm{cl}_\sigma A \cap B_2 = A \cap B_2$. Hence $W \cap B_2$ is countable. On the other hand, since B_2 is a dense Baire subspace of (R, τ), $W \cap B_2$ has to be uncountable, a contradiction. So we have shown that any set which is closed and Lindelöf in (X, σ) has to be countable, and consequently every Lindelöf F_σ-set in (X, σ) is countable and thus closed in (X, σ). So (X, σ) is an L_1-space and we have shown

Example 2.8. Under (CH) there exists a Hausdorff L_1-space (X, σ) which is not an LC-space. Also, one easily checks that (X, σ) is not an L_4-space, hence neither an L_2-space nor an L_3-space.

Example 2.9. The set of reals with the "right ray" topology is a P-space and thus an L_1-space. The rationals form a Lindelöf non-closed subset and so this space is not an LC-space.

Question. Does there exist a Hausdorff L_1-space which fails to be an LC-space without any set-theoretic assumptions?

3. Characterizations

In this section we will provide characterizations of L_1-spaces and additional characterizations of LC-spaces.

In 1984, Gauld, Mrsevic, Reilly and Vamanamurthy [**8**] introduced the co-Lindelöf topology of a given space (X, τ). They showed that $l(\tau) = \{\emptyset\} \cup \{G \in \tau : X - G$ is Lindelöf in $(X, \tau)\}$ is a topology on X with $l(\tau) \subseteq \tau$, called the co-Lindelöf topology of (X, τ).

Theorem 3.1. *For a space (X, τ) the following are equivalent:*
(1) *(X, τ) is an L_1-space,*
(2) *$(X, l(\tau))$ is a P-space.*

Proof. (1) $\Rightarrow$ (2): For each $n \in \omega$, let A_n be closed in $(X, l(\tau))$ and let $A = \cup\{A_n : n \in \omega\}$. If $A = X$ we are done. Otherwise each A_n is closed and Lindelöf in (X, τ) and thus A is closed and Lindelöf in (X, τ). Hence A is closed in $(X, l(\tau))$.

(2) $\Rightarrow$ (1): For each $n \in \omega$, let A_n be closed and Lindelöf in (X, τ) and let $A = \cup\{A_n : n \in \omega\}$. Then each A_n is closed in $(X, l(\tau))$ and so A is also closed in $(X, l(\tau))$. Hence A is closed in (X, τ) and so (X, τ) is an L_1-space. □

Theorem 3.2. *For a Hausdorff space (X, τ) the following are equivalent:*
(1) *(X, τ) is an LC-space,*
(2) *(X, τ) is an L_1-space and an L_2-space.*

Proof. (1) $\Rightarrow$ (2): This is obvious.

(2) $\Rightarrow$ (1): Let L be a Lindelöf subset of (X, τ) and let $x \notin L$. Since (X, τ) is Hausdorff, for each $y \in L$ there exist an open set V_y containing y with $x \notin \operatorname{cl} V_y$. Clearly $\{V_y : y \in L\}$ is a cover of L and so there exists a countable set $C \subseteq L$ such that $L \subseteq \cup\{V_y : y \in C\} \subseteq \cup\{\operatorname{cl} V_y : y \in C\}$. For each $y \in C$, $L \cap \operatorname{cl} V_y$ is Lindelöf and so $\operatorname{cl}(L \cap \operatorname{cl} V_y)$ is Lindelöf since (X, τ) is an L_2-space. Furthermore, if $W = \cup\{\operatorname{cl}(L \cap \operatorname{cl} V_y) : y \in C\}$ then W is a Lindelöf F_σ-set and, since (X, τ) is an L_1-space, W is a closed Lindelöf set not containing x. Thus $x \notin \operatorname{cl} L$. This shows that L is closed in (X, τ). □

Remark 3.3. Note that the Hausdorff condition cannot be removed as the following example shows. Let X be any countably infinite set with a distinguished point p and let τ be the point excluded topology on X (see e.g. [**4**]), i.e. $\tau = \{X\} \cup \{U \subseteq X : p \notin U\}$.

Clearly (X, τ) is a countable, thus hereditarily Lindelöf, non-Hausdorff space but not an LC-space since it is not discrete. If A is a nonempty F_σ-set then $p \in A$ and so (X, τ) is an L_1-space. Since (X, τ) is Lindelöf it is also an L_2-space.

Theorem 3.4. *For a Hausdorff space (X, τ) the following are equivalent:*
(1) *(X, τ) is an LC-space.*
(2) *Every locally countable family of Lindelöf sets is closure preserving.*

(3) *Every countable family of Lindelöf sets is closure preserving.*

Proof. (1) $\Rightarrow$ (2): Let $\{L_i : i \in I\}$ be a locally countable family of Lindelöf subsets, i.e. each $x \in X$ has a neighbourhood U_x intersecting at most countably many sets L_i. Since each L_i is closed we need to show that $L = \cup\{L_i : i \in I\}$ is closed. Let $x \in X - L$ and let U_x be a neighbourhood of x such that $I_0 = \{i \in I : U_x \cap L_i \text{ is nonempty}\}$ is at most countable. This implies that $\cup\{L_i : i \in I_0\}$ is Lindelöf and thus closed. Hence there is a neighbourhood V_x of x with $V_x \subseteq U_x$ and $V_x \cap (\cup\{L_i : i \in I_0\})$ is empty. So we have $V_x \cap (\cup\{L_i : i \in I\}) = \emptyset$. This shows that L is closed.

(2) $\Rightarrow$ (3): This is obvious.

(3) $\Rightarrow$ (1): Let L be a Lindelöf subset and let $x \notin L$. Since (X, τ) is Hausdorff, for each $y \in L$ there exists an open neighbourhood V_y of y with $x \notin \operatorname{cl} V_y$. Choose a countable set $C \subseteq Y$ with $L \subseteq \cup\{V_y : y \in C\}$. Since $\{L \cap \operatorname{cl} V_y : y \in C\}$ is a countable family of Lindelöf sets, we have $\operatorname{cl} L \subseteq \cup\{\operatorname{cl}(L \cap \operatorname{cl} V_y) : y \in C\}$ and so $x \notin \operatorname{cl} L$. Hence L is closed. □

Note, however that the Sierpinski space (X, τ) where $X = \{0, 1\}$ and $\tau = \{\emptyset, \{0\}, X\}$ is non-Hausdorff and every countable family of Lindelöf sets is closure preserving. Obviously, (X, τ) fails to be an LC-space.

Theorem 3.5. *For a space (X, τ) the following are equivalent:*

(1) *(X, τ) is an L_1-space.*

(2) *Every locally countable family of closed Lindelöf sets is closure preserving.*

(3) *Every countable family of closed Lindelöf sets is closure preserving.*

Proof. (1) $\Rightarrow$ (2): This is very similar to the proof of (1) $\Rightarrow$ (2) in Theorem 3.4.

(2) $\Rightarrow$ (3): This is obvious.

(3) $\Rightarrow$ (1): Let L be a Lindelöf F_σ-set, i.e. $L = \cup\{L_n : n \in \omega\}$ where each L_n is closed. By assumption, $\operatorname{cl} L = \cup\{\operatorname{cl} L_n : n \in \omega\} = \cup\{L_n : n \in \omega\} = L$. Thus L is closed. □

4. Local Lindelöfness and Generalized LC-spaces

In this section we consider locally Lindelöf spaces and their relationships to generalized LC-spaces.

Definition 3. A topological space (X, τ) is called locally Lindelöf (resp. weakly locally Lindelöf) if each point has a closed Lindelöf (resp. Lindelöf) neighbourhood.

Note that a weakly locally Lindelöf space need not be a locally Lindelöf space as any uncountable point generated space [**4**] shows.

Our first result is immediate and so its proof is omitted.

Proposition 4.1. *(i) Every weakly locally Lindelöf L_2-space is locally Lindelöf, and so every weakly locally Lindelöf space which is L_1 and L_4 is locally Lindelöf.*

(ii) Every F_σ-subspace of a (weakly) locally Lindelöf space is (weakly) locally Lindelöf.

(iii) Every locally Lindelöf Q-set space is hereditarily locally Lindelöf.

Theorem 4.2. *Every locally Lindelöf space (X, τ) is an L_1-space if and only if it is a P-space.*

Proof. We already know that a P-space is an L_1-space. Now let F be an F_σ-set in (X, τ). If $x \notin F$ choose a closed Lindelöf neighbourhood U of x. Then $U \cap F$ is a Lindelöf F_σ-set in (X, τ) and so closed, since (X, τ) is an L_1-space. Hence $U - (U \cap F)$ is a neighbourhood of x disjoint from F. This shows that F is closed and so (X, τ) is a P-space. □

We note that if we replace the 'L_1-condition' in Theorem 4.2 by any of the other 'L_1-conditions' then the space (X, τ) need not even be a cid-space. If we take a non-discrete, countable, zero-dimensional Hausdorff space (X, τ) then (X, τ) clearly is an L_i-space for $i = 2, 3, 4$ since it is countable and thus hereditarily Lindelöf. Since (X, τ) is not discrete it is not a cid-space and hence not a P-space. A space (X, τ) satisfying the hypothesis above is, for example, the space $\mathrm{Seq}\,(\xi)$ discussed in [**14**].

Corollary 4.3. *Every Hausdorff, locally Lindelöf L_1-space (X, τ) is an LC-space.*

Corollary 4.4. *Every weakly locally Lindelöf LC-space (X, τ) is a P-space.*

Recall that a space (X, τ) is said to be a weak P-space if any countable union of regular closed sets is closed. One can show easily that (X, τ) is a weak P-space if and only if for every countable family $\{U_n : n \in \omega\}$ of open sets, $\mathrm{cl}(\cup\{U_n : n \in \omega\}) = \cup\{\mathrm{cl}\, U_n : n \in \omega\}$.

Theorem 4.5. *Let (X, τ) be a weak P-space. Then the following are equivalent:*
(1) (X, τ) *is locally Lindelöf,*
(2) (X, τ) *is a weakly locally Lindelöf L_2-space.*

Proof. (1) $\Rightarrow$ (2): Let L be a Lindelöf subset of (X, τ). Each point of L has an open neighbourhood U_x such that $\mathrm{cl}\, U_x$ is Lindelöf. Pick a countable subset C of L such that $L \subseteq \cup\{U_x : x \in C\}$. Since (X, τ) is a weak P-space we have $\mathrm{cl}\, L \subseteq \cup\{\mathrm{cl}\, U_x : x \in C\} = W$. Since W is Lindelöf we conclude that $\mathrm{cl}\, L$ is Lindelöf and so (X, τ) is an L_2-space.

(2) $\Rightarrow$ (1): This is Proposition 4.1. □

Recall that a subset A of a space (X, τ) is called locally closed if A is the intersection of an open set and a closed set, or, equivalently, if each point $x \in A$ has a neighbourhood V such that $A \cap V$ is a closed subset of V. (X, τ) is said to be submaximal if every dense set is open or, equivalently, if every subset of (X, τ) is locally closed.

Theorem 4.6. *Let (X, τ) be an LC-space and let A be a weakly locally Lindelöf subspace. Then A is locally closed in (X, τ).*

Proof. Choose $x \in A$. Then x has a neighbourhood V in (X, τ) such that $A \cap V$ is Lindelöf in A and thus also in X. So $A \cap V$ is closed in (X, τ) and thus also in V. This shows that A is locally closed in (X, τ). □

Corollary 4.7. *Every hereditarily weakly locally Lindelöf LC-space is submaximal.*

Remark 4.8. It is well known that maximally connected spaces are submaximal. A hereditarily weakly locally Lindelöf LC-space is not necessarily maximally connected, in fact such a space might be hereditarily disconnected as the One-point-Lindelöfication of an uncountable discrete space shows.

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