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WEAK EQUIVALENCE CLASSES OF COMPLEX VECTOR BUNDLES

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Abstract. For any complex vector bundle E^k of rank k over a manifold M^m with Chern classes $c_i \in H^{2i}(M^m, \mathbb{Z})$ and any non-negative integers $l_1, \cdots, l_k$ we show the existence of a positive number $p(m, k)$ and the existence of a complex vector bundle $\hat{E}^k$ over M^m whose Chern classes are $p(m, k) \cdot l_i \cdot c_i \in H^{2i}(M^m, \mathbb{Z})$. We also discuss a version of this statement for holomorphic vector bundles over projective algebraic manifolds.

1. Introduction.

The study of complex vector bundles of rank k over a manifold M^m can be reduced to the study of mappings from M^m to the classifying space $Gr_k(\mathbb{C}^\infty) = BU_k$. Certain equivalence relations of complex vector bundles lead us to study *stable mappings* of BU_k to itself.

We call *a map* $g : BU_k \to BU_k$ *stable*, if the restriction of g to any subspace $Gr_k(\mathbb{C}^N)$ sends $Gr_k(\mathbb{C}^N)$ to some Grassmannian $Gr_k(\mathbb{C}^{f(N)})$, moreover $g^*(c_k) = \lambda \cdot c_k$ for some positive λ. Here c_k denotes the top Chern class of the universal bundle over BU_k.

Two maps $f_1, f_2 : M^m \to BU_k$ are said to be in one *stable equivalence class*, if $f_1 = g \circ f_2$ for a stable map $g : BU_k \to BU_k$. Two complex vector bundles E_1^k and E_2^k are said to be in the same *weak equivalence class*, if the corresponding homotopy classes of classifying maps contain maps in the same stable equivalence class. Two complex vector bundles E_1^k and E_2^k are called *Chern weakly equivalent*, if their top Chern classes are related by a positive scalar factor. Clearly vector bundles in the same weak equivalence class are Chern weakly equivalent. Zero sections of Chern weakly equivalent vector bundles realize the same homology classes up to a positive constant.

Theorem 1.1. *For any complex vector bundle E^k of rank k over a manifold M^m with the Chern classes $c_i \in H^{2i}(M^m, \mathbb{Z})$ and any non-negative integers $l_1, \cdots, l_k$, $l_k > 0$, there exists a vector bundle $\hat{E}^k$ in the same weak equivalence*

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class with E^k, and a positive number $p(m,k)$ such that the Chern classes $c_i(\hat{E}^k)$ are $p(m,k) \cdot l_i \cdot c_i \in H^{2i}(M, \mathbb{Z})$.

As a corollary of Theorem 1.1 and Thom's theorem [**9**, Theorem II.25] (a detailed proof of this theorem is given in [**6**] and in the Appendix below) we get

Corollary 1.2. [**5**, Proposition 2.7] *Suppose that M^m is an orientable differentiable manifold. For any $c \in H^{2k}(M^m, \mathbb{Z})$ there exists a number $N > 0$ such that there exists a complex vector bundle E^k of rank k over M whose a top Chern class is $N \cdot c$ and all other lower Chern classes are zero.*

We can think of Theorem 1.1 together with the Thom theorem as a version of the Atiyah-Hirzebruch theorem about isomorphism between the two rings $K(M^m) \otimes \mathbb{Q}$ and $H^{\mathrm{even}}(M^m, \mathbb{Q})$ via the Chern character [**1**] which implies that giving an element of $K(M^m) \otimes \mathbb{Q}$ is the same as giving an element in $H^{\mathrm{even}}(M^m, \mathbb{Q})$. Our Theorem 1.1 concerns vector bundles with a given dimension on M^m. I did not give enough details to the proof of Proposition 2.7 in [**5**], so now the proof of Theorem 1.1 should compensate that deficit.

For $i \le k$ there is a projection map p from BU_i to BU_k with the fiber $U(k)/U(i)$ such that $p^*(c_i) = c_i$. Here c_i also denotes the i-th Chern class of the universal bundle over BU_k. Another consequence of a proof of Theorem 1.1 is

Corollary 1.3. *There are maps $g^m_{k,i} : Gr_k(\mathbb{C}^m) \to Gr_i(\mathbb{C}^l) \to BU_i \to BU_k$ such that $g^N_{k,i}(c_j) = p(k, \binom{m}{k}) \cdot \delta^i_j \cdot c_j$ for $1 \le i, j \le k$.*

In the third section of this note we discuss the problem of extending the notion of weak equivalence to the category of holomorphic vector bundles over projective algebraic manifolds.

Two holomorphic vector bundles E^k and F^k over a complex manifold M^m are said to be *Kähler weakly equivalent*, if there are two holomorphic line bundles L_1 and L_2 over M^m such that $E^k \otimes L_1$ and $F^k \otimes L_2$ are Chern weakly equivalent.

It is well-known that the Hodge conjecture is equivalent to the statement that the Hodge group $H^{p,p}(M, \mathbb{Q}) := H^{2p}(M, \mathbb{Q}) \cap H^{p,p}(M, \mathbb{C})$ is generated by the top Chern classes of a holomorphic vector bundle of rank p on a projective algebraic manifold M^n (see e.g. [**10**]). A motivation for the notion of Kähler weakly equivalence is Lemma 3.1 below which states that any holomorphic vector bundle on a projective algebraic manifold is Kähler weak equivalent to a holomorphic vector bundle such that the homotopy class of its classifying maps contains a holomorphic map. With this on hand, we speculate about a reduction of the Hodge conjecture to the existence of certain holomorphic maps which may be obtained by using on the one hand Zucker and Saito results for the existence of normal functions associated to primitive Hodge cocycles in middle dimensions and on the other hand Siu's technique for harmonic maps.

For the convenience of the reader in this note I include an appendix which re-exposes the detailed proof of Thom's theorem in [**6**], which now has a simpler form, since this proof is very close to our proof of Theorem 1.1.

2. Proof of Theorem 1.1.

Proof of Theorem 1.1. Denote by γ^k the universal bundle over the Grassmannian $Gr_k(\mathbb{C}^N)$ (we assume that $N=\infty$ or N is sufficiently large as it will be specified later). Since E^k is the pull-back of γ via a classifying map $f: M^m \to Gr_k(\mathbb{C}^N)$, it suffices to prove Theorem 1.1 for the case $M = Gr_k(\mathbb{C}^N)$, $E^k = \gamma^k$ and N is sufficiently large, and after that we use the classifying map f to take back the obtained bundle to M^m.

Let us denote by $K(\mathbb{Z}, n)$ the Eilenberg-McLane space and by τ^n the fundamental class of $K(\mathbb{Z}, n)$. Let $f_k^N : Gr_k(\mathbb{C}^N) \to K(\mathbb{Z}, 2k)$ be a classifying map for $c_k(\gamma) \in H^{2k}(M, \mathbb{Z})$, i.e. $(f_k^N)^*(\tau^{2k}) = c_k(\gamma) \in H^{2i}(M, \mathbb{Z})$.

Let F_N^n be a map from $K(\mathbb{Z}, n) \to K(\mathbb{Z}, n)$ such that $F_N^n(\tau^n) = N\tau^n$. The existence of a map F_N^n is ensured by the fact that $K(\mathbb{Z}, n)$ is the classifying space for $(H^n, \mathbb{Z})$. Clearly F_N^n is defined uniquely up to homotopy.

Lemma 2.1. [**9**, Lemma II.22] *For any finite abelian group G of order N the endomorphism $(F_N^n)^* : H^*(K(\mathbb{Z}, n), G)$ is trivial.*

Lemma 2.1 follows directly from Cartan's result which states that the algebra $H^*(K(\mathbb{Z}, n), \mathbb{Z}_p)$ is generated by iteration of the Steenrod squares of τ^n.

Denote by Y^q the q-skeleton of a CW-complex Y. Clearly $\pi_k(K^q(\mathbb{Z}, n)) = \pi_k(K(\mathbb{Z}, n))$ for any $k < q$.

Proposition 2.2. *Suppose that Y is a simplicial space whose q-skeleton Y^q is compact for each q. Let the free component of $\pi_k(Y)$ be isomorphic to $\mathbb{Z}$ with a generator t and let Q be an integer such that $Q \ge k$. If for all $Q \ge q \ge k$ the group $H^{q+1}(K(\mathbb{Z}, k), \pi_q(Y))$ is finite, then there exists a map $G_Q : K^Q(\mathbb{Z}, k) \to Y$ such that $(G_Q)_*(\pi_k(K^Q(\mathbb{Z}, k))) = \langle N(Q, k)t\rangle_{\otimes\mathbb{Z}} \subset \pi_k(Y)$.*

Proposition 2.2 is a reformulation of Lemma II.24 in [**9**], where Thom did not introduce the parameter Q explicitly. We quickly recall his argument, adapted to this new reformulation. We prove Proposition 2.2 by induction on the dimension $Q \ge k$. Clearly Proposition 2.2 for $Q = k$ is trivial, since $K^k(\mathbb{Z}, k) = S^k$.

Suppose that we have constructed a map G_Q for $Q \ge k$.

Now we put

$$G_Q^1 = F_N^k \circ G_Q,$$

where F_N^k is the map in Lemma 2.1. By theorem of simplicial approximation we can assume that F_N^k sends $K^q(\mathbb{Z}, k)$ to $K^q(\mathbb{Z}, k)$ for each $Q \ge q \ge k$.

Since the obstruction to an extension of G_Q^1 to $K^{Q+1}(\mathbb{Z}, k)$ lies in the group $F_N^*(H^{q+1}(K(\mathbb{Z}, k), \pi_q(Y))$ which is trivial by Lemma 2.1, we shall put G_{Q+1} as an extension of G_q^1 to $K^{Q+1}(\mathbb{Z}, k)$. This completes the induction step for the proof of Proposition 2.2.

Lemma 2.3. *Suppose that $N \ge 2l + 1$. Then the space $Gr_l(\mathbb{C}^N)$ satisfies the condition for Y with $Q = N$ and for all $k = 2r$, if $1 \le r \le l$, in Proposition 2.2.*

Proof. To prove Lemma 2.3 it suffices to verify the following three identities

$$\pi_{2r}(Gr_l(\mathbb{C}^N)) \otimes \mathbb{Q} = \mathbb{Q}, \qquad \text{for all } 1 \le r \le l \tag{2.3.1}$$

$$\pi_q(Gr_l(\mathbb{C}^N)) \otimes \mathbb{Q} = 0, \qquad \text{for all other } q \le N \tag{2.3.2}$$

$$H^{q+1}(K(\mathbb{Z}, 2l), \pi_q(Gr_l(\mathbb{C}^N)) \otimes \mathbb{Q} = 0, \qquad \forall q. \tag{2.3.3}$$

To prove (2.3.1) we consider the following exact sequence

$$\pi_q(U_l \times U_{N-l}) \to \pi_q(U_N) \to \pi_q(Gr_l(\mathbb{C}^N)) \to \pi_{q-1}(U_l \times U_{N-l}) \tag{2.3.4}$$

which also remains exact after tensoring with $\mathbb{Q}$. To save the notation we shall consider this exact sequence as of that of rational homotopy groups.

For $2 \le q = 2r \le 2l$ the exact sequence (2.3.4) implies the equality (2.3.1), since $\pi_{2r}(U_N) \otimes \mathbb{Q} = 0$, $\pi_{2r-1}(U_m) \times \mathbb{Q} = \mathbb{Q}$ for $r \le m$, and taking into account the fact that the kernel of the map

$$i : \mathbb{Q} \oplus \mathbb{Q} = \pi_{2r-1}(U_l \times U_{N-l}) \otimes \mathbb{Q} \to \pi_{2r-1}(U_N) \times \mathbb{Q} = \mathbb{Q}$$

is equal to $\mathbb{Q}$.

To prove (2.3.2) we have to consider several cases for q. First let us consider the exact sequence (2.3.4) for $2l + 1 \le q \le N$. We know [**8**, 9.7], that $\pi_q(U_l \otimes U_{N-l}) \otimes \mathbb{Q} = \pi_q(U_{N-l}) \otimes Q$ and taking into account the fact that the map

$$i : \mathbb{Q} = \pi_q(U_l \times U_{N-l}) \otimes \mathbb{Q} \to \pi_q(U_N) \otimes \mathbb{Q}$$

is isomorphism. Taking into account the fact that $\pi_q(U_{N-l}) \otimes Q$ vanishes if q is even, we get

$$\pi_q(Gr_l(\mathbb{C}^N)) = \ker(\pi_{q-1}(U_l \times U_N) \to \pi_{q-1}(U_N)) = 0$$

which implies (2.3.2) for $2l + 1 \le q \le N$.

Finally to verify (2.3.2) for q odd and less than $2l$ we notice that the map $\pi_q(U_l \times U_{N-l}) \to \pi_q(U_N)$ is surjective, hence $\pi_q(Gr_l(\mathbb{C}^N)) = \pi_{q-1}(U_l \times U_{N-l}) = 0$.

The last statement (2.3.3) follows from (2.3.1) for $q = 2l$, and it follows from (2.3.2) for all others and taking into account the fact that $H^*(K(\mathbb{Z}, 2l), \mathbb{Q}) = \mathbb{Q}[x]$, $\dim x = 2l$. The last fact is obtained by Serre and Cartan (see e.g.[**3**, 3.25] for an exposition. In fact this computation of $H^*(K(\mathbb{Z}, 2l), \mathbb{Q})$ can be easily obtained by using induction method and by using the cohomology spectral sequence associated with the fibration $K(\mathbb{Z}, n-1) \cong \Omega K(\mathbb{Z}, n) \to K(\mathbb{Z}, n)$, whose fiber is contractible.) □

Continuation of the proof of Theorem 1.1. For $N \ge 2k+1$ and for all $2 \le i \le k$ Proposition 2.2 and Lemma 2.3 give us a map

$$G^N_{k,i} : K^N(\mathbb{Z}, 2i) \to Gr_k(\mathbb{C}^N)$$

such that $(G^N_{k,i})_*(w_i) = \alpha(N, k, i)t_i$, where w_i is a generator of $\pi_{2k}(K^N(\mathbb{Z}, 2i)) = \mathbb{Z}$ and t_i is a generator of $\pi_i(Gr_k(\mathbb{C}^N)) \otimes \mathbb{Q}$. Since $H^*(Gr_k(\mathbb{C}^N), \mathbb{Z})$ is generated by $c_i(\gamma), i = \overline{1, k}$, [**2**], we have

$$\langle c_i, t_i \rangle = A_i \ne 0 \tag{2.4}$$

because t_i is the generator of the free part of $\pi_{2i}(Gr_k(\mathbb{C}^N))$. (To show that $A_i \neq 0$ we consider the exact sequence (2.3.4). We see easily that the image of $\rho(t_i)$ via embedding $G_k(\mathbb{C}^N) \to G_k(\mathbb{C}^\infty)$ is also a generator of $\pi_i(Gr_k(\mathbb{C}^\infty))$. Applying the $\mathcal{C}$-version of the Whitehead theorem [**7**, Theorem III.3] to BU_k and the product $K(\mathbb{Z},2) \times \cdots \times K(\mathbb{Z},2k)$ we notice that

$$\langle c_i, i(t_i)\rangle = A_i \neq 0$$

which must also hold on $G_k(\mathbb{C}^N)$ after pulling back.

Thus

$$(G^N_{k,i})^*(c_i) = \alpha(N,k,i) \cdot A_i \cdot \tau^{2i}. \tag{2.5}$$

We can assume that A_i is positive by choosing appropriate orientation of the generator t_i.

Completion of the proof of Theorem 1.1. Denote by $\lambda^N_{k,i}$ the classifying map from $G_k(\mathbb{C}^N)$ to $K(\mathbb{Z},2i)$ for $c_i \in H^{2i}(G_l(\mathbb{C}^N),\mathbb{Z})$, i.e.

$$\lambda^N_{k,i}(\tau^{2i}) = c_i.$$

We can assume that $\lambda^N_{k,i}(G_k(\mathbb{C}^N)) \subset K^{2k(N-k)}(\mathbb{Z},2i)$. For each i denote by $s(2i)$ the smallest positive number such that for any $j \leq i-1$ and any $c \in H^{2j}(K(\mathbb{Z},2i),\mathbb{Z})$ we have $s(2i)\cdot z = 0$. By the theorem of Serre and Cartan mentioned above (see [**3**, 3.25]) there exists such a number $s(2i)$ for all i. Let $p(N,k)$ be the smallest integer, such that for all $1 \leq i \leq k$ we have

$$p(N,k) = \alpha(N,k,i)\cdot A_i \cdot s(2i) \cdot \beta(N,k,i)$$

for a positive integer $\beta(N,k,i)$. We shall construct a map $T : Gr_k(\mathbb{C}^N) \to Gr_k(\mathbb{C}^{2k(N-k)})$ such that

$$T^*(c_k) = p(N,k)\cdot l_k \cdot c_k. \tag{2.6}$$

Then, taking into account of the functoriality of the Chern classes, the bundle $\hat{E}^k$ defined by $\hat{E}^k = (f\circ T)^*\gamma_k$ satisfies the condition of Theorem 1.1. Our map T is the composition of

$$(f^N_{k,1}, f^N_{k,2}, \cdots, f^N_{k,k})$$

where

$$f^N_{k,i} = G^N_{k,i} \circ \overline{F^{2i}_{l_i\cdot\beta(N,k,i)}} \circ \lambda^N_{k,i},$$

where $\overline{F^{2i}_{l_i\cdot s(2i)\cdot\beta(N,k,i)}}$ denotes the restriction of the map $F^{2i}_{l_i\cdot s(2i)\cdot\beta(N,k,i)}$ to $K^{2k(N-k)}(\mathbb{Z},2i)$ (see Lemma 2.1). Because of our choice of $s(2i)$ and taking into account of Lemma 2.1, the map T satisfies the condition (2.6). $\square$

3. Kähler weak equivalence

In this section we discuss some problems which arise from extending the results in the previous section to the category of holomorphic bundles over complex or projective algebraic manifolds.

We would like to show another necessity for the notion of Kähler weak equivalence notion. Let E^k be a complex vector bundle over a complex manifold M^n and $[f_{E^k}]$ be the homotopy class of a classifying map $M \to Gr_k(\mathbb{C}^N)$ for E^k. Clearly if $[f_{E^k}]$ contains a holomorphic map, then E^k has a holomorphic structure. But the converse statement is not true, because the pull back of of any positive $(1,1)$-cohomology classes via a holomorphic map is also a non-negative $(1,1)$-cohomology class. On the other hand there are many holomorphic vector bundles whose first Chern class is a negative $(1,1)$-class. We shall say that a holomorphic vector bundle is *positive*, if its classifying class contains a holomorphic map.

Lemma 3.1. *Suppose that M^m is a projective algebraic manifold and E^k is a holomorphic vector bundle over M^m. Then E^k is Kähler weakly equivalent to a positive holomorphic vector bundle.*

Proof. This Lemma is a consequence of a well-known fact (see e.g. [**4**, Chapter 1, Section 5]) that a tensor of E^k with the power $L^{\otimes l}$ of a Kähler line bundle L admits enough holomorphic sections which serve as a holomorphic map from M^m to $Gr_k(\mathbb{C}^N)$, where $\mathbb{C}^N$ is a subspace in $H^0(M^m, \mathcal{O}(E^k \otimes L^{\otimes l}))$. Furthermore, this holomorphic map is a classifying map for the holomorphic bundle $E^k \otimes L^{\otimes l}$, see e.g. [**4**, Chapter 3, Section 3]. □

We are lead by Lemma 3.1 to study the space $\mathrm{Hol}(M^m, BU_k)$ of holomorphic maps from $M^m \to BU_k$. Denote by Hodge(M) the group $H^{p,p}(M, \mathbb{Q})$ and by $[Hodge(M)]$ the quotient class Hodge (M)/$\mathbb{Q}$ by multiplication. This quotient space is provided with induced topology by embedding Hodge(M)/$\mathbb{Q}$ into the projecive space $H^*(M, \mathbb{C})/\mathbb{C}$. Then we define a map $C : \mathrm{Hol}(M^m, BU_k) \to [\mathrm{Hodge}^k(M)]$ by $C(f) = [f^*(c_k)]$. Then the Hodge conjecture in dimension p is true, if and only if the image of the map C contains some neighborhood of a point $[P] \in [\mathrm{Hodge}^k(M)]$ where P is the p-th power of a Kähler class. The problem in this naive thinking is that, C maps a connected component of $\mathrm{Hol}(M^m, BU_k)$ onto one point. It seems that we need to work every thing up (including the Hodge theory) from the beginning in the field of rationals. Another possible way to do it is mentioned in the introduction.

4. Appendix: A proof of a theorem of Thom.

Let M^n be an orientable differentiable manifold.

Theorem A.1 ([**9**, Theorem II.25)]).

a) *For each cohomology class $z \in H^k(M^n, \mathbb{Z})$ there exists a number $N(k,n)$ such that the class $N(k,n) \cdot z$ is the Euler class of an orientable vector bundle on M^n.*

b) *If* $k = 2l$, *then there exists a number* $N_1(k, n) \geq N(k, n)$ *such that the class* $N_1(k, n) \cdot z$ *is a top Chern class of a complex vector bundle on* M^n.

Thom gave a detailed proof of Theorem A.1.a. He noticed that his proof also works for the statement b. Since we use this statement in [**5**] as well as for our statement in the introduction on the relation with Atiyah-Bott theorem, we feel a need for a detailed proof of Thom's theorem A.1.b.

Proof of Theorem A.1.b. Suppose that $u \in H^{2k}(M^m, \mathbb{Z})$. Then there is a map

$$f : M^m \to K(\mathbb{Z}, 2k)$$

such that $f^*(\tau^{2k}) = u$, where τ^{2k} is the fundamental class of $H^k(K(\mathbb{Z}, 2k), \mathbb{Z})$. Moreover we can assume that $f(M^m) \subset K^m(\mathbb{Z}, 2k)$, where $K^q(\mathbb{Z}, 2k)$ is the q-skeleton of the Eilenberg-McLane space $K(\mathbb{Z}, 2k)$. To prove Theorem A.1 it suffices to find a map

$$h : K^m(\mathbb{Z}, 2k) \to BU_k$$

such that for a positive number $N_1(k, m)$ we have

$$h^*(c_k) = N_1(k, m) j^*(\tau^{2k}), \tag{4.1}$$

where c_k is the top Chern class of the universal bundle γ^k over BU_k and j is the embedding $K^m(\mathbb{Z}, 2k) \to K(\mathbb{Z}, 2k)$.

To find a map h we apply Proposition 2.2. The main issue is to verify that the space BU_k satisfies the condition for the space Y in Proposition 2.2. We use the same argument as that in our proof of Lemma 2.3, actually the case of BU_k is easier, since the related exact sequences are simpler. The required map h can be constructed in the same way as we did in our proof of Proposition 2.2. □

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