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ON CERTAIN OPERATIONAL FORMULA FOR MULTIVARIABLE BASIC HYPERGEOMETRIC FUNCTIONS

S. D. PUROHIT and R. K. RAINA

Abstract. In the present paper certain operational formulae involving Riemann-Liouville and Kober fractional q-integral operators for an analytic function are derived. The usefulness of the main results are exhibited by considering some examples which also yield q-extensions of some known results for ordinary hypergeometric functions of one and more variables.

1. Introduction

Several authors have used certain fractional q-calculus operators to obtain various operational and transformation formulae involving basic hypergeometric functions (see, for instance, [**1**]–[**4**], [**7**], [**10**]–[**12**]). Motivated by the interesting outcome of some of the earlier works and a possible scope for their applications in evaluation and solution of the types of q-integral equations, we further determine certain operational formulae involving the Riemann-Liouville and Kober type fractional q-integral operators.

A q-analogue of the familiar Riemann-Liouville fractional integral operator of a function $f(x)$ is defined by ([**1**])

$$I_q^{\mu}\left\{f(x)\right\} = \frac{1}{\Gamma_q(\mu)} \int_0^x (x-tq)_{\mu-1} f(t) \mathrm{d}(t;q) \tag{1.1}$$

$$(\mathrm{Re}(\mu) > 0; \quad |q| < 1).$$

Also, in [**1**] the basic analogue of the Kober fractional integral operator is defined by

$$I_q^{\eta,\mu}\left\{f(x)\right\} = \frac{x^{-\eta-\mu}}{\Gamma_q(\mu)} \int_0^x (x-tq)_{\mu-1} t^{\eta} f(t) \mathrm{d}(t;q) \tag{1.2}$$

$$(\mathrm{Re}(\mu) > 0; \quad |q| < 1; \quad \eta \in \mathbb{R}).$$

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We shall make use of the following notations and definitions in the sequel. For real or complex a and $|q|<1$, the q-shifted factorial is defined by

$$(a;q)_n = \begin{cases} 1, & \text{if } n=0 \\ (1-a)(1-aq)\dots(1-aq^{n-1}) & \text{if } n\in\mathbb{N}, \end{cases} \tag{1.3}$$

and in terms of the q-gamma function (1.3) can be expressed as

$$(q^a;q)_n \;=\; \frac{\Gamma_q(a+n)(1-q)^n}{\Gamma_q(a)}, \qquad n>0, \tag{1.4}$$

where the q-gamma function (cf. Gasper and Rahman [**3**]) is given by

$$\Gamma_q(a) \;=\; \frac{(q;q)_\infty}{(q^a;q)_\infty(1-q)^{a-1}} = \frac{(1;q)_{a-1}}{(1-q)^{a-1}}, \tag{1.5}$$

provided that $a\neq 0,-1,-2,\dots$
Further,

$$(x;y)_\nu = x^\nu \prod_{n=0}^{\infty}\left[\frac{1-(y/x)q^n}{1-(y/x)q^{\nu+n}}\right] = x^\nu\ {}_1\Phi_0\left[\begin{matrix} q^{-\nu} \\ - \end{matrix};q,yq^\nu/x\right]. \tag{1.6}$$

The multiple basic hypergeometric function (cf. Srivastava and Karlsson [**9**]) is defined by
(1.7)

$$\Phi^{A:B';\dots;B^{(n)}}_{C:D';\dots;D^{(n)}}\left(\begin{matrix}[(a):\theta',\dots,\theta^{(n)}]:[(b'):\phi'];\dots;[(b^{(n)}):\phi^{(n)}] \\ [(c):\psi',\dots,\psi^{(n)}]:[(d'):\delta'];\dots;[(d^{(n)}):\delta^{(n)}]\end{matrix};q,z_1,\dots,z_n\right)$$

$$= \sum_{m_1,\cdots,m_n=0}^{\infty} \frac{\prod_{j=1}^{A}(a_j;q)_{m_1\theta'_j+\dots+m_n\theta^{(n)}_j}\prod_{j=1}^{B'}(b'_j;q)_{m_1\phi'_j}\cdots\prod_{j=1}^{B^{(n)}}(b^{(n)}_j;q)_{m_n\phi^{(n)}_j}}{\prod_{j=1}^{C}(c_j;q)_{m_1\psi'_j+\dots+m_n\psi^{(n)}_j}\prod_{j=1}^{D'}(d'_j;q)_{m_1\delta'_j}\cdots\prod_{j=1}^{D^{(n)}}(d^{(n)}_j;q)_{m_n\delta^{(n)}_j}}$$

$$\cdot\frac{z_1^{m_1}}{(q;q)_{m_1}}\cdots\frac{z_n^{m_n}}{(q;q)_{m_n}},$$

where the arguments $z_1,\cdots,z_n$, and the complex parameters

$$\begin{cases} a_j,\ j=1,\dots,A; & b^{(k)}_j,\ j=1,\dots,B^{(k)}_j; \\ c_j,\ j=1,\dots,C; & d^{(k)}_j,\ j=1,\dots,D^{(k)}_j; \quad k=1,\dots,n \ \text{(primes)}, \end{cases}$$

and the associated coefficients

$$\begin{cases} \theta^{(k)}_j,\ j=1,\dots,A; & \phi^{(k)}_j,\ j=1,\dots,B^{(k)}_j; \\ \psi^{(k)}_j,\ j=1,\dots,C; & \delta^{(k)}_j,\ j=1,\dots,D^{(k)}_j \quad k=1,\dots,n\text{(primes)}, \end{cases}$$

are so constrained that the multiple series (1.7) converges.

For $\theta^{(k)}_j=1$ $(j=1,\dots,A)$, $\phi^{(k)}_j=1$ $(j=1,\dots,B^{(k)}_j)$, $\psi^{(k)}_j=1$ $(j=1,\dots,C)$, $\delta^{(k)}_j=1$ $(j=1,\dots,D^{(k)}_j)$ for all $k=1,\dots,n$ (primes) the definition (1.7) reduces to the q-analogue of the generalized Kampé de Fériet function of n variables given by

$$\Phi^{A:B';\ldots;B^{(n)}}_{C:D';\ldots;D^{(n)}}\left(\begin{array}{l}(a):(b');\ldots;(b^{(n)})\\(c):(d');\ldots;(d^{(n)})\end{array};q,z_1,\ldots,z_n\right)$$

$$= \sum_{m_1,\cdots,m_n=0}^{\infty}\frac{\prod_{j=1}^{A}(a_j;q)_{m_1+\ldots+m_n}\prod_{j=1}^{B'}(b_j';q)_{m_1}\cdots\prod_{j=1}^{B^{(n)}}(b_j^{(n)};q)_{m_n}}{\prod_{j=1}^{C}(c_j;q)_{m_1+\ldots+m_n}\prod_{j=1}^{D'}(d_j';q)_{m_1}\cdots\prod_{j=1}^{D^{(n)}}(d_j^{(n)};q)_{m_n}} \cdot\frac{z_1^{m_1}}{(q;q)_{m_1}}\cdots\frac{z_n^{m_n}}{(q;q)_{m_n}}. \tag{1.8}$$

The generalized basic hypergeometric series (cf. Slater [**7**] is given by

$$ {}_r\Phi_s\left[\begin{array}{l}a_1,\ldots,a_r\\b_1,\ldots,b_s\end{array};q,z\right]=\sum_{n=0}^{\infty}\frac{(a_1,\ldots,a_r;q)_n}{(q,b_1,\ldots,b_s;q)_n}\,z^n, \tag{1.9}$$

where, for convergence, $|q|<1$ and $|z|<1$ if $r=s+1$, and for any z if $r\le s$.

The purpose of this paper is to obtain certain operational formulae involving the Riemann-Liouville and Kober type of fractional q-integral operators of an analytic function. The applications yield examples of image formula under these above operators, thereby illustrating the usefulness of the main results.

2. Main Results

Suppose that a function $f(z_1,\ldots,z_n)$ is analytic in the domain $D=D_1\times D_2\times\cdots\times D_n$ ($z_i\in D_i$, $i=1,\cdots,n$) possessing the power series expansion

$$f(z_1,\ldots,z_n)=\sum_{m_1,\ldots,m_n=0}^{\infty}C(m_1,\ldots,m_n)\prod_{i=1}^{n}z_i^{m_i}, \tag{2.1}$$

where $|z_i|<R_i$ ($R_i>0$, $i=1,\ldots,n$), and $C(m_1,\ldots,m_n)$ is a bounded sequence of real or complex numbers.

For an analytic function $f(z_1,\ldots,z_n)$ defined by (2.1), we derive the following two operational formula involving the fractional q-integral operators for a real variable x and complex variables $z_1,\ldots,z_n$.

Theorem 1. *Corresponding to the bounded sequence $C(m_1,\ldots,m_n)$ let the function $f(z_1,\ldots,z_n)$ be defined by (2.1), then*

$$\begin{aligned}\Omega\left\{f(x^{k_1}z_1,\ldots,x^{k_n}z_n)\right\}&=x^{\alpha_p-1}\sum_{m_1,\ldots,m_n=0}^{\infty}C(m_1,\ldots,m_n)\prod_{i=1}^{n}(x^{k_i}z_i)^{m_i}\\&\quad\cdot\prod_{j=1}^{p}\left\{\frac{\Gamma_q(\alpha_j+\mu_j+M)}{\Gamma_q(\alpha_j+\mu_j+\lambda_j+M)}\right\},\end{aligned} \tag{2.2}$$

where $\mathrm{Re}(\alpha_j+\mu_j)>0$ $(j=1,\ldots,p)$, $\max\left\{\left|x^{k_1}z_1\right|,\ldots,\left|x^{k_n}z_n\right|\right\}<R$ $(R>0)$, *for arbitrary* k_i $(i=1,\ldots,n)$, Ω *is a chain of fractional* q*-calculus operators defined by*

$$\Omega = I_q^{\mu_p,\lambda_p} x^{\alpha_p-\alpha_{p-1}} \ldots I_q^{\mu_2,\lambda_2} x^{\alpha_2-\alpha_1} I_q^{\mu_1,\lambda_1} x^{\alpha_1-1}, \tag{2.3}$$

provided that both sides of (2.2) *exist, and*

$$M = k_1 m_1 + \ldots + k_n m_n. \tag{2.4}$$

Proof. In view of (2.1) and (2.3), we obtain by replacing each z_i by $x^{k_i}z_i$ $(i=1,\ldots,n)$:

$$\Omega\left\{f(x^{k_1}z_1,\ldots,x^{k_n}z_n)\right\} = \Omega\left\{\sum_{m_1,\ldots,m_n=0}^{\infty} C(m_1,\ldots,m_n)x^M \prod_{i=1}^{n} z_i^{m_i}\right\}.$$

On interchanging the order of summation and the chain of fractional q-integral operator Ω (which is valid under the conditions given in (2.1) and in the hypothesis of Theorem 1), we get

$$\Omega\left\{f(x^{k_1}z_1,\ldots,x^{k_n}z_n)\right\} = \sum_{m_1,\ldots,m_n=0}^{\infty} C(m_1,\ldots,m_n)\prod_{i=1}^{n} z_i^{m_i}\,\Omega\left\{x^M\right\}. \tag{2.5}$$

Applying the fractional q-integral formula due to Yadav and Purohit [**12**, p. 440, eqn. (19)]

$$I_q^{\eta,\mu}\left\{x^{\nu-1}\right\} = \frac{\Gamma_q(\nu+\eta)}{\Gamma_q(\nu+\eta+\mu)}x^{\nu-1}, \tag{2.6}$$

$$(\mathrm{Re}(\nu+\eta)>0,\ |q|<1)$$

succesively p times on the right-hand side of (2.5), we arrive at the desired result (2.2) of Theorem 1. □

If we set $k_1=\ldots=k_n=1$ in the Theorem 1, then we obtain the following corollary:

Corollary 1. *Corresponding to the bounded sequence* $C(m_1,\ldots,m_n)$, *let the function* $f(z_1,\ldots,z_n)$ *be defined by* (2.1), *then*

$$\begin{aligned}\Omega\left\{f(xz_1,\ldots,xz_n)\right\} = x^{\alpha_p-1} &\sum_{m_1,\ldots,m_n=0}^{\infty} C(m_1,\ldots,m_n)\prod_{i=1}^{n}(xz_i)^{m_i}\\ &\cdot\prod_{j=1}^{p}\left\{\frac{\Gamma_q(\alpha_j+\mu_j+M_1)}{\Gamma_q(\alpha_j+\mu_j+\lambda_j+M_1)}\right\},\end{aligned} \tag{2.7}$$

where $\mathrm{Re}(\alpha_j+\mu_j)>0$ $(j=1,\ldots,p)$, $\max\left\{|xz_1|,\ldots,|xz_n|\right\}<R_1$ $(R_1>0)$, Ω *is defined by equation* (2.3), *and*

$$M_1 = m_1 + \ldots + m_n. \tag{2.8}$$

Theorem 2. *For the bounded sequence* $C(m_1, \ldots, m_n)$*, let the function* $f(z_1, \ldots, z_n)$ *be defined by* (2.1)*, then*

$$\begin{aligned}\Omega^* \left\{f(x^{k_1}z_1, \ldots, x^{k_n}z_n)\right\} = x^{\beta_p - 1} \sum_{m_1, \ldots, m_n = 0}^{\infty} C(m_1, \ldots, m_n) \prod_{i=1}^{n} (x^{k_i} z_i)^{m_i} \\ \cdot \prod_{j=1}^{p} \left\{\frac{\Gamma_q(\alpha_j + M)}{\Gamma_q(\beta_j + M)}\right\},\end{aligned} \tag{2.9}$$

where $\mathrm{Re}(\alpha_j) > 0$ $(j = 1, \ldots, p)$*,* $\max\left\{\left|x^{k_1}z_1\right|, \ldots, \left|x^{k_n}z_n\right|\right\} < R$ $(R > 0)$*, for arbitrary* k_i $(i = 1, \ldots, n)$*,* Ω^* *is a chain of fractional* q*-calculus operators defined by*

$$\Omega^* = I_q^{\beta_p - \alpha_p} x^{\alpha_p - \beta_{p-1}} \ldots I_q^{\beta_2 - \alpha_2} x^{\alpha_2 - \beta_1} I_q^{\beta_1 - \alpha_1} x^{\alpha_1 - 1}, \tag{2.10}$$

provided that both sides of (2.9) *exist, and* M *is given by* (2.4).

Proof. Proceeding as in Theorem 1, we can write

$$\Omega^* \left\{f(x^{k_1}z_1, \ldots, x^{k_n}z_n)\right\} = \sum_{m_1, \ldots, m_n = 0}^{\infty} C(m_1, \ldots, m_n) \prod_{i=1}^{n} z_i^{m_i} \Omega^* \left\{x^M\right\}. \tag{2.11}$$

Then applying the formula due to Agarwal [**1**]:

$$I_q^{\mu} \left\{x^{\nu - 1}\right\} = \frac{\Gamma_q(\nu)}{\Gamma_q(\nu + \mu)} x^{\mu + \nu - 1}, \tag{2.12}$$

$$(\mathrm{Re}(\nu) > 0;\ |q| < 1)$$

successively p times on the right-hand side of (2.11), we obtain (2.9) of Theorem 2. □

For $k_1 = \ldots = k_n = 1$, the Theorem 2 reduces to the following corollary:

Corollary 2. *For the bounded sequence* $C(m_1, \ldots, m_n)$*, let the function* $f(z_1, \ldots, z_n)$ *be defined by* (2.1)*, then*

$$\begin{aligned}\Omega^* \left\{f(xz_1, \ldots, xz_n)\right\} = x^{\beta_p - 1} \sum_{m_1, \ldots, m_n = 0}^{\infty} C(m_1, \ldots, m_n) \prod_{i=1}^{n} (xz_i)^{m_i} \\ \cdot \prod_{j=1}^{p} \left\{\frac{\Gamma_q(\alpha_j + M_1)}{\Gamma_q(\beta_j + M_1)}\right\},\end{aligned} \tag{2.13}$$

where $\mathrm{Re}(\alpha_j) > 0$ $(j = 1, \ldots, p)$*,* $\max\left\{|xz_1|, \ldots, |xz_n|\right\} < R_1$ $(R_1 > 0)$*,* Ω^* *is a chain of fractional* q*-calculus operators defined by equation* (2.10)*, and* M_1 *is given by* (2.8).

3. Applications of the Main Results

In this section, we consider some consequences and applications of the results derived in Section 2.

It is interesting to observe that in view of the following limiting cases:

$$\lim_{q\to 1^-}\Gamma_q(a)=\Gamma(a) \qquad \text{and} \qquad \lim_{q\to 1^-}\frac{(q^a;q)_n}{(1-q)^n}=(a)_n, \tag{3.1}$$

where

$$(a)_n=a(a+1)\dots(a+n-1), \tag{3.2}$$

the operational formulae (2.7) of Corollary 1 and (2.13) of Corollary 2 above provide, respectively, the q-extensions of the known results due to Raina [**5**, p. 52, eqns. (27) and (26)].

By assigning suitable special values to the arbitrary sequence $C(m_1,\dots,m_n)$, our main results (Theorems 1 and 2) can be applied to derive certain operational formulae for a basic hypergeometric function of several variables involving Riemann- -Liouville and Kober fractional q-integral operators. To illustrate that we consider the following examples.

Example 1. Let us set

(3.3)

$$C(m_1,\dots,m_n)=\frac{\prod_{j=1}^{A}(a_j;q)_{m_1\theta'_j+\dots+m_n\theta_j^{(n)}}\prod_{j=1}^{B'}(b'_j;q)_{m_1\phi'_j}\cdots\prod_{j=1}^{B^{(n)}}(b_j^{(n)};q)_{m_n\phi_j^{(n)}}}{\prod_{j=1}^{C}(c_j;q)_{m_1\psi'_j+\dots+m_n\psi_j^{(n)}}\prod_{j=1}^{D'}(d'_j;q)_{m_1\delta'_j}\cdots\prod_{j=1}^{D^{(n)}}(d_j^{(n)};q)_{m_n\delta_j^{(n)}}}\cdot\frac{1}{(q;q)_{m_1}}\cdots\frac{1}{(q;q)_{m_n}},$$

in (2.1), then in view of (1.7), Theorems 1 and 2 yield the following operational formulae involving the multivariable basic hypergeometric function:

(3.4)

$$\Omega\left\{\Phi^{A:B';\dots;B^{(n)}}_{C:D';\dots;D^{(n)}}\left(\begin{matrix}[(a):\theta',\dots,\theta^{(n)}]:[(b'):\phi'];\dots;[(b^{(n)}):\phi^{(n)}]\\ [(c):\psi',\dots,\psi^{(n)}]:[(d'):\delta'];\dots;[(d^{(n)}):\delta^{(n)}]\end{matrix};q,x^{k_1}z_1,\dots,x^{k_n}z_n\right)\right\}$$

$$=\prod_{j=1}^{p}\left\{\frac{\Gamma_q(\alpha_j+\mu_j)}{\Gamma_q(\alpha_j+\mu_j+\lambda_j)}\right\}x^{\alpha_p-1}\Phi^{A+p:B';\dots;B^{(n)}}_{C+p:D';\dots;D^{(n)}}\left(\begin{matrix}[(a):\theta',\dots,\theta^{(n)}];\\ [(c):\psi',\dots,\psi^{(n)}];\end{matrix}\right.$$

$$\left.\begin{matrix}[(\alpha_p+\mu_p):k_1,\dots,k_n]:[(b'):\phi'];\dots;[(b^{(n)}):\phi^{(n)}]\\ [(\alpha_p+\mu_p+\lambda_p):k_1,\dots,k_n]:[(d'):\delta'];\dots;[(d^{(n)}):\delta^{(n)}]\end{matrix};q,x^{k_1}z_1,\dots,x^{k_n}z_n\right),$$

and

(3.5)

$$\Omega^*\left\{\Phi^{A:B';\ldots;B^{(n)}}_{C:D';\ldots;D^{(n)}}\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}]:[(b'):\phi'];\ldots;[(b^{(n)}):\phi^{(n)}]\\ [(c):\psi',\ldots,\psi^{(n)}]:[(d'):\delta'];\ldots;[(d^{(n)}):\delta^{(n)}]\end{matrix};q,x^{k_1}z_1,\ldots,x^{k_n}z_n\right)\right\}$$

$$= \prod_{j=1}^{p}\left\{\frac{\Gamma_q(\alpha_j}{\Gamma_q(\beta_j)}\right\}x^{\beta_p-1}\Phi^{A+p:B';\ldots;B^{(n)}}_{C+p:D';\ldots;D^{(n)}}\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(n)}];\\ [(c):\psi',\ldots,\psi^{(n)}];\end{matrix}\right.$$

$$\left.\begin{matrix}[(\alpha_p):k_1,\ldots,k_n]:[(b'):\phi'];\ldots;[(b^{(n)}):\phi^{(n)}]\\ [(\beta_p):k_1,\ldots,k_n]:[(d'):\delta'];\ldots;[(d^{(n)}):\delta^{(n)}]\end{matrix};q,x^{k_1}z_1,\ldots,x^{k_n}z_n\right).$$

It is may be noted that for $p=1$ the results (3.4) and (3.5) correspond respectively to the known results due to Yadav, Purohit and Kalla [**13**, p. 60, eqn. (17) and p. 70, eqn. (47)].

Example 2. Next, if we set

$$C(m_1,\ldots,m_n)=\prod_{j=1}^{n}\left\{\frac{(\gamma_j;q)_{m_j}}{(q;q)_{m_j}}\right\} \tag{3.6}$$

in (2.1), then

$$f(xz_1,\ldots,xz_n)=\prod_{j=1}^{n}{}_1\Phi_0\left[\begin{matrix}\gamma_j\\ -\end{matrix};q,xz_j\right], \tag{3.7}$$

then the results (2.7) of Corollary 1 and (2.13) of Corollary 2 yield respectively the following operational formulae involving the basic generalized Kampé de Fériet function of n variables (1.8)

(3.8)

$$\Omega\left\{\prod_{j=1}^{n}{}_1\Phi_0\left[\begin{matrix}\gamma_j\\ -\end{matrix};q,xz_j\right]\right\}=\prod_{j=1}^{p}\left\{\frac{\Gamma_q(\alpha_j+\mu_j)}{\Gamma_q(\alpha_j+\mu_j+\lambda_j)}\right\}x^{\alpha_p-1}$$

$$\Phi^{p:1;\ldots;1}_{p:0;\ldots;0}\left(\begin{matrix}(\alpha_p+\mu_p):\gamma_1;\ldots;\gamma_n\\ (\alpha_p+\mu_p+\lambda_p):\_;\ldots;\_\end{matrix};q,xz_1,\ldots,xz_n\right),$$

and

(3.9)

$$\Omega^*\left\{\prod_{j=1}^{n}{}_1\Phi_0\left[\begin{matrix}\gamma_j\\ -\end{matrix};q,xz_j\right]\right\}=\prod_{j=1}^{p}\left\{\frac{\Gamma_q(\alpha_j)}{\Gamma_q(\beta_j)}\right\}x^{\beta_p-1}$$

$$\Phi^{p:1;\ldots;1}_{p:0;\ldots;0}\left(\begin{matrix}(\alpha_p):\gamma_1;\ldots;\gamma_n\\ (\beta_p):\_;\ldots;\_\end{matrix};q,xz_1,\ldots,xz_n\right).$$

Further, if we put $p = 1$ in (3.9) and replace α_1 and β_1 by $1 + k$ and $1 + k + \alpha$, respectively, then we are led to the result

$$I_q^{\alpha}\left\{x^k \prod_{j=1}^{n} {}_1\Phi_0\left[\begin{array}{c}\gamma_j\\ -\end{array}; q, xz_j\right]\right\} = \frac{\Gamma_q(1+k)}{\Gamma_q(1+k+\alpha)} x^{k+\alpha} \tag{3.10}$$
$$\Phi_D^{(n)}\left[1+k : \gamma_1; \ldots; \gamma_n; 1+k+\alpha;\ q, xz_1, \ldots, xz_n\right],$$

provided that $\mathrm{Re}(\alpha) > 0$, $q < 1$ and $\max\{|xz_1|, \ldots, |xz_n|\} < 1$, where the function $\Phi_D^{(n)}(\cdot)$ denotes the basic Lauricella function defined by

$$\Phi_D^{(n)}\left[a, b_1, \ldots, b_n; c; q; z_1, \ldots, z_n\right] = \sum_{m_1,\ldots,m_n \geq 0} \frac{(a;q)_{m_1+\ldots+m_n}}{(c;q)_{m_1+\ldots+m_n}} \prod_{j=1}^{n}\left\{\frac{(b_j;q)_{m_j}\ z_j^{m_j}}{(q;q)_{m_j}}\right\}, \tag{3.11}$$

where for convergence $|z_1| < 1, \ldots, |z_n| < 1, |q| < 1$. The result (3.10) is the q-extension of the known result due to Srivastava and Goyal [**8**, p. 649, eqn. (3.6)] (see also Saigo and Raina [**6**]).

Example 3. Finally, if we set

$$C(m_1, \ldots, m_n) = \frac{(\alpha;q)_{M_1}}{(\mu;q)_{M_1}} \prod_{j=1}^{n}\left\{\frac{(\sigma_j;q)_{m_j}}{(q;q)_{m_j}}\right\}, \tag{3.12}$$

where as before M_1 is given by (2.8), then (2.7) of Corollary 1 and (2.13) of Corollary 2 yield, respectively the following operational formulae involving the basic Lauricella function $\Phi_D^{(n)}(\cdot)$ defined by (3.11) and the basic Kampé de Fériet function of n variables defined by (1.8)

(3.13)
$$\Omega\left\{\Phi_D^{(n)}\left[\alpha, \sigma_1, \ldots, \sigma_n; \mu; q; xz_1, \ldots, xz_n\right]\right\} = \prod_{j=1}^{p}\left\{\frac{\Gamma_q(\alpha_j+\mu_j)}{\Gamma_q(\alpha_j+\mu_j+\lambda_j)}\right\} x^{\alpha_p - 1}$$
$$\Phi^{p+1:1;\ldots;1}_{p+1:0;\ldots;0}\left(\begin{array}{l}(\alpha_p+\mu_p), \alpha : \sigma_1; \ldots; \sigma_n\\ (\alpha_p+\mu_p+\lambda_p), \mu : \_; \ldots; \_\end{array}; q, xz_1, \ldots, xz_n\right)$$

and

$$\Omega^*\left\{\Phi_D^{(n)}\left[\alpha, \sigma_1, \ldots, \sigma_n; \mu; q; xz_1, \ldots, xz_n\right]\right\} = \prod_{j=1}^{p}\left\{\frac{\Gamma_q(\alpha_j)}{\Gamma_q(\beta_j)}\right\} x^{\beta_p - 1} \tag{3.14}$$
$$\Phi^{p+1:1;\ldots;1}_{p+1:0;\ldots;0}\left(\begin{array}{l}(\alpha_p), \alpha : \sigma_1; \ldots; \sigma_n\\ (\beta_p), \mu : \_; \ldots; \_\end{array}; q, xz_1, \ldots, xz_n\right),$$

provided that both sides of (3.13) and (3.14) exist.

We conclude by the remark that the results established in this paper are in general forms and one can deduce several operational formulae involving the Riemann-Liouville and Kober type fractional q-integral operators associated with the basic

Lauricella functions, basic Kampé de Fériet function, basic Appell functions, basic Horn's functions and basic confluent hypergeometric functions.

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S. D. Purohit, Department of Basic-Sciences (Mathematics), College of Technology and Engineering, M.P. University of Agriculture and Technology, Udaipur-313001, India,
e-mail: `sunil_a_purohit@yahoo.com`

R. K. Raina, 10/11, Ganpati Vihar, Opposite Sector-5, Udaipur-313001, India,
e-mail: `rkraina_7@hotmail.com`