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ALMOST STABLE ITERATION SCHEMES FOR LOCAL STRONGLY PSEUDOCONTRACTIVE AND LOCAL STRONGLY ACCRETIVE OPERATORS IN REAL UNIFORMLY SMOOTH BANACH SPACES

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Abstract. In this paper we establish the strong convergence and almost stability of the Ishikawa iteration methods with errors for the iterative approximations of either fixed points of local strongly pseudocontractive operators or solutions of nonlinear operator equations with local strongly accretive type in real uniformly smooth Banach spaces. Our convergence results extend some known results in the literature.

1. Introduction

Let X be a real Banach space, X^* be its dual space and $\langle x, f\rangle$ be the generalized duality pairing between $x \in X$ and $f \in X^*$. The mapping $J : X \to 2^{X^*}$ defined by

$$J(x) = \{f \in X^* : \langle x, f\rangle = \|x\|\|f\|, \|f\| = \|x\|\}, \quad \forall\, x \in X,$$

is called the *normalized duality mapping*. In the sequel, we denote by I and $F(T)$ the identity mapping on X and the set of all fixed points of T, respectively.

Let T be an operator on X. Assume that $x_0 \in X$ and $x_{n+1} = f(T, x_n)$ defines an iteration scheme which produces a sequence $\{x_n\}_{n=0}^{\infty} \subset X$. Suppose, furthermore, that $\{x_n\}_{n=0}^{\infty}$ converges strongly to $q \in F(T) \neq \emptyset$. Let $\{y_n\}_{n=0}^{\infty}$ be any sequence in X and put $\varepsilon_n = \|y_{n+1} - f(T, y_n)\|$ for $n \geq 0$.

Definition 1.1. ([**13**]–[**15**], [**50**]).

1. The iteration scheme $\{x_n\}_{n=0}^{\infty}$ defined by $x_{n+1} = f(T, x_n)$ is said to be *T-stable* if $\lim_{n\to\infty} \varepsilon_n = 0$ implies that $\lim_{n\to\infty} y_n = q$.
2. The iteration scheme $\{x_n\}_{n=0}^{\infty}$ defined by $x_{n+1} = f(T, x_n)$ is said to be *almost T-stable* if $\sum_{n=0}^{\infty} \varepsilon_n < \infty$ implies that $\lim_{n\to\infty} y_n = q$.

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Note that $\{y_n\}_{n=0}^\infty$ is bounded provided that the iteration scheme $\{x_n\}_{n=0}^\infty$ defined by $x_{n+1} = f(T, x_n)$ is either T-stable or almost T-stable. Therefore we revise Definition 1.1 as follows:

Definition 1.2.

1. The iteration scheme $\{x_n\}_{n=0}^\infty$ defined by $x_{n+1} = f(T, x_n)$ is said to be *T-stable* if $\{y_n\}_{n=0}^\infty$ is bounded and $\lim_{n\to\infty} \varepsilon_n = 0$ imply that $\lim_{n\to\infty} y_n = q$.
2. The iteration scheme $\{x_n\}_{n=0}^\infty$ defined by $x_{n+1} = f(T, x_n)$ is said to be *almost T-stable* if $\{y_n\}_{n=0}^\infty$ is bounded and $\sum_{n=0}^\infty \varepsilon_n < \infty$ imply that $\lim_{n\to\infty} y_n = q$.

Definition 1.3. ([**1**]–[**12**], [**18**], [**19**], [**53**]–[**55**]). Let X be a real Banach space and $T : D(T) \subseteq X \to X$ be an operator, where $D(T)$ and $R(T)$ denote the domain and range of T, respectively.

1. T is said to be *local strongly pseudocontractive* if for each $x \in D(T)$ there exists $t_x > 1$ such that for all $y \in D(T)$ and $r > 0$

$$\|x - y\| \le \|(1 + r)(x - y) - rt_x(Tx - Ty)\|. \tag{1.1}$$

2. T is called *local strongly accretive* if for given $x \in D(T)$ there exists $k_x \in (0, 1)$ such that for each $y \in D(T)$ there is $j(x - y) \in J(x - y)$ satisfying

$$\langle Tx - Ty, j(x - y)\rangle \ge k_x\|x - y\|^2. \tag{1.2}$$

3. T is called *strongly pseudocontractive* (respectively, *strongly accretive*) if it is local strongly pseudocontractive (respectively, local strongly accretive) and $t_x \equiv t$ (respectively, $k_x \equiv k$) is independent of $x \in D(T)$.
4. T is said to be *accretive* for if given $x, y \in D(T)$ there is $j(x-y) \in J(x-y)$ satisfying

$$\langle Tx - Ty, j(x - y)\rangle \ge 0.$$

5. T is said to be *m-accretive* if it is accretive and $(I + rT)D(T) = X$ for all $r > 0$.

Clearly, each strongly pseudocontractive operator is local strongly pseudocontractive and each strongly accretive operator is local strongly accretive. It is known (see [**54**]) that T is local strongly pseudocontractive if and only if $I - T$ is local strongly accretive and $k_x = 1 - \frac{1}{t_x}$, where t_x and k_x are the constants appearing in (1.1) and (1.2), respectively.

The concept of accretive operators was introduced independently by Browder [**1**] and Kato [**17**] in 1967. An early fundamental result in the theory of accretive operators, due to Browder, states that the initial value problem

$$\frac{\mathrm{d}u(T)}{\mathrm{d}t} + Tu(T) = 0, \qquad u(0) = u_0,$$

is *solvable* if T is locally Lipschitzian and accretive on X. It is well known that if $T : X \to X$ is strongly accretive and demi-continuous, then for any $f \in X$, the equation

$$Tx = f \tag{1.3}$$

has a solution in X. Martin [**50**] proved that if T is a continuous accretive operator, then T is m-accretive. Thus for any $f \in X$, the equation

$$x + Tx = f \tag{1.4}$$

has a solution in X.

Recently several researches introduced and studied the iterative approximation methods to find either fixed points of ϕ-hemicontractive, strictly hemicontractive, strictly successively hemicontractive, strongly pseudocntractive, generalized asymptotically contractive and generalized hemicontractive, nonexpansive, asymptotically nonexpansive mappings, local strictly pseudocontractive and local strongly pseudocntractive operators or solutions of ϕ-strongly accretive, strongly quasiaccretive, strongly accretive, local strongly accretive and m-accretive operators equations (1.3) and (1.4) (see, for example, [**1**]–[**55**]).

Rhoades [**52**] proved that the Mann and Ishikawa iteration methods may exhibit different behaviors for different classes of nonlinear operators. A few stability results for certain classes of nonlinear operators have been established by several authors in [**13**]–[**15**], [**23**]–[**25**], [**30**], [**32**], [**33**], [**38**], [**40**]–[**43**], [**48**], [**51**]. Harder and Hicks [**14**] revealed that the importance of investigating the stability of various iteration procedures for various classes of nonlinear operators. Harder [**13**] obtained applications of stability results to first order differential equations. Osilike [**51**] obtained the stability of certain Mann and Ishikawa iteration sequences for fixed points of Lipschitz strong pseudocontractions and solutions of nonlinear accretive operator equations in real q-uniformly smooth Banach spaces.

The purpose of this paper is to establish the strong convergence and almost stability of the Ishikawa iteration methods with errors for either fixed point of local strongly pseudocontractive operators or solutions of nonlinear operator equations with local strongly accretive type in uniformly smooth Banach spaces. The convergence results presented in this paper are generalizations and improvements of the results in [**3**]–[**8**], [**10**], [**12**], [**53**], [**55**].

2. Preliminaries

The following results shall be needed in the sequel.

Lemma 2.1. ([**56**]). *Let X be a real uniformly smooth Banach space. Then there exists a nondecreasing continuous function $b : [0, +\infty) \to [0, +\infty)$ satisfying the conditions*

(a) $b(0) = 0$, $b(ct) \le cb(t)$, $\forall t \ge 0$, $c \ge 1$;
(b) $\|x + y\|^2 \le \|x\|^2 + 2\langle y, j(x)\rangle + \max\{\|x\|, 1\}\|y\|b(\|y\|)$, $\forall x, y \in X$.

Lemma 2.2. ([**4**]). *Let X be a real Banach space. Then the following conditions are equivalent.*

(a) *X is uniformly smooth;*
(b) *X^* is uniformly convex;*
(b) *J is single valued and uniformly continuous on any bounded subset of X.*

Lemma 2.3. ([**18**]). *Suppose that* $\{\alpha_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$, $\{\gamma_n\}_{n=0}^{\infty}$ *and* $\{\omega_n\}_{n=0}^{\infty}$ *are nonnegative sequences such that*

$$\alpha_{n+1} \le (1-\omega_n)\alpha_n + \beta_n\omega_n + \gamma_n, \quad \forall\, n \ge 0$$

with $\{\omega_n\}_{n=0}^{\infty} \subset [0,1]$, $\sum_{n=0}^{\infty} \omega_n = \infty$, $\sum_{n=0}^{\infty} \gamma_n < \infty$ *and* $\lim_{n\to\infty} \beta_n = 0$. *Then* $\lim_{n\to\infty} \alpha_n = 0$.

3. Main results

In this section, put $d_n = b_n + c_n$ and $d'_n = b'_n + c'_n$ for $n \ge 0$. Let b, k_q and t_q are the function and constants appearing in Lemma 2.1 and Definition 1.3, respectively, where $q \in F(T)$.

Theorem 3.1. *Let* X *be a real uniformly smooth Banach space and* $T : X \to X$ *be a local strongly pseudocontractive operator. Let* $R(T)$ *be bounded and* $F(T) \ne \emptyset$. *Suppose that* $\{u_n\}_{n=0}^{\infty}, \{v_n\}_{n=0}^{\infty}$ *are arbitrary bounded sequences in* X *and* $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$, $\{c'_n\}_{n=0}^{\infty}$ *and* $\{r_n\}_{n=0}^{\infty}$ *are any sequences in* $[0,1]$ *satisfying*

$$a_n + b_n + c_n = a'_n + b'_n + c'_n = 1, \quad \forall n \ge 0; \tag{3.1}$$

$$c_n(1-r_n) = r_n b_n, \quad \forall n \ge 0; \tag{3.2}$$

$$\lim_{n\to\infty} b(d_n) = \lim_{n\to\infty} r_n = \lim_{n\to\infty} b'_n = \lim_{n\to\infty} c'_n = 0; \tag{3.3}$$

$$\sum_{n=0}^{\infty} d_n = \infty. \tag{3.4}$$

For any $x_0 \in X$, *the Ishikawa iteration sequences with errors* $\{x_n\}_{n=0}^{\infty}$ *are defined by*

$$z_n = a'_n x_n + b'_n T x_n + c'_n v_n, \quad x_{n+1} = a_n x_n + b_n T z_n + c_n u_n, \quad \forall\, n \ge 0. \tag{3.5}$$

Let $\{y_n\}_{n=0}^{\infty}$ *be any bounded sequence in* X *and define* $\{\varepsilon_n\}_{n=0}^{\infty}$ *by*

$$w_n = a'_n y_n + b'_n T y_n + c'_n v_n, \quad \varepsilon_n = \|y_{n+1} - a_n y_n - b_n T w_n - c_n u_n\| \tag{3.6}$$

for all $n \ge 0$. *Then there exist nonnegative sequences* $\{s_n\}_{n=0}^{\infty}$, $\{t_n\}_{n=0}^{\infty}$ *and a constant* $M > 0$ *such that* $\lim_{n\to\infty} s_n = \lim_{n\to\infty} t_n = 0$ *and*

(a) $\{x_n\}_{n=0}^{\infty}$ *converges strongly to the unique fixed point* q *of* T *and*

$$\|x_{n+1} - q\|^2 \le (1 - d_n k_q)\|x_n - q\|^2 + M d_n (d'_n b(d'_n) + c'_n + s_n + b(d_n) + r_n), \quad \forall\, n \ge 0;$$

(b) *For all* $n \ge 0$

$$\|y_{n+1} - q\|^2 \le (1 - d_n k_q)\|y_n - q\|^2 + M d_n (d'_n b(d'_n) + c'_n + t_n + b(d_n) + r_n) + M\varepsilon_n;$$

(c) $\sum_{n=0}^{\infty} \varepsilon_n < \infty$ *implies that* $\lim_{n\to\infty} y_n = q$, *so that* $\{x_n\}_{n=0}^{\infty}$ *is almost* T*-stable;*

(d) $\lim_{n\to\infty} y_n = q$ *implies that* $\lim_{n\to\infty} \varepsilon_n = 0$.

Proof. Since T is local strongly pseudocontractive and $F(T) \neq \emptyset$, it follows from (1.1) that $F(T)$ is a singleton, say $F(T) = \{q\}$. Thus there exists $k_q \in (0,1)$ such that

$$\langle Tx - Tq, j(x-q)\rangle \leq (1-k_q)\|x-q\|^2, \quad \forall\, x \in X. \tag{3.7}$$

Set, for all $n \geq 0$,

$$p_n = a_n y_n + b_n S w_n + c_n u_n, \tag{3.8}$$

$$\begin{aligned} D = 2 &+ 2\|x_0 - q\| + 2\sup\{\|Tx - q\| : x \in X\} \\ &+ \sup\{\|y_n - q\| : n \geq 0\} + \sup\{\|u_n - q\| : n \geq 0\} \\ &+ \sup\{\|v_n - q\| : n \geq 0\}, \end{aligned} \tag{3.9}$$

$$s_n = \|j(x_n - q) - j(z_n - q)\|, \quad t_n = \|j(y_n - q) - j(w_n - q)\|. \tag{3.10}$$

It is easy to show that for all $n \geq 0$

$$\max\{\|x_n - q\|, \|z_n - q\|, \|p_n - q\|, \|y_n - q\|, \|w_n - q\|,\} \leq \tfrac{D}{2} < D, \tag{3.11}$$

$$\varepsilon_n \leq \|y_{n+1} - q\| + \|p_n - q\| \leq D. \tag{3.12}$$

In view of Lemma 2.1, (3.1), (3.5), (3.7) and (3.11), we infer that

$$\begin{aligned} &\|z_n - q\|^2 \\ &\quad = \|(1-d_n')(x_n - q) + d_n'(Tx_n - q) + c_n'(v_n - Tx_n)\|^2 \\ &\quad \leq \|(1-d_n')(x_n - q) + d_n'(Tx_n - q)\|^2 \\ &\qquad + 2c_n'\langle v_n - Tx_n, j((1-d_n')(x_n - q) + d_n'(Tx_n - q))\rangle \\ &\qquad + \max\{\|(1-d_n')(x_n - q) + d_n'(Tx_n - q)\|, 1\} \\ &\qquad\quad \times c_n'\|v_n - Tx_n\| b(c_n'\|v_n - Tx_n\|) \\ &\quad \leq (1-d_n')^2\|x_n - q\|^2 + 2d_n'\langle Tx_n - q, j((1-d_n')(x_n - q))\rangle \\ &\qquad + \max\{(1-d_n')\|x_n - q\|, 1\} d_n'\|Tx_n - q\| b(d_n'\|Tx_n - q\|) \\ &\qquad + 2c_n'\|v_n - Tx_n\|\|(1-d_n')(x_n - q) + d_n'(Tx_n - q)\| + D^3 c_n' b(c_n') \\ &\quad \leq (1-d_n')^2\|x_n - q\|^2 + 2d_n'(1-d_n')\langle Tx_n - q, j((x_n - q))\rangle \\ &\qquad + D^3 d_n' b(d_n') + 2D^2 c_n' + D^3 c_n' b(c_n') \\ &\quad \leq \{(1-d_n')^2 + 2d_n'(1-d_n')(1-k_q)\}\|x_n - q\|^2 + 2D^3(c_n' + d_n' b(d_n')) \\ &\quad = \{1 - k_q d_n' + {d_n'}^2(k_q - 1) + k_q d_n(d_n - 1)\}\|x_n - q\|^2 \\ &\qquad + 2D^3(c_n' + d_n' b(d_n')) \\ &\quad \leq (1 - k_q d_n')\|x_n - q\|^2 + 2D^3(c_n' + d_n' b(d_n')) \end{aligned} \tag{3.13}$$

for all $n \geq 0$. Observe that

$$\begin{aligned} \|(x_n - q) - (z_n - q)\| &\leq b_n'\|x_n - Tx_n\| + c_n'\|x_n - v_n\| \\ &\leq D d_n' \to 0 \quad \text{as } n \to \infty \end{aligned} \tag{3.14}$$

and

$$\begin{aligned}\|(y_n-q)-(w_n-q)\| &\le b'_n\|y_n-Ty_n\|+c'_n\|y_n-v_n\| \\ &\le Dd'_n \to 0 \quad \text{as } n\to\infty.\end{aligned} \tag{3.15}$$

Using Lemma 2.2, (3.14) and (3.15), we have

$$s_n, t_n \to 0 \quad \text{as } n\to\infty. \tag{3.16}$$

Using again Lemma 2.1, (3.1), (3.2), (3.5), (3.7), (3.11) and (3.13), we obtain that

$$\begin{aligned}
&\|x_{n+1}-q\|^2 \\
&= \|(1-d_n)(x_n-q)+d_n(Tz_n-q)+c_n(u_n-Tz_n)\|^2 \\
&\le (1-d_n)^2\|x_n-q\|^2+2d_n(1-d_n)\langle Tz_n-q, j(x_n-q)\rangle \\
&\quad +\max\{(1-d_n)\|x_n-q\|,1\}d_n\|Tz_n-q\|b(d_n\|Tz_n-q\|) \\
&\quad +2c_n\langle u_n-Tz_n, j((1-d_n)(x_n-q)+d_n(Ty_n-q))\rangle \\
&\quad +\max\{\|(1-d_n)(x_n-q)+d_n(Tz_n-q)\|,1\} \\
&\qquad \times c_n\|u_n-Tz_n\|b(c_n\|u_n-Tz_n\|) \\
&\le (1-d_n)^2\|x_n-q\|^2+2d_n(1-d_n)[\langle Tz_n-q, j(z_n-q)\rangle \\
&\quad +\langle Tz_n-q, j(x_n-q)-j(z_n-q)\rangle]+D^3(d_nb(d_n)+c_nb(c_n)) \\
&\quad +2c_n\|u_n-Tz_n\|\|(1-d_n)(x_n-q)+d_n(Ty_n-q)\| \\
&\le (1-d_n)^2\|x_n-q\|^2+2d_n(1-d_n)(1-k_q)\|z_n-q\|^2 \\
&\quad +2d_n(1-d_n)\|Tz_n-q\|\|j(x_n-q)-j(z_n-q)\| \\
&\quad +D^3(d_nb(d_n)+c_nb(c_n))+2c_nD^2 \\
&\le \{(1-d_n)^2+2d_n(1-d_n)(1-k_q)(1-k_qd'_n)\}\|x_n-q\|^2 \\
&\quad +2Dd_n(1-d_n)s_n+4D^3d_n(1-d_n)(1-k_q)(c'_n+d'_nb(d'_n)) \\
&\quad +D^3(d_nb(d_n)+c_nb(c_n))+2D^2c_n \\
&\le (1-k_qd_n)\|x_n-q\|^2+D^5d_n(d'_nb(d'_n)+c'_n+s_n+b(d_n))+2D^2c_n \\
&\le (1-k_qd_n)\|x_n-q\|^2+Md_n(d'_nb(d'_n)+c'_n+s_n+b(d_n)+r_n)
\end{aligned} \tag{3.17}$$

for all $n \ge 0$, where $M = D^5$. Let

$$\alpha_n = \|x_n-q\|^2, \quad \omega_n = k_qd_n, \quad \gamma_n = 0,$$
$$\beta_n = k_q^{-1}M(d'_nb(d'_n)+c'_n+s_n+b(d_n)+r_n)$$

for all $n \ge 0$. Thus (3.17) can be written as

$$\alpha_{n+1} \le (1-\omega_n)\alpha_n+\omega_n\beta_n+\gamma_n, \quad \forall n\ge 0. \tag{3.18}$$

It follows from (3.3), (3.4), (3.16), (3.18) and Lemma 2.3 that $\alpha_n \to 0$ as $n\to\infty$. That is, $x_n \to q$ as $n\to\infty$.

Observe that Lemma 2.1 and (3.1), (3.6), (3.7) and (3.9) ensure that

$$
\begin{aligned}
\|w_n-q\|^2 & \\
&= \|(1-d'_n)(y_n-q)+d'_n(Ty_n-q)+c'_n(v_n-Ty_n)\|^2\\
&\le (1-d'_n)^2\|y_n-q\|^2+2d'_n(1-d'_n)\langle Ty_n-q,j(y_n-q)\rangle\\
&\quad+\max\{(1-d'_n)\|y_n-q\|,1\}d'_n\|Ty_n-q\|b(\|Ty_n-q\|)\\
&\quad+2c'_n\langle v_n-Ty_n,j((1-d'_n)(y_n-q)+d'_n(Ty_n-q))\rangle\\
&\quad+\max\{\|(1-d'_n)(y_n-q)+d'_n(Ty_n-q)\|,1\}\\
&\qquad\times c'_n\|v_n-Ty_n\|b(c'_n\|v_n-Ty_n\|)\\
&\le \{(1-d'_n)^2+2d'_n(1-d'_n)(1-k_q)\}\|y_n-q\|^2+D^3d'_nb(d'_n)\\
&\quad+2c'_n\|v_n-Ty_n\|\|(1-d'_n)(y_n-q)+d'_n(Ty_n-q)\|+D^3c'_nb(c'_n)\\
&\le (1-k_qd'_n)\|y_n-q\|^2+2D^3d'_nb(d'_n)+2D^2c'_n
\end{aligned}
\tag{3.19}
$$

for all $n\ge 0$. In view of Lemma 2.1, (3.1), (3.8) and (3.11), we get that

$$
\begin{aligned}
\|p_n-q\|^2 & \\
&= \|(1-d_n)(y_n-q)+d_n(Tw_n-q)+c_n(u_n-Tw_n)\|^2\\
&\le (1-d_n)^2\|y_n-q\|^2+2\langle d_n(Tw_n-q),j((1-d_n)(y_n-q))\rangle\\
&\qquad\times\max\{(1-d_n)\|y_n-q\|,1\}d_n\|Tw_n-q\|b(d_n\|Tw_n-q\|)\\
&\quad+2\langle c_n(u_n-Tw_n),j((1-d_n)(y_n-q)+d_n(Tw_n-q))\rangle\\
&\quad+\max\{\|(1-d_n)(y_n-q)+d_n(Tw_n-q)\|,1\}\\
&\qquad\times c_n\|u_n-Tw_n\|b(c_n\|u_n-Tw_n\|)\\
&\le (1-d_n)^2\|y_n-q\|^2+2d_n(1-d_n)(1-k_q)\|w_n-q\|^2\\
&\quad+2d_n(1-d_n)\langle Tw_n-q,j(y_n-q)-j(w_n-q)\rangle\\
&\quad+D^3d_nb(d_n)+2c_n\|u_n-Tw_n\|\|(1-d_n)(y_n-q)\\
&\quad+d_n(Tw_n-q)\|+D^3c_nb(c_n)\\
&\le \{(1-d_n)^2+2d_n(1-d_n)(1-k_q)(1-k_qd'_n)\}\|y_n-q\|^2\\
&\quad+2d_n(1-d_n)Dt_n+2d_n(1-d_n)(1-k_q)[2D^3d'_nb(d'_n)+2D^2c'_n]\\
&\quad+2D^3d_nb(d_n)+2c_nD^2\\
&\le (1-k_qd_n)\|y_n-q\|^2+Md_n(d'_nb(d'_n)+c'_n+t_n+b(d_n)+r_n)
\end{aligned}
\tag{3.20}
$$

for any $n\ge 0$ It follows from (3.2), (3.12) and (3.20) that

$$
\begin{aligned}
\|y_{n+1}-q\|^2 &\le (\|p_n-q\|+\varepsilon_n)^2\le\|p_n-q\|^2+M\varepsilon_n\\
&\le (1-k_qd_n)\|y_n-q\|^2\\
&\quad+Md_n(d'_nb(d'_n)+c'_n+t_n+b(d_n)+r_n)+M\varepsilon_n
\end{aligned}
\tag{3.21}
$$

for any $n\ge 0$.

Suppose that $\sum_{n=0}^{\infty} \varepsilon_n < \infty$. Put $\alpha_n = \|y_n - q\|^2$, $\omega_n = k_q d_n$, $\gamma_n = M\varepsilon_n$ $\beta_n = M(d'_n b(d'_n) + c'_n + t_n + b(d_n) + r_n)k_q^{-1}$ for all $n \geq 0$. Using Lemma 2.3, (3.3), (3.4), (3.16) and (3.21), we conclude immediately that $\alpha_n \to 0$ as $n \to \infty$. That is, $y_n \to q$ as $n \to \infty$. Therefore $\{x_n\}_{n=0}^{\infty}$ is almost S-stable. Suppose that $\lim_{n\to\infty} y_n = q$. It follows from (3.20), (3.16) and (3.3) that

$$\begin{aligned}\varepsilon_n &\leq \|y_{n+1} - q\| + \|p_n - q\| \\ &\leq \|y_{n+1} - q\| + \big[(1 - k_q d_n)\|y_n - q\|^2 \\ &\quad + M d_n (d'_n b(d'_n) + c'_n + t_n + b(d_n) + r_n)\big]^{1/2} \to 0\end{aligned}$$

as $n \to \infty$. That is, $\varepsilon_n \to 0$ as $n \to \infty$. This completes the proof. □

Theorem 3.2. *Let X, T, $R(T)$, q, $\{u_n\}_{n=0}^{\infty}$, $\{v_n\}_{n=0}^{\infty}$, $\{x_n\}_{n=0}^{\infty}$, $\{z_n\}_{n=0}^{\infty}$, $\{y_n\}_{n=0}^{\infty}$, $\{w_n\}_{n=0}^{\infty}$ and $\{\varepsilon_n\}_{n=0}^{\infty}$ be as in Theorem 3.1. Suppose that $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$ and $\{c'_n\}_{n=0}^{\infty}$ are any sequences in $[0,1]$ satisfying (3.1) and*

$$\lim_{n\to\infty} b(d_n) = \lim_{n\to\infty} b'_n = \lim_{n\to\infty} c'_n = 0; \tag{3.22}$$

$$\sum_{n=0}^{\infty} c_n < \infty; \tag{3.23}$$

$$\sum_{n=0}^{\infty} b_n = \infty. \tag{3.24}$$

Then the conclusions of Theorem 3.1 hold.

Proof. Let

$$\begin{aligned}&\alpha_n = \|x_n - q\|^2, \quad \omega_n = k_q d_n, \quad \gamma_n = 2D^2 + r_n, \\ &\beta_n = k_q^{-1} M(d'_n b(d'_n) + c'_n + s_n + b(d_n))\end{aligned}$$

for all $n \geq 0$. As in the proof of (3.17), we conclude that $x_n \to q$ as $n \to \infty$.

Put $\alpha_n = \|y_n - q\|^2$, $\omega_n = k_q d_n$, $\gamma_n = M(r_n + \varepsilon_n)$ and $\beta_n = M(d'_n b(d'_n) + c'_n + t_n + b(d_n))k_q^{-1}$ for all $n \geq 0$. It follows from (3.21) that $y_n \to q$ as $n \to \infty$. The rest of the proof is similar to that of Theorem 3.1, and is omitted. This completes the proof. □

The proof of Theorem 3.3 below is similar to that of Theorem 3.1, so we omit the details.

Theorem 3.3. *Let K be a nonempty bounded closed convex subset of a real uniformly smooth Banach space X and $T : K \to K$ be a local strongly pseudo-contractive operator. Let $q \in K$ be a fixed point of T and $\{u_n\}_{n=0}^{\infty}$, $\{v_n\}_{n=0}^{\infty}$ be arbitrary sequences in K. Suppose that $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$, $\{c'_n\}_{n=0}^{\infty}$ and $\{r_n\}_{n=0}^{\infty}$ are any sequences in $[0,1]$ satisfying (3.1)–(3.4). If $\{x_n\}_{n=0}^{\infty}$ is the Ishikawa iteration sequence with errors generated from an arbitrary $x_0 \in K$ by (3.5), then it converges strongly to the unique fixed point q of T.*

Theorem 3.4. *Let X, K, T, q, $\{u_n\}_{n=0}^{\infty}$, $\{v_n\}_{n=0}^{\infty}$, $\{x_n\}_{n=0}^{\infty}$, $\{z_n\}_{n=0}^{\infty}$ be as in Theorem 3.2 and $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$ and $\{c'_n\}_{n=0}^{\infty}$ be any sequences in $[0,1]$ satisfying (3.1) and (3.22)–(3.24). Then $\{x_n\}_{n=0}^{\infty}$ converges strongly to the unique fixed point q of T.*

Remark. Theorem 3.3 extends, improves and unifies Theorems 3.2 and 4.1 of Chang [**3**], Theorems 3.3 and 4.1 of Chang et al. [**4**], the Theorem Chidume [**5**], Theorems 1 and 2 of Chidume [**6**], Theorems 3 and 4 of Chidume [**7**], Theorem 4 of Chidume and Osilike [**10**], Theorem 4.2 of Tan and Xu [**53**] and Theorem 3.3 of Xu [**55**].

Theorem 3.5. *Let X be a real uniformly smooth Banach space and $T : X \to X$ be a local strongly accretive operator. Define $G : X \to X$ by $Gx = f - Tx$ for all $x \in X$. Suppose that $R(T)$ is bounded and the equation $x + Tx = f$ has a solution q for some $f \in X$. Suppose that $\{u_n\}_{n=0}^{\infty}$, $\{v_n\}_{n=0}^{\infty}$ are arbitrary bounded sequences in X and $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$, $\{c'_n\}_{n=0}^{\infty}$ and $\{r_n\}_{n=0}^{\infty}$ are any sequences in $[0,1]$ satisfying (3.1)–(3.4). For arbitrary $x_0 \in X$, the Ishikawa iteration sequence with errors $\{x_n\}_{n=0}^{\infty}$ is defined by*

$$z_n = a'_n x_n + b'_n G x_n + c'_n v_n, \quad x_{n+1} = a_n x_n + b_n G z_n + c_n u_n \tag{3.25}$$

for all $n \geq 0$. Let $\{y_n\}_{n=0}^{\infty}$ be any bounded sequence in X and define $\{\varepsilon_n\}_{n=0}^{\infty}$ by

$$\begin{aligned} w_n &= a'_n y_n + b'_n G y_n + c'_n v_n, \\ \varepsilon_n &= \|y_{n+1} - a_n y_n - b_n G w_n - c_n u_n\| \end{aligned} \tag{3.26}$$

for all $n \geq 0$. Then there exist nonnegative sequences $\{s_n\}_{n=0}^{\infty}$, $\{t_n\}_{n=0}^{\infty}$ and a constant $M > 0$ such that $\lim_{n\to\infty} s_n = \lim_{n\to\infty} t_n = 0$ and

(a) *$\{x_n\}_{n=0}^{\infty}$ converges strongly to the unique solution q of the equation $x+Tx=f$ and*

$$\begin{aligned} \|x_{n+1} - q\|^2 \leq (1 - d_n k_q)\|x_n - q\|^2 + M d_n (d'_n b(d'_n) \\ + c'_n + s_n + b(d_n) + r_n), \quad \forall\, n \geq 0; \end{aligned}$$

(b) *for all $n \geq 0$*

$$\begin{aligned} \|y_{n+1} - q\|^2 \leq (1 - d_n k_q)\|y_n - q\|^2 + M d_n (d'_n b(d'_n) + c'_n \\ + t_n + b(d_n) + r_n) + M\varepsilon_n; \end{aligned}$$

(c) *$\sum_{n=0}^{\infty} \varepsilon_n < \infty$ implies that $\lim_{n\to\infty} y_n = q$, so that $\{x_n\}_{n=0}^{\infty}$ is almost G-stable;*

(d) *$\lim_{n\to\infty} y_n = q$ implies that $\lim_{n\to\infty} \varepsilon_n = 0$.*

Proof. It follows from (1.2) that for given $x \in X$ there exists $k_x \in (0,1)$ such that

$$\langle Tx - Ty, j(x-y)\rangle \geq k_x \|x - y\|^2, \quad \forall\, y \in X,$$

which implies that

$$\begin{aligned}\langle (I-G)x-(I-G)y, j(x-y)\rangle &= \|x-y\|^2 - \langle Gx-Gy, j(x-y)\rangle \\ &= \|x-y\|^2 + \langle Tx-Ty, j(x-y)\rangle \\ &\geq k_x\|x-y\|^2, \quad \forall\, y \in X.\end{aligned}$$

That is, $I-G$ is local strongly accretive. Thus G is local strongly pseudocontractive. It is easy to see that q is a unique fixed point of G. Therefore, q is the unique solution of the equation $x+Tx=f$. The rest of the argument uses the same ideas as that of Theorem 3.1 and is thus omitted. This completes the proof. □

Theorem 3.6. *Let X, T, G, $R(T)$, f, q, $\{u_n\}_{n=0}^{\infty}$, $\{v_n\}_{n=0}^{\infty}$, $\{x_n\}_{n=0}^{\infty}$, $\{z_n\}_{n=0}^{\infty}$, $\{y_n\}_{n=0}^{\infty}$, $\{w_n\}_{n=0}^{\infty}$ and $\{\varepsilon_n\}_{n=0}^{\infty}$ be as in Theorem 3.3. Suppose that $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$ and $\{c'_n\}_{=0}^{\infty}$ are any sequences in $[0,1]$ satisfying (3.1) and (3.22)–(3.24). Then the conclusions of Theorem 3.5 hold.*

Remark. The convergence result in Theorem 3.6 generalizes Theorems 11 and 12 of Chidume [**8**].

Theorem 3.7. *Let X be a real uniformly smooth Banach space and $T: X \to X$ be a local strongly accretive operator. Define $S: X \to X$ by $Sx = f + x - Tx$ for all $x \in X$. Suppose that the equation $Tx = f$ has a solution q for some $f \in X$ and either $R(T)$ or $R(I-T)$ is bounded. Assume that $\{u_n\}_{n=0}^{\infty}$, $\{v_n\}_{n=0}^{\infty}$ are arbitrary bounded sequences in X and $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$, $\{c'_n\}_{n=0}^{\infty}$ and $\{r_n\}_{n=0}^{\infty}$ are any sequences in $[0,1]$ satisfying (3.1)–(3.4). For arbitrary $x_0 \in X$, the Ishikawa iteration sequence with errors $\{x_n\}_{n=0}^{\infty}$ is defined by*

$$z_n = a'_n x_n + b'_n S x_n + c'_n v_n, \quad x_{n+1} = a_n x_n + b_n S z_n + c_n u_n, \quad \forall\, n \geq 0. \tag{3.27}$$

Let $\{y_n\}_{n=0}^{\infty}$ be any bounded sequence in X and define $\{\varepsilon_n\}_{n=0}^{\infty}$ by

$$\begin{aligned} w_n &= a'_n y_n + b'_n S y_n + c'_n v_n, \\ \varepsilon_n &= \|y_{n+1} - a_n y_n - b_n S w_n - c_n u_n\| \end{aligned} \tag{3.28}$$

for all $n \geq 0$. Then there exist nonnegative sequences $\{s_n\}_{n=0}^{\infty}$, $\{t_n\}_{n=0}^{\infty}$ and a constant $M > 0$ such that $\lim_{n\to\infty} s_n = \lim_{n\to\infty} t_n = 0$ and

(a) *$\{x_n\}_{n=0}^{\infty}$ converges strongly to the unique solution q of the equation $Tx = f$ and*

$$\begin{aligned}\|x_{n+1}-q\|^2 \leq (1-d_n k_q)\|x_n - q\|^2 &+ M d_n (d'_n b(d'_n) \\ &+ c'_n + s_n + b(d_n) + r_n)\end{aligned}$$

for all $n \geq 0$;

(b) *for all $n \geq 0$*

$$\begin{aligned}\|y_{n+1}-q\|^2 \leq (1-d_n k_q)\|y_n - q\|^2 &+ M d_n (d'_n b(d'_n) \\ &+ c'_n + t_n + b(d_n) + r_n) + M\varepsilon_n;\end{aligned}$$

(c) $\sum_{n=0}^{\infty} \varepsilon_n < \infty$ *implies that* $\lim_{n\to\infty} y_n = q$, *so that* $\{x_n\}_{n=0}^{\infty}$ *is almost* S*-stable*;

(d) $\lim_{n\to\infty} y_n = q$ *implies that* $\lim_{n\to\infty} \varepsilon_n = 0$.

Proof. Since T is local strongly accretive and q is a solution of the equation $Tx = f$, it follows that q is a unique solution of the equation $Tx = f$ and there exists $k_q \in (0,1)$ such that

$$\langle Tx - Tq, j(x-q)\rangle \geq k_q \|x-q\|^2, \quad \forall\, x \in X,$$

which implies that

$$\|x - q\| \leq k_q^{-1}\|Tx - Tq\|, \quad \forall\, x \in X \tag{3.29}$$

and

$$\langle Sx - Sq, j(x-q)\rangle \leq (1-k_q)\|x-q\|^2, \quad \forall\, x \in X. \tag{3.30}$$

We now claim that $R(S)$ is bounded. Suppose that $R(I-T)$ is bounded. It is clear that $R(S)$ is bounded. Suppose that $R(T)$ is bounded. From (3.29), we have

$$\begin{aligned}\|Sx - Sy\| &\leq \|x-y\| + \|Tx - Ty\| \\ &\leq \|x-q\| + \|y-q\| + \|Tx-Tq\| + \|Ty - Tq\| \\ &\leq (1+k_q^{-1})(\|Tx-Tq\| + \|Ty-Tq\|), \quad \forall\, x, y \in X,\end{aligned}$$

which implies that $R(S)$ is bounded. Note that S is local strongly pseudocontractive and $F(S) = \{q\}$. The rest of the proof follows immediately as in the proof of Theorem 3.1, and is therefore omitted. This completes the proof. □

Theorem 3.8. *Let* X, T, S, $R(T)$, $R(I-T)$, f, q, $\{u_n\}_{n=0}^{\infty}$, $\{v_n\}_{n=0}^{\infty}$, $\{x_n\}_{n=0}^{\infty}$, $\{z_n\}_{n=0}^{\infty}$, $\{y_n\}_{n=0}^{\infty}$, $\{w_n\}_{n=0}^{\infty}$ *and* $\{\varepsilon_n\}_{n=0}^{\infty}$ *be as in Theorem 3.4. Suppose that* $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$, $\{c_n\}_{n=0}^{\infty}$, $\{a'_n\}_{n=0}^{\infty}$, $\{b'_n\}_{n=0}^{\infty}$ *and* $\{c'_n\}_{n=0}^{\infty}$ *are any sequences in* $[0,1]$ *satisfying* (3.1) *and* (3.22)–(3.24). *Then the conclusions of Theorem 3.4 hold.*

Remark. The convergence result in Theorem 3.8 extends Theorem 1 of Chidume [**7**], Theorems 7 and 8 of Chidume [**8**], Theorem 3.2 of Ding [**12**], Theorem 4.1 of Tan and Xu [**53**] and Theorem 3.1 of Xu [**55**].

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