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ASCOLI'S THEOREM IN ALMOST QUIET QUASI-UNIFORM SPACE

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ABSTRACT. In this paper we have generalized Ascoli's theorem on almost quiet quasi-uniform space. We have also discussed some properties of the collection of all δ-continuous functions and the collection of all δ-equicontinuous functions.

1. INTRODUCTION

In [**1**] it is shown that Doitchinov's concept of quietness is sufficient to extend some classical results regarding uniform spaces to the much broader setting of quasi-uniform spaces. In [**2**] almost quiet quasi-uniform space has been introduced and it has been shown that a topological space is almost quiet quasi uniformizable if and only if it is almost regular.

In this paper, endeavour has been made to generalize Ascoli's theorem in almost quiet quasi-uniform spaces.

Throughout this paper, for $\operatorname{int}(\operatorname{cl}(A))$ where $A \subset X$ (where X is a topological space), we shall use the notation $\dot{\overline{A}}$.

A quasi-uniformity on a set X is a filter $\mathcal{U}$ on $X \times X$ such that (a) each member of $\mathcal{U}$ contains the diagonal of $X \times X$ and (b) if $U \in \mathcal{U}$, then $V_\circ V \subset U$ for some $V \in \mathcal{U}$. The pair $(X,\mathcal{U})$ is called a quasi-uniform space. $\mathcal{U}$ generates a topology $\tau(\mathcal{U})$ containing all subsets G of X such that for each $x \in G$, there exists $U \in \mathcal{U}$ such that $U[x] \subset G$.

Definition 1.1. [**2**] A topological space (X, τ) is said to be almost quiet quasi-uniformizable iff there exists a compatible quasi-uniformity $\mathcal{U}$ with the following properties: for $U \in \mathcal{U}$ and $x \in X$, there exists $V_x \in \mathcal{U}$ for which the following conditions hold: if $\{x_\alpha : \alpha \in A\}$ & $\{y_\beta : \beta \in B\}$ be two nets such that $(x, x_\alpha) \in V_x$ for $\alpha \in A$, $(y_\beta, y) \in V_x$ (for some $y \in X$), for $\beta \in B$, and $(y_\beta, x_\alpha) \to 0$ (i.e., for any $V \in \mathcal{U}$, $\exists\ \beta_V$ & α_V belonging to B and A respectively such that $(y_\beta, x_\alpha) \in V$ for $\beta \geq \beta_V$ & $\alpha \geq \alpha_V$), then $y \in \dot{\overline{U[x]}}$, where the closure and the interior of $U[x]$

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and $\overline{U[x]}$ respectively are taken under the topology τ; we call V_x subordinated to U with respect to x.

Definition 1.2. [**6**] A topological space (X, τ) is almost regular if for every point $x \in X$ and each neighbourhood M of x, there exists an open set U such that $x \in U \subset \overline{U} \subset \dot{\overline{M}}$, where $\overline{M} = \mathrm{cl}\,(M)$ and $\dot{\overline{M}} = \mathrm{int}(\mathrm{cl}\,M)$.

Definition 1.3. [**5**] Let X be a topological space. A subset $S \subset X$ is said to be regular open (respectively, regular closed) if $\mathrm{int}\,(\mathrm{cl}\,S) = S$ (respectively, $\mathrm{cl}\,(\mathrm{int}\,S) = S$). A point $x \in S$ is said to be a δ-cluster point of S if $S \cap U \neq \emptyset$, for every regular open set U containing x. The set of all δ-cluster points of S is called the δ-closure of S and is denoted by $[S]_\delta$. If $[S]_\delta = S$, then S is said to be δ-closed. The complement of a δ-closed set is called a δ-open set.

For every topological space (X, τ), the collection of all δ-open sets forms a topology for X, which is weaker than τ. This topology τ^* has a base consisting of all regular open sets in (X, τ).

Definition 1.4. [**5**] A function $f : X \to Y$ is said to be δ-continuous at a point $x \in X$, if for every regular open neighbourhood V of $f(x)$ in Y, $\exists$ a δ-open neighbourhood U of x such that $f(U) \subseteq V$.

The collection of all δ-continuous functions from X to Y is denoted by $D(X, Y)$.

Definition 1.5. [**2**] Let $\mathcal{F}$ be a family of functions from a topological space X to a quasi-uniform space $(Y, \mathcal{U})$. Then $\mathcal{F}$ is called δ-equicontinuous at $x \in X$, if for $V \in \mathcal{U}$, there exists a regular open neighbourhood N of x such that $f(N) \subset \dot{\overline{V[f(x)]}}$, for every $f \in F$.

Definition 1.6. [**7**] A set $A \subset (X, \tau)$ is said to be N-closed in X or simply N-closed, if for any cover of A by τ-open sets, there exists a finite subcollection the interiors of the closures of which cover A; interiors and closures are of course w.r.t. τ.

A set (X, τ) is said to be nearly compact iff it is N-closed in X.

Definition 1.7. [**3**] The N-R topology on Y^X denoted by $N_{\Re}$ is generated by the sets of the form $\{T(C, U) : C \text{ is N-closed in } X \text{ and } U \text{ is regular open in } Y\}$, where $T(C, U) = \{f \in Y^X : f(C) \subseteq U\}$.

Theorem 1.8. [**3**] *Let $Z \subset Y^X$ be endowed with the N-R topology $N_{\Re}$. Then $T(x, U)$ is δ-open in $(Z, N_{\Re})$, where U is regular open in Y and Y is almost regular.*

Definition 1.9. [**3**] Let $Z \subset Y^X$; if τ is such a topology on Z such that $P : Z \times X \to Y : (f, x) \to f(x)$ is δ-continuous, then we say that τ is δ-admissible.

For a topological space X and a quasi-uniform space $(Y, \mathcal{U})$, the quasi-uniformity Q of quasi-uniform convergence on Y^X is defined by the collection $\{L_V : V \in \mathcal{U}\}$ where $L_V = \{(f, g) \in Y^X \times Y^X : (f(x), g(x)) \in V, \text{ for each } x \in X\}$; the topology $\tau(Q)$ generated by Q is called the topology of quasi-uniform convergence. The basic $\tau(Q)$ neighbourhood of an arbitrary $f \in Y^X$ is of the form $L_V[f] = \{g \in Y^X : (f, g) \in L_V\}$.

Another quasi-uniformity on Y^X can be constructed by considering quasi-uniform convergence on each member of a family $\wp$ of subsets of the domain space. Explicitly, if F is a family of functions on a set X to a quasi-uniform space $(Y,\mathcal{U})$ and $\wp$ is a family of subsets of X, then the quasi-uniformity of quasi-uniform convergence on members of $\wp$ abbreviated as $\mathcal{U}|_\wp$ has for a subbase, the family of all sets of the form $\{(f,g) : (f(x),g(x)) \in V$ for all $x \in A; V \in \mathcal{U}, A \in \wp\}$. We denote it by L_V^A.

Lemma 1.10. [**3**] If $\mathcal{F} \subset Y^X$ be endowed with a topology $\wp$ where the subbase for $\wp$ is $\{T(x,U) : x \in X, U$ is regular open in $Y\}$, then each $T(x,U)$ is δ-open in $\wp$ if Y is almost regular.

Note 1.11 ([**2**])**.** If $W \in \mathcal{U}$ is a regular open surrounding in a uniform space $(X,\mathcal{U})$ then $W[x]$ is a regular open subset of X.

2. Main Results

Proposition 2.1. [**3**] *Let X be a topological space and let $(Y,\mathcal{U})$ be an almost quiet quasi-uniform space. If $\mathcal{H}$ is a δ-equicontinuous collection of functions, then its closure $\overline{\mathcal{H}}^\wp$ relative to the topology $\wp$ is also δ-equicontinuous.*

Lemma 2.2. [**4**] *Let H be an N-closed subset of an almost quiet quasi-uniform space $(X,\mathcal{U})$. Then for some regular open set U of X, $\exists$ a surrounding $D \in \mathcal{U}$ such that $D[H] \subset U$.*

Theorem 2.3. [**5**] *The image of an N-closed set under a δ-continuous map is N-closed.*

Proposition 2.4. *Let X be a topological space and let $(Y,\mathcal{U})$ be an almost quiet quasi-uniform space. Then the topology of quasi-uniform convergence on N-closed sets coincides with the N-R topology on $D(X,Y)$.*

Proof. Let τ denotes the topology of quasi-uniform convergence on N-closed sets and σ denotes the N-R topology on $D(X,Y)$. Consider $T(K,U) \in \sigma$ and let $f \in T(K,U)$, then $f(K) \subset U$. Since $f(K)$ is N-closed and U is regular open in $(Y,\mathcal{U})$, by Lemma 2.2 there exists a surrounding $V \in \mathcal{U}$ such that $V[f(K)] \subset U$. Choose

$$L_V^K = \{(f,g) : (f(x),g(x)) \in V,\ \forall x \in K\}.$$

Then $L_V^K \in \mathcal{U}|_\infty$ (where ∞ is the collection of all N-closed sets in X). We show that for any $f \in T(K,U)$, $f \in L_V^K[f] \subset T(K,U)$ showing that $T(K,U) \in \tau$, i.e., $\sigma \subset \tau$: in fact, let $g \in L_V^K[f]$; then $(f,g) \in L_V^K$, i.e., $(f(x),g(x)) \in V$ for all x in K which implies that $g(x) \in V[f(x)]$ for all x in K, i.e., $g(K) \subset V[f(K)] \subset U$. Thus $g \in T(K,U)$.

Now, let $S \in \tau$ and let $f \in S$ where $f \in D(X,Y)$. Then there is a $L_V^K \in \mathcal{U}|_\infty$ $(K \in \infty)$ such that $f \in L_V^K[f] \subset S$. We show that $\bigcap_{i=1}^{n} T(K_i,U_i)$, for N-closed sets $K_i \subset X$; $i = 1,2,\ldots,n$ and regular open sets U_i, $i = 1,2,\ldots,n$ in Y contains f and is contained in $L_V^K[f]$. Choose a regular open symmetric $W \in \mathcal{U}$ such that $W \circ W \circ W \circ W \subset V$, $K \subset X$ being N-closed, $f(K)$ is N-closed in Y and $\{W[f(x)] : x \in K\}$ is a cover of $f(K)$ and has a finite subcover say,

$$\{W[f(x_i)] : i = 1, 2, \ldots, n\}, \qquad x_i \in K. \tag{1}$$

Obviously, $W[f(x_i)]$ are regular open neighbourhoods of $f(x_i)$, $i = 1, 2, \ldots, n$ (by Note 1.11); $f : X \to Y$ being δ-continuous, $f^{-1}[W[f(x_i)]]$, $i = 1, 2, \ldots, n$ are regular open neighbourhoods of x_i in X, $i = 1, 2, \ldots, n$. Choose, $K_i = K \cap f^{-1}[W[f(x_i)]]$. Then K_i's are N-closed in X.

Now,

$$W \subset W \circ W \circ W \quad \text{implies} \quad W \circ W \circ W \in \mathcal{U}.$$

Choose, $U_i = (W \circ W \circ W)[f(x_i)]$. We show that, for regular open W, $(W \circ W \circ W)[x]$ is regular open. Let $y \in \overline{(W \circ W \circ W)[x]}$ and we show that $\overline{W[x]} \times \overline{W[x]} \subset \overline{W}$. Let $(x, y) \notin \overline{(W \circ W \circ W)}$. Since $\overline{W} \subset \overline{(W \circ W \circ W)}$, $(x, y) \notin \overline{W}$. Then there exists neighbourhoods U_x and U_y of x and y respectively such that

$$(U_x \times U_y) \cap W = \phi.$$

If $t \in U_y$, then $(x, t) \notin W$ implies $t \notin W[x]$, i.e., $U_y \cap W[x] = \phi$, i.e., $y \notin \overline{W[x]}$. Hence, $(x, y) \notin \overline{W[x]} \times \overline{W[x]}$. Thus,

$$\overline{W[x]} \times \overline{W[x]} \subset \overline{W},$$

i.e.,

$$(x, y) \in \operatorname{int}(\overline{W}) = W \subset W \circ W \circ W,$$

i.e.,

$$y \in (W \circ W \circ W)[x].$$

Therefore,

$$\overline{(W \circ W \circ W)[x]} \subset (W \circ W \circ W)[x].$$

Hence, $(W \circ W \circ W)[x]$ is regular open. Thus, U_i's are regular open in Y for $i = 1, 2, \ldots, n$. Let $g \in \bigcap_{i=1}^{n} T(K_i, U_i)$, let $x \in K$. Then $f(x) \in W[f(x_i)]$ for some $i : 1 \le i \le n$ by (1), i.e., $x \in f^{-1}[W[f(x_i)]]$, i.e., $x \in K_i$.

Now

$$\begin{aligned} g \in T(K_i, U_i) \quad &\Rightarrow \quad g(K_i) \subset U_i \Rightarrow g(x) \in U_i \\ &\Rightarrow \quad g(x) \in (W \circ W \circ W)[f(x_i)] \\ &\Rightarrow \quad (f(x_i), g(x)) \in W \circ W \circ W. \end{aligned} \tag{2}$$

Also,

$$f(x) \in W[f(x_i)] \;\Rightarrow\; (f(x_i), f(x)) \in W. \tag{3}$$

By (2) and (3),

$$(f(x), g(x)) \in W \circ W \circ W \circ W \subset V.$$

Since x is any point of K, $(f(x), g(x)) \in V$ for all

$$x \in K \Rightarrow (f,g) \in L_V^K \Rightarrow g \in L_V^K[f] \Rightarrow \bigcap_{i=1}^{n} T(K_i, U_i) \subset L_V^K[f].$$

We now show that $f \in T(K_i, U_i)$ for each $i = 1, 2, \ldots, n$, i.e., $f(K_i) \subset U_i$ for each $i = 1, 2, \ldots, n$.

Now, $f(K_i) \subset W[f(x_i)]$, $i = 1, 2, \ldots, n$ implies

$$f(K_i) \subset (W \circ W \circ W)[f(x_i)] = U_i, \qquad i = 1, 2, \ldots, n.$$

Hence

$$f \in T(K_i, U_i), \qquad \text{for } i = 1, 2, \ldots, n.$$

Thus the proposition is proved. □

Lemma 2.5. *Each jointly δ-continuous topology on N-closed sets is larger than the N-R topology.*

Proof. Suppose that a topology τ for $Z \subset Y^X$ is jointly δ-continuous on N-closed sets, U is a regular open subset of Y, K is an N-closed subset of X and P is the map such that $P(f,x) = f(x)$. It must be shown that $T(K,U)$ is open to show that $\tau \supset N_{\mathfrak{R}}$.

The set $V = (Z \times K) \cap P^{-1}(U)$ is regular open in $Z \times K$ because $P|_{Z\times K}$ is δ-continuous for any N-closed $K \subset X$. If $f \in T(K,U)$, then

$$f(K) \subset U, \quad \text{i.e.,} \quad \{f\} \times K \subset P^{-1}(U) \quad \text{i.e.,} \quad \{f\} \times K \subset V.$$

Now $\{f\}$ is N-closed in Z and K is so in X. Cover $\{f\} \times K$ by basis elements $U \times W$ lying in V. The space $\{f\} \times K$ is N-closed, since it is δ-homeomorphic to K. Therefore we can choose finitely many U_i, W_i, $i = 1, 2, \ldots, n$ such that

$$\{f\} \times K \subset \dot{\overline{U_i}} \times \dot{\overline{W_i}}.$$

Then $\operatorname{int}(\operatorname{cl}(U_i))$, $i = 1, 2, \ldots, n$ are open sets. Let $N = \bigcap_{i=1}^{n} \dot{\overline{U_i}}$. Thus N is open and contains f. We assert that the sets $\dot{\overline{U_i}} \times \dot{\overline{W_i}}$, which were chosen to cover $\{f\} \times K$ actually cover $N \times K$. Let $(g,y) \in N \times K$. Consider $(f,y) \in \{f\} \times K$. Then $(f,y) \in \dot{\overline{U_i}} \times \dot{\overline{W_i}}$ for some i, i.e., $y \in \dot{\overline{W_i}}$. Because $g \in N$, $g \in \dot{\overline{U_i}}$, for each $i = 1, 2, \ldots, n$. Therefore, $(g,y) \in \dot{\overline{U_i}} \times \dot{\overline{W_i}}$. Since all the sets $\dot{\overline{U_i}} \times \dot{\overline{W_i}}$ lie in V and cover $N \times K$, $N \times K \subset V$. Hence there exists a τ-neighbourhood N of f such that $N \times K \subset P^{-1}(U)$. For each $f \in N$, $f(K) \subset U$, i.e., $N \subset T(K,U)$. Thus

$$f \in N \subset T(K,U)$$

gives $T(K,U)$ is open in τ and hence $\tau \supset N_{\mathfrak{R}}$. □

Proposition 2.6. *Let X be a topological space and $(Y, \mathcal{U})$ be an almost quiet quasi-uniform space. If F is a δ-equicontinuous collection of functions, then the N-R topology coincides with the topology $\wp$.*

Proof. We consider $P : F \times X \to Y : (f,x) \to f(x)$. We show that if F has the topology $\wp$, then P is δ-continuous. Let $W \in \mathcal{U}$ be regular open. Choose $V \in \mathcal{U}$ such that $V \circ V \subset U$. Consider the set

$$T = \{h : h(x) \in V[f(x)]\}. \tag{4}$$

By Lemma 1.10 T is a neighbourhood of f in $(F, \wp)$. F being δ-equicontinuous, there exists a regular open neighbourhood U of x such that

$$f^*(U) \subset V[f^*(x)] \qquad \text{for all } f^* \in F. \tag{5}$$

Consider the neighbourhood $T \times U$ of (f,x) and let $(g,y) \in T \times U$. Then $g(x) \in V[f(x)]$ by (4) and $g(y) \in V[g(x)]$ by (5). Hence, $(f(x), g(x)) \in V$ and $(g(x), g(y)) \in V$ giving $(f(x), g(y)) \in V \circ V \subset W$, i.e., $g(y) \in W[f(x)]$, i.e., $P(g,y) \in W[f(x)]$, i.e.,

$$P(T \times U) \subset W[f(x)].$$

Hence P is δ-continuous and thus joint δ-continuity of $\wp$ follows. Now each jointly δ-continuous topology is larger than the N-R topology and the N-R topology coincides with the topology of quasi-uniform convergence on N-closed sets since $F \subset D(X,Y)$.

Now we show that $\tau_\wp \subset N_{\Re}$. For each $x \in X$, $\{x\}$ is N-closed in X and thus

$$\begin{aligned}&\{T(x,U) : x \in X, U \text{ is regular open in } Y\}\\ &\qquad \subset \{T(C,U) : C \text{ is N-closed in } X \text{ and } U \text{ is regular open in } Y\}\end{aligned}$$

and thus $\tau_\wp \subset N_{\Re}$ in $Z \subset Y^X$. Thus we can conclude that if F is a δ-equicontinuous collection of functions, then the N-R topology coincides with the topology $\wp$. $\square$

3. Ascoli's theorem in almost quiet quasi-uniform space

In this section we generalize Ascoli's theorem in almost quiet quasi-uniform space.

Theorem 3.1. *Let X be a nearly compact topological space and $(Y,\mathcal{U})$ be an almost quiet quasi-uniform T_2 space. Let τ_N denote the topology of quasi-uniform convergence on N-closed sets. Then a subset $H \subset D(X,Y)$ is τ_N-compact iff*

(a) *H is τ_N-closed.*

(b) *$\overline{\Pi_x(H)}$ is compact for each $x \in X$ and*

(c) *H is δ-equicontinuous.*

Proof. Since H is δ-equicontinuous by Proposition 2.1, its $\tau_\wp$ closure $\overline{H}^\wp$ is also δ-equicontinuous. But $\overline{H}^\wp$ is a $\tau_\wp$-closed subset of the $\tau_\wp$-compact product set $\Pi\{\overline{\Pi_x(H)} : x \in X\}$ and thus $\overline{H}^\wp$ is itself $\tau_\wp$-compact. Using Proposition 2.4 and Proposition 2.6 above we conclude that $\overline{H}^\wp$ is τ_N-compact. Now, the τ_N-closed subset H of the τ_N-compact subset $\overline{H}^\wp$ is also τ_N-compact. Hence H is τ_N-compact.

Conversely, let $H \subset D(X,Y)$ be τ_N-compact. Since Y is T_2, we first show that $(D(X,Y), \tau_N)$ is also so. Let $f, g \in D(X,Y)$ be such that $f \neq g$. Then $\exists\, x \in X$ such that $f(x) \neq g(x)$. Since Y is T_2, there exists disjoint open neighbourhoods

U and V such that $f(x) \in U$, $g(x) \in V$. Hence, $f(x) \in U = \operatorname{int} U \subset \operatorname{int}(\overline{U}) = \dot{\overline{U}}$. Now,

$$U \cap V = \phi \Rightarrow \overline{U} \cap V = \phi \Rightarrow \dot{\overline{U}} \cap V = \phi \Rightarrow V \subseteq Y \setminus \dot{\overline{U}},$$

i.e.,

$$V = \operatorname{int} V \subseteq \operatorname{int}(Y \setminus \dot{\overline{U}}) = M.$$

Then M is regular open and $g(x) \in M$ with $\dot{\overline{U}} \cap M = \phi$. Now, $\{x\}$ is N-closed in X and $f \in L^{\{x\}}_{\dot{\overline{U}}}$, $g \in L^{\{x\}}_M$ with $L^{\{x\}}_{\dot{\overline{U}}} \cap L^{\{x\}}_M = \phi$. Hence $(D(X,Y), \tau_N)$ is T_2. If H is τ_N-compact, then H is τ_N-closed and $\Pi_x(H)$ is compact for each $x \in X$ and hence closed in Y. Thus $\overline{\Pi_x(H)}$ is compact in Y for each $x \in X$.

Now if $Z \subset Y^X$ and $P \subset X$, then $Z|_P = \{f|_P : f \in Z\}$. Let $\mathcal{C}$ denote the collection of all N-closed sets in X and let $P \in \mathcal{C}$. We show that $H|_P$ is δ-equicontinuous on P. Let $x_0 \in P$ and $W \in \mathcal{U}$ be regular open. Choose regular open symmetric $V \in \mathcal{U}$ such that $V \circ V \circ V \subset W$. Then $\{L^P_V[f] : f \in H\}$ is a cover of H by neighbourhoods of members of H in the topology of quasi-uniform convergence on N-closed sets and by the given condition of τ_N-compactness, there exists f_i, $i = 1, 2, \ldots, n$ (belonging to H) such that $H \subset \bigcup_{i=1}^{n} L^P_V[f_i]$. Let $f \in H$. Then

$$f \in L^P_V[f_i] \qquad \text{for some } i. \tag{6}$$

Since each $f_i|_P$ is δ-continuous at x_0, there is a regular open neighbourhood U_i of x_0 in P such that $f_i|_P(U_i) \subset V[f_i(x_0)]$, i.e.

$$x \in U_i \Rightarrow (f_i(x_0), f_i(x)) \in V. \tag{7}$$

Let $U = \bigcap_{i=1}^{n} U_i$. Obviously U is a regular open neighbourhood of x_0 in P. We show that $f|_P(U) \subset W(f(x_0))$ for all $f \in H$. Let $f \in H$. By (6), $f \in L^P_V[f_i]$ for some i, i.e., $(f_i(x), f(x)) \in V$, for all $x \in P$ and hence

$$(f_i(x_0), f(x_0)) \in V \quad [\text{since } x_0 \in P]. \tag{8}$$

Again

$$x \in U \Rightarrow x \in P \Rightarrow (f_i(x), f(x)) \in V. \tag{9}$$

From (7), (8) and (9) we get, $x \in U \Rightarrow (f(x_0), f(x)) \in V \circ V \circ V \subset W$ for each $f \in H \Rightarrow f(x) \in W[f(x_0)]$ for all $f \in H$, i.e.,

$$f(U) \subset W[f(x_0)] \qquad \text{for all } f \in H,$$

i.e.,

$$f|_P(U) \subset W[f(x_0)] \qquad \text{for each } f \in H.$$

Since X is nearly compact, X is N-closed in X. Hence $f(U) \subset W[f(x_0)]$ for all $f \in H$, i.e., H is δ-equicontinuous. □

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