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UNSTEADY FLOW OF A CONDUCTING DUSTY FLUID BETWEEN TWO CIRCULAR CYLINDERS

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Abstract. The present analysis deals with the study of laminar flow of a conducting dusty fluid with uniform distribution of dust particles between two circular cylinders. Initially the fluid and dust particles are at rest. The flow is due to the influence of time dependent pressure gradient and the differential rotations of the circular cylinders. The exact solutions for both fluid and dust velocities are obtained using Variable Separable method. Further the skin friction at the boundaries is calculated. Finally the changes in the velocity profiles with R are shown graphically.

1. Introduction

The influence of dust particles on viscous flows has great importance in petroleum industry and in the purification of crude oil. Other important applications of dust particles in a boundary layer include soil erosion by natural winds and dust entrainment in a cloud during nuclear explosion. Also such flows occur in a wide range of areas of technical importance like fluidization, flow in rocket tubes, combustion, paint spraying and more recently blood flows in capillaries.

P. G. Saffman [**14**] formulated the equations for dusty fluid flow and studied the laminar flow of a dusty gas. Michael and Miller [**13**] investigated the motion of dusty gas with uniform distribution of the dust particles occupied in a cylinder and between two rotating cylinders. Samba Siva Rao [**15**] obtained unsteady flow of a dusty viscous liquid through circular cylinder. E. Amos [**1**] studied magnetic effect on pulsatile flow in a constricted axis-symmetric tube. A. J. Chamkha [**5**] obtained unsteady hydromagnetic flow and heat transfer from a non-isothermal stretching sheet immersed in a porous medium. Datta and Dalal [**6**] obtained solutions for pulsatile flow and heat transfer of a dusty fluid through an infinitely long annular pipe. Liu [**12**] studied flow induced by an oscillating infinite flat plate in a dusty gas. Indrasena [**10**] made the solution of steady rotating hydrodynamic-flows. Girishwar Nath [**9**] studied the dusty viscous Fluid Flow between Rotating Coaxial Cylinders.

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Yang Lei and Bakhtier Farouk [**17**] investigated three-dimensional mixed convection flows in a horizontal annulus with a heated rotating inner circular cylinder. Colette Calmelet-Eluhu and Philip Crooke [**4**] tudied unsteady conducting dusty gas flow through a circular pipe in the presence of an applied and induced magnetic field. The authors Bagewadi and Gireesha [**2**], [**3**] studied two-dimensional dusty fluid flow in Frenet frame field system and recently the authors [**7**], [**8**] obtained solutions for the flow of unsteady dusty fluid under varying time dependent pressure gradients through different regions like parallel plates, rectangular channel and open rectangular channel.

The present investigation deals with the study of unsteady flow of a conducting dusty fluid between two circular cylinders. Here the flow is due to the influence of time dependent pressure gradient and differential rotations of the cylinders. The fluid and dust particles are assumed to be at rest initially. The analytical expressions are obtained for velocities of fluid and dust particles. Further the skin friction at the boundaries is calculated and graphical representation of the velocity profiles versus R is given.

2. Equations of Motion

The equations of motion of conducting unsteady viscous incompressible fluid with uniform distribution of dust particles are given by [**14**]:
For fluid phase

$$\nabla \cdot \overrightarrow{u} = 0 \qquad \text{(Continuity)} \tag{2.1}$$

$$\frac{\partial \overrightarrow{u}}{\partial t} + (\overrightarrow{u} \cdot \nabla)\overrightarrow{u} = -\rho^{-1}\nabla p + \nu \nabla^2 \overrightarrow{u} + \frac{kN}{\rho}(\overrightarrow{v} - \overrightarrow{u}) + \frac{1}{\rho}(\overrightarrow{J} \times \overrightarrow{B}) \tag{2.2}$$
(Linear Momentum)

For dust phase

$$\nabla \cdot \overrightarrow{v} = 0 \qquad \text{(Continuity)} \tag{2.3}$$

$$\frac{\partial \overrightarrow{v}}{\partial t} + (\overrightarrow{v} \cdot \nabla)\overrightarrow{v} = \frac{k}{m}(\overrightarrow{u} - \overrightarrow{v}) \tag{2.4}$$
(Linear Momentum)

We have following nomenclature: $\overrightarrow{u}-$velocity of the fluid phase, $\overrightarrow{v}-$velocity of dust phase, $\rho-$density of the gas, $p-$pressure of the fluid, $N-$number density of dust particles, $\nu-$kinematic viscosity, $k = 6\pi a\mu-$Stoke's resistance (drag coefficient), $a-$spherical radius of dust particle, $m-$mass of the dust particle, $\mu-$the coefficient of viscosity of fluid particles, $t-$time and $\overrightarrow{J}$ and $\overrightarrow{B}$ given by Maxwell's equations and Ohm's law, namely,

$$\nabla \times \overrightarrow{H} = 4\pi \overrightarrow{J}, \quad \nabla \times \overrightarrow{B} = 0, \quad \nabla \times \overrightarrow{E} = 0, \quad \overrightarrow{J} = \sigma[\overrightarrow{E} + \overrightarrow{u} \times \overrightarrow{B}] \tag{2.5}$$

Here $\overrightarrow{H}-$magnetic field, $\overrightarrow{J}-$current density, $\overrightarrow{B}-$magnetic flux, $\overrightarrow{E}-$electric field and $\sigma-$the electrical conductivity of the fluid.

It is assumed that the effect of induced magnetic fields produced by the motion of the electrically conducting gas is negligible and no external electric field is applied. With those assumptions the magnetic field $\overrightarrow{J} \times \overrightarrow{B}$ of the body force in (2.2) reduces simply to $-\sigma B_0^2 \overrightarrow{u}$, where B_0 is the intensity of the imposed transverse magnetic field.

3. Formulation of the Problem

Consider a flow of viscous incompressible, conducting dusty fluid between two circular cylinders. The inner cylinder is of unit radius and outer cylinder is of radius b. The flow is due to the influence of time dependent pressure gradient and differential rotations of the cylinders. It is assumed that the inner and outer cylinders rotate with the different angular velocities. Both the fluid and the dust particle clouds are supposed to be static at the beginning. The dust particles are assumed to be spherical in shape and uniform in size. The number density of the dust particles is taken as a constant throughout the flow. As Figure 1 shows, the axis of the channel is along z-axis and the velocity components of both fluid and dust particles are respectively given by:

$$\left.\begin{aligned} u_r = 0; \quad u_\theta = 0; \quad u_z = u_z(r,t); \\ v_r = 0; \quad v_\theta = 0; \quad v_z = v_z(r,t) \end{aligned}\right\} \tag{3.1}$$

where (u_r, u_θ, u_z) and (v_r, v_θ, v_z) are velocity components of fluid and dust particles, respectively.

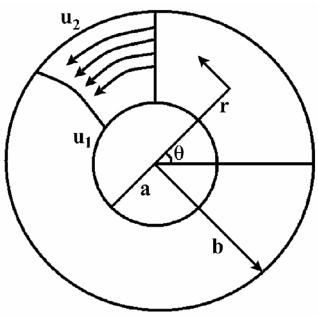

Figure 1. Schematic diagram of the flow.

By virtue of equation (3.1) the intrinsic decomposition of equations (2.1) to (2.4) in cylindrical polar coordinates give the following forms:

$$-\frac{1}{\rho}\frac{\partial p}{\partial r} = 0, \tag{3.2}$$

$$\frac{\partial u_z}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left[\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r}\frac{\partial u_z}{\partial r}\right] + \frac{kN}{\rho}(v_z - u_z) - \frac{\sigma B_0^2}{\rho}u_z, \tag{3.3}$$

$$\frac{\partial v_z}{\partial t} = \frac{k}{m}(u_z - v_z), \tag{3.4}$$

Let us introduce the following non-dimensional quantities:

$$
\begin{aligned}
&R=\frac{r}{a},\quad \bar z=\frac{z}{a},\quad \bar p=\frac{pa^2}{\rho\nu^2},\quad T=\frac{t\nu}{a^2},\quad u=\frac{u_z a}{\nu},\quad v=\frac{v_z a}{\nu},\\
&\beta=\frac{l}{\gamma}=\frac{Nka^2}{\rho\nu},\quad l=\frac{Nm}{\rho},\quad \gamma=\frac{\nu m}{ka^2}.
\end{aligned}
\tag{3.5}
$$

Transform the equations (3.2)–(3.4) to the non-dimensional forms as

$$
-\frac{\nu^2}{a^3}\frac{\partial p}{\partial R}=0,
\tag{3.6}
$$

$$
\frac{\partial u}{\partial T}=-\frac{\partial p}{\partial \bar z}+\left[\frac{\partial^2 u}{\partial R^2}+\frac{1}{R}\frac{\partial u}{\partial R}\right]+\beta(v-u)-M^2u,
\tag{3.7}
$$

$$
\gamma\frac{\partial v}{\partial T}=(u-v),
\tag{3.8}
$$

where $M=B_0a\sqrt{(\sigma/\mu)}=$ Hartmann number.

Since we have assumed that the time dependent pressure gradient is impressed on the system for $t>0$, so we can write

$$
-\frac{1}{\rho}\frac{\partial p}{\partial z}=c+d\,\mathrm{e}^{\mathrm{i}\,\alpha t},
$$

where $c,\ d$ and α are reals.

Eliminating v from (3.7) and (3.8) and then substituting the expression for pressure gradient, one can get

$$
\begin{aligned}
&\gamma\frac{\partial^2 u}{\partial T^2}+(l+1+M^2\gamma)\frac{\partial u}{\partial T}-\gamma\frac{\partial}{\partial T}\left[\frac{\partial^2 u}{\partial R^2}+\frac{1}{R}\frac{\partial u}{\partial R}\right]\\
=&c+d\,\mathrm{e}^{\mathrm{i}\,\alpha t}+\left[\frac{\partial^2 u}{\partial R^2}+\frac{1}{R}\frac{\partial u}{\partial R}\right]-M^2u.
\end{aligned}
\tag{3.9}
$$

4. Solution Part

Let the solution of the equation (3.9) be written in the form [**16**], [**11**]

$$
u=U(R)+V(R,T),
\tag{4.1}
$$

where U is the steady part and V is the unsteady part of the fluid velocity

Separating the steady part from the unsteady part of the equation (3.9),we get

$$
\frac{\partial^2 U}{\partial R^2}+\frac{1}{R}\frac{\partial U}{\partial R}-M^2U=-c,
\tag{4.2}
$$

$$
\begin{aligned}
&\gamma\frac{\partial^2 V}{\partial T^2}+(l+1+M^2\gamma)\frac{\partial V}{\partial T}-\gamma\frac{\partial}{\partial T}\left[\frac{\partial^2 V}{\partial R^2}+\frac{1}{R}\frac{\partial V}{\partial R}\right]\\
&=d\,\mathrm{e}^{\mathrm{i}\,\alpha t}+\left[\frac{\partial^2 V}{\partial R^2}+\frac{1}{R}\frac{\partial V}{\partial R}\right]-M^2V.
\end{aligned}
\tag{4.3}
$$

Case 1. Periodic Motion.

Consider the boundary conditions

$$
\begin{aligned}
u &= u_1 \sin(\alpha T), && \text{at } R = 1,\\
u &= u_2 \sin(\alpha T), && \text{at } R = b,
\end{aligned}
$$

where u_1 and u_2 are uniform angular velocities.

Since $u = U(R) + V(R, T)$, one can see that the boundary conditions become as follows:

$$
\begin{aligned}
U &= 0 \quad \text{and} \quad V = u_1 \sin(\alpha t) && \text{at } R = 1,\\
U &= 0 \quad \text{and} \quad V = u_2 \sin(\alpha t) && \text{at } R = b.
\end{aligned} \tag{4.4}
$$

Now, by solving equation (4.2) using the boundary conditions (4.4), one can get

$$
U = \frac{c}{M^2}\left(\frac{T_1 J_0(MR) + T_2 K_0(MR)}{T_0} - 1\right), \tag{4.5}
$$

where J_0 and K_0 are Bessel's functions of the first and the second kind, respectively, of order zero.

Assume the solution of the equation (4.3) is in the form

$$
V = g(R)\, \mathrm{e}^{\mathrm{i}\,\alpha t}, \tag{4.6}
$$

where $g(R)$ is an unknown function to be determined.

Using equation (4.6) in (4.3), one can obtain

$$
\frac{\partial^2 g}{\partial R^2} + \frac{1}{R}\frac{\partial g}{\partial R} - \lambda_1^2 g = -\lambda_2, \tag{4.7}
$$

where $\lambda_1 = \frac{(M^2 - \gamma\alpha^2) + i\alpha(1+l)}{(1+i\alpha\gamma)}$ and $\lambda_2 = \frac{d}{(1+i\alpha\gamma)}$.

Using the boundary conditions (4.4) one can obtain the solution of (4.7)

$$
\begin{aligned}
g(R) = {} & \frac{\lambda_2}{\lambda_1^2}\left[\frac{Q_1 J_0(\lambda_1 R) + Q_3 K_0(\lambda_1 R)}{Q_0} - 1\right]\\
& + \frac{\sin(\alpha T)}{\mathrm{e}^{\mathrm{i}\,\alpha t}}\left[\frac{Q_2 J_0(\lambda_1 R) + Q_4 K_0(\lambda_1 R)}{Q_0}\right],
\end{aligned} \tag{4.8}
$$

Using this in (4.6), we get

$$
\begin{aligned}
V = {} & \frac{\lambda_2\, \mathrm{e}^{\mathrm{i}\,\alpha t}}{\lambda_1^2}\left[\frac{Q_1 J_0(\lambda_1 R) + Q_3 K_0(\lambda_1 R)}{Q_0} - 1\right]\\
& + \sin(\alpha T)\left[\frac{Q_2 J_0(\lambda_1 R) + Q_4 K_0(\lambda_1 R)}{Q_0}\right].
\end{aligned} \tag{4.9}
$$

Now, using equations (4.9) and (4.5) in (4.1), we obtain the fluid velocity u in the form

$$\begin{aligned} u = {} & \frac{c}{M^2}\left[\frac{T_1 J_0(MR)+T_2K_0(MR)}{T_0}-1\right] \\ & +\frac{\lambda_2\,\mathrm{e}^{\mathrm{i}\,\alpha t}}{\lambda_1^2}\left[\frac{Q_1J_0(\lambda_1 R)+Q_3K_0(\lambda_1 R)}{Q_0}-1\right] \\ & +\sin(\alpha T)\left[\frac{Q_2J_0(\lambda_1 R)+Q_4K_0(\lambda_1 R)}{Q_0}\right], \end{aligned} \tag{4.10}$$

Also, the dust phase velocity is obtained from equation (3.8) as

$$\begin{aligned} v = {} & A\,\mathrm{e}^{-\frac{1}{\gamma}T}+\frac{c}{M^2}\left[\frac{T_1 J_0(MR)+T_2K_0(MR)}{T_0}-1\right] \\ & +\frac{\lambda_2\,\mathrm{e}^{\mathrm{i}\,\alpha t}}{\lambda_1^2(1+\mathrm{i}\,\alpha\gamma)}\left[\frac{Q_1J_0(\lambda_1 R)+Q_3K_0(\lambda_1 R)}{Q_0}-1\right] \\ & +\frac{1}{1+\alpha^2\gamma^2}[\sin\alpha T-\alpha\gamma\cos\alpha T]\left[\frac{Q_2J_0(\lambda_1 R)+Q_4K_0(\lambda_1 R)}{Q_0}\right], \end{aligned} \tag{4.11}$$

where

$$\begin{aligned} A = {} & \frac{\alpha\gamma}{1+\alpha^2\gamma^2}\left[\frac{Q_2J_0(\lambda_1 R)+Q_4K_0(\lambda_1 R)}{Q_0}\right] \\ & -\frac{c}{M^2}\left[\frac{T_1 J_0(MR)+T_2K_0(MR)}{T_0}-1\right] \\ & -\frac{\lambda_2(1-\mathrm{i}\,\alpha\gamma)}{\lambda_1^2(1+\alpha^2\gamma^2)}\left[\frac{Q_1J_0(\lambda_1 R)+Q_3K_0(\lambda_1 R)}{Q_0}-1\right]. \end{aligned}$$

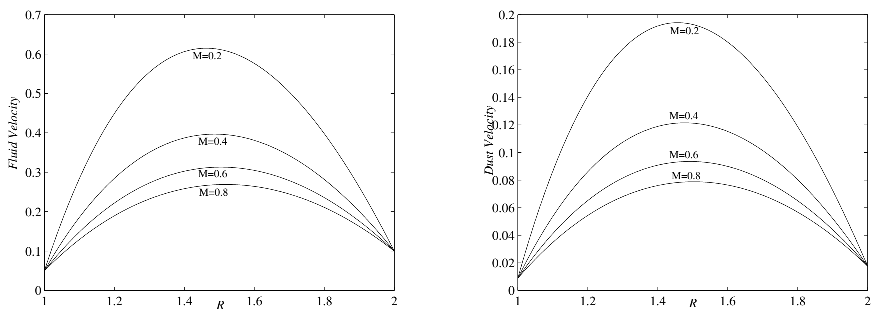

Figure 2. Variation of fluid and dust velocities with R for Case 1.

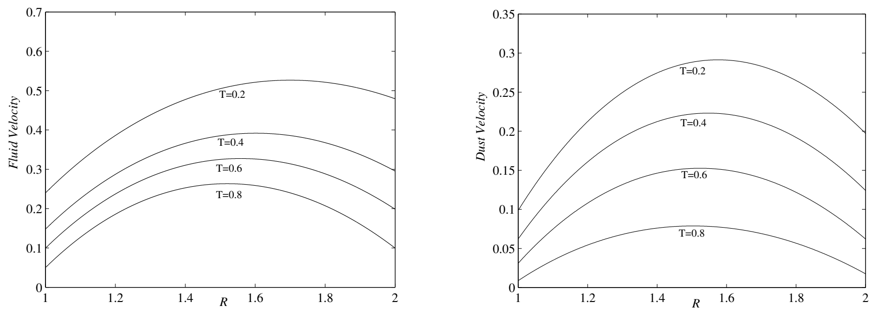

Figure 3. Variation of fluid and dust velocities with R for Case 1.

Shearing Stress (Skin Friction).

The Shear stress at the boundaries $R = 1$ and $R = b$, respectively, is given by

$$D_1 = \frac{\mu c}{M}\left[\frac{T_1 J_0'(M) + T_2 K_0'(M)}{T_0}\right] + \frac{\mu\lambda_2\, \mathrm{e}^{\mathrm{i}\,\alpha t}}{\lambda_1}\left[\frac{Q_1 J_0'(\lambda_1) + Q_3 K_0'(\lambda_1)}{Q_0}\right] + \mu\lambda_1 \sin(\alpha T)\left[\frac{Q_2 J_0'(\lambda_1) + Q_4 K_0'(\lambda_1)}{Q_0}\right]$$

$$D_b = \frac{\mu c}{M}\left[\frac{T_1 J_0'(Mb) + T_2 K_0'(Mb)}{T_0}\right] + \frac{\mu\lambda_2\, \mathrm{e}^{\mathrm{i}\,\alpha t}}{\lambda_1}\left[\frac{Q_1 J_0'(\lambda_1 b) + Q_3 K_0'(\lambda_1 b)}{Q_0}\right] + \mu\lambda_1 \sin(\alpha T)\left[\frac{Q_2 J_0'(\lambda_1 b) + Q_4 K_0'(\lambda_1 b)}{Q_0}\right]$$

Case 2. Impulsive Motion.

In impulsive motion, we consider the boundary conditions

$$u = u_1\delta(T), \qquad \text{at } \ R = 1,$$
$$u = u_2\delta(T), \qquad \text{at } \ R = b,$$

where $\delta(T)$ is the Dirac delta function.

Using these boundary conditions, one can see that the solution for velocities of fluid and dust phases is obtained as

$$u = \frac{c}{M^2}\left[\frac{T_1 J_0(MR) + T_2 K_0(MR)}{T_0} - 1\right] + \frac{\lambda_2\, \mathrm{e}^{\mathrm{i}\,\alpha t}}{\lambda_1^2}\left[\frac{Q_1 J_0(\lambda_1 R) + Q_3 K_0(\lambda_1 R)}{Q_0} - 1\right] + \delta(T)\left[\frac{Q_2 J_0(\lambda_1 R) + Q_4 K_0(\lambda_1 R)}{Q_0}\right], \tag{4.12}$$

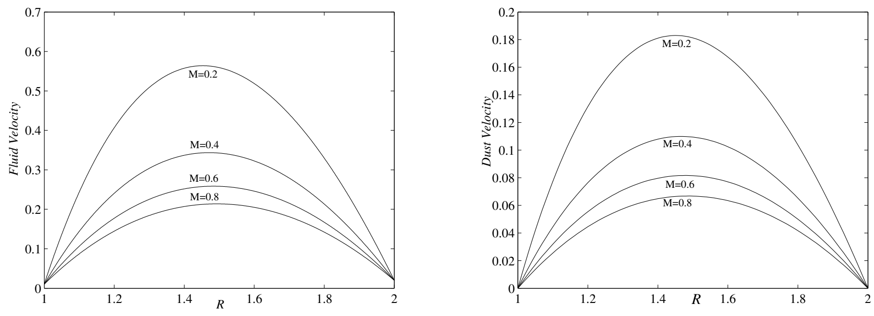

Figure 4. Variation of fluid and dust velocities with R for Case 2.

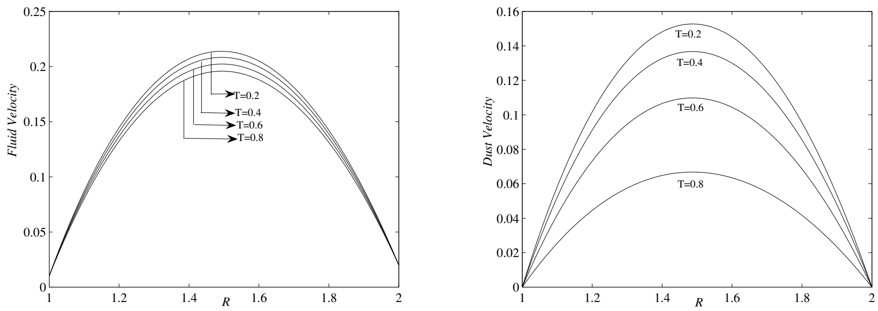

Figure 5. Variation of fluid and dust velocities with R for Case 2.

and

$$
\begin{aligned}
v = {} & \frac{c}{M^2}\left[\frac{T_1 J_0(MR) + T_2 K_0(MR)}{T_0} - 1\right] \\
& + \frac{\lambda_2\, \mathrm{e}^{\mathrm{i}\,\alpha t}}{(1+i\alpha\gamma)\lambda_1^2}\left[\frac{Q_1 J_0(\lambda_1 R) + Q_3 K_0(\lambda_1 R)}{Q_0} - 1\right] \\
& + \frac{\mathrm{e}^{\frac{-1}{\gamma}T}}{\gamma}\left[\frac{Q_2 J_0(\lambda_1 R) + Q_4 K_0(\lambda_1 R)}{Q_0}\right] + A_1\, \mathrm{e}^{-\frac{1}{\gamma}T},
\end{aligned} \tag{4.13}
$$

where

$$
\begin{aligned}
A_1 = {} & -\frac{1}{\gamma}\left[\frac{Q_2 J_0(\lambda_1 R) + Q_4 K_0(\lambda_1 R)}{Q_0}\right] \\
& - \frac{\lambda_2}{\lambda_1^2(1+i\alpha\gamma)}\left[\frac{Q_1 J_0(\lambda_1 R) + Q_3 K_0(\lambda_1 R)}{Q_0} - 1\right] \\
& - \frac{c}{M^2}\left[\frac{T_1 J_0(MR) + T_2 K_0(MR)}{T_0} - 1\right].
\end{aligned}
$$

Shearing Stress (Skin Friction).

The Shear stress, i.e. the skin friction at $R = 1$ and $R = b$, respectively, is given by

$$D_1 = \frac{\mu c}{M}\left[\frac{T_1 J_0'(M) + T_2 K_0'(M)}{T_0}\right] + \frac{\mu\lambda_2 \,\mathrm{e}^{\mathrm{i}\,\alpha t}}{\lambda_1}\left[\frac{Q_1 J_0'(\lambda_1) + Q_3 K_0'(\lambda_1)}{Q_0}\right] + \mu\delta(T)\lambda_1\left[\frac{Q_2 J_0'(\lambda_1) + Q_4 K_0'(\lambda_1)}{Q_0}\right],$$

$$D_b = \frac{\mu c}{M}\left[\frac{T_1 J_0'(Mb) + T_2 K_0'(Mb)}{T_0}\right] + \frac{\mu\lambda_2 \mu}{\lambda_1}\left[\frac{Q_1 J_0'(\lambda_1 b) + Q_3 K_0'(\lambda_1 b)}{Q_0}\right] + \mu\delta(T)\lambda_1\left[\frac{Q_2 J_0'(\lambda_1 b) + Q_4 K_0'(\lambda_1 b)}{Q_0}\right].$$

Case 3. Transition Motion.

For transition motion, we consider the boundary conditions

$$u = u_1 H(T)\,\mathrm{e}^{\alpha T}, \qquad \text{at } R = 1,$$
$$u = u_2 H(T)\,\mathrm{e}^{\alpha T}, \qquad \text{at } R = b,$$

where $H(T)$ is the Heaviside's unit step function.

Using these boundary conditions, the solution for velocities of fluid and dust phases can be written as

$$(4.14)\qquad u = \frac{c}{M^2}\left[\frac{T_1 J_0(MR) + T_2 K_0(MR)}{T_0} - 1\right] + \frac{\lambda_2\,\mathrm{e}^{\mathrm{i}\,\alpha t}}{\lambda_1^2}\left[\frac{Q_1 J_0(\lambda_1 R) + Q_3 K_0(\lambda_1 R)}{Q_0} - 1\right] + H(T)\,\mathrm{e}^{\alpha T}\left[\frac{Q_2 J_0(\lambda_1 R) + Q_4 K_0(\lambda_1 R)}{Q_0}\right]$$

and

$$(4.15)\qquad v = A_2\,\mathrm{e}^{-\frac{1}{\gamma}T} + \frac{c}{M^2}\left[\frac{T_1 J_0(MR) + T_2 K_0(MR)}{T_0} - 1\right] + \frac{\lambda_2\,\mathrm{e}^{\mathrm{i}\,\alpha t}}{\lambda_1^2(1 + i\alpha\gamma)}\left[\frac{Q_1 J_0(\lambda_1 R) + Q_3 K_0(\lambda_1 R)}{Q_0} - 1\right] + \frac{\mathrm{e}^{\frac{-1}{\gamma}T}\left[\mathrm{e}^{(\frac{1}{\gamma}+\alpha)T} - 1\right]}{(1 + \alpha\gamma)} H(T)\left[\frac{Q_2 J_0(\lambda_1 R) + Q_4 K_0(\lambda_1 R)}{Q_0}\right],$$

where

$$A_2 = -\frac{\lambda_2}{\lambda_1^2(1 + i\alpha\gamma)}\left[\frac{Q_1 J_0(\lambda_1 R) + Q_3 K_0(\lambda_1 R)}{Q_0} - 1\right] - \frac{c}{M^2}\left[\frac{T_1 J_0(MR) + T_2 K_0(MR)}{T_0} - 1\right].$$

Shearing Stress (Skin Friction).

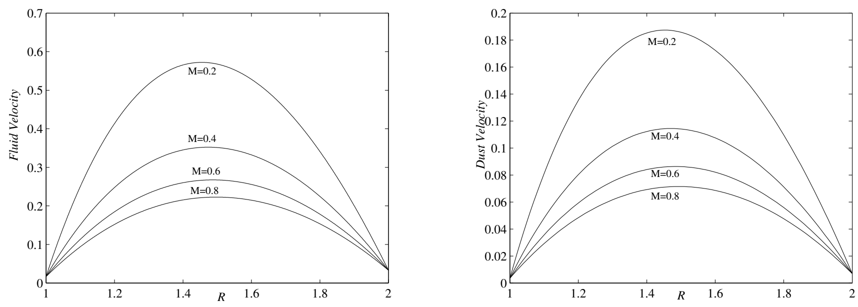

Figure 6. Variation of fluid and dust velocities with R for Case 3.

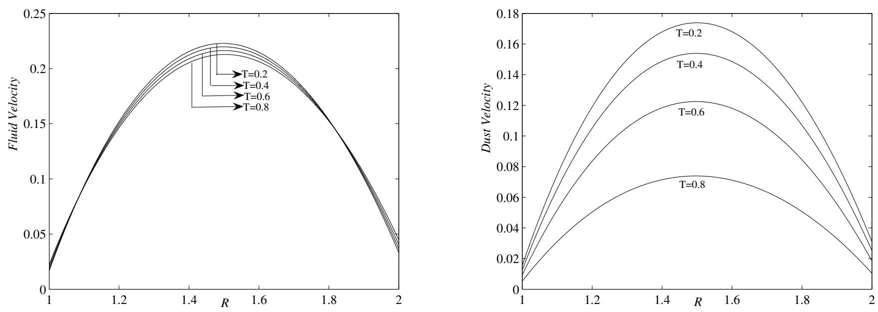

Figure 7. Variation of fluid and dust velocities with R for Case 3.

The skin friction at $R = 1$ and $R = b$, respectively, is given by

$$\begin{aligned} D_1 = {} & \frac{\mu c}{M}\left[\frac{T_1 J_0'(M) + T_2 K_0'(M)}{T_0}\right] + \frac{\mu\lambda_2\, \mathrm{e}^{\mathrm{i}\,\alpha t}}{\lambda_1}\left[\frac{Q_1 J_0'(\lambda_1) + Q_3 K_0'(\lambda_1)}{Q_0}\right] \\ & + \mu\lambda_1 H(T) e^{\alpha T}\left[\frac{Q_2 J_0'(\lambda_1) + Q_4 K_0'(\lambda_1)}{Q_0}\right], \end{aligned}$$

$$\begin{aligned} D_b = {} & \frac{\mu c}{M}\left[\frac{T_1 J_0'(Mb) + T_2 K_0'(Mb)}{T_0}\right] + \frac{\mu\lambda_2\, \mathrm{e}^{\mathrm{i}\,\alpha t}}{\lambda_1}\left[\frac{Q_1 J_0'(\lambda_1 b) + Q_3 K_0'(\lambda_1 b)}{Q_0}\right] \\ & + \mu\lambda_1 H(T) e^{\alpha T}\left[\frac{Q_2 J_0'(\lambda_1 b) + Q_4 K_0'(\lambda_1 b)}{Q_0}\right]. \end{aligned}$$

Case 4. Motion for a Finite Time.

For this case, we consider the boundary conditions

$$\begin{aligned} u &= u_1[H(T) - H(T-t)], \qquad \text{at} \;\; R = 1, \\ u &= u_2[H(T) - H(T-t)], \qquad \text{at} \;\; R = b, \end{aligned}$$

where $H(T)$ is the Heaviside step function.

Using these boundary conditions, we found the solution for velocities of fluid and dust phases as follows:

$$\begin{aligned} u = {} & \frac{c}{M^2}\left[\frac{T_1 J_0(MR) + T_2 K_0(MR)}{T_0} - 1\right] \\ & + \frac{\lambda_2 \, \mathrm{e}^{\mathrm{i}\,\alpha t}}{\lambda_1^2}\left[\frac{Q_1 J_0(\lambda_1 R) + Q_3 K_0(\lambda_1 R)}{Q_0} - 1\right] \\ & + [H(T) - H(T-t)]\left[\frac{Q_2 J_0(\lambda_1 R) + Q_4 K_0(\lambda_1 R)}{Q_0}\right] \end{aligned} \tag{4.16}$$

and

$$\begin{aligned} v = {} & A_3 \, \mathrm{e}^{-\frac{1}{\gamma}T} + \frac{c}{M^2}\left[\frac{T_1 J_0(MR) + T_2 K_0(MR)}{T_0} - 1\right] \\ & + \frac{\lambda_2 \, \mathrm{e}^{\mathrm{i}\,\alpha t}}{\lambda_1^2(1 + i\alpha\gamma)}\left[\frac{Q_1 J_0(\lambda_1 R) + Q_3 K_0(\lambda_1 R)}{Q_0} - 1\right] \\ & + \frac{\mathrm{e}^{\frac{-1}{\gamma}T}[\mathrm{e}^{(\frac{1}{\gamma}+\alpha)T} - 1]}{(1+\alpha\gamma)}[H(T) - H(T-t)]\left[\frac{Q_2 J_0(\lambda_1 R) + Q_4 K_0(\lambda_1 R)}{Q_0}\right], \end{aligned} \tag{4.17}$$

where

$$\begin{aligned} A_3 = {} & -\frac{\lambda_2}{\lambda_1^2(1 + i\alpha\gamma)}\left[\frac{Q_1 J_0(\lambda_1 R) + Q_3 K_0(\lambda_1 R)}{Q_0} - 1\right] \\ & - \frac{c}{M^2}\left[\frac{T_1 J_0(MR) + T_2 K_0(MR)}{T_0} - 1\right]. \end{aligned}$$

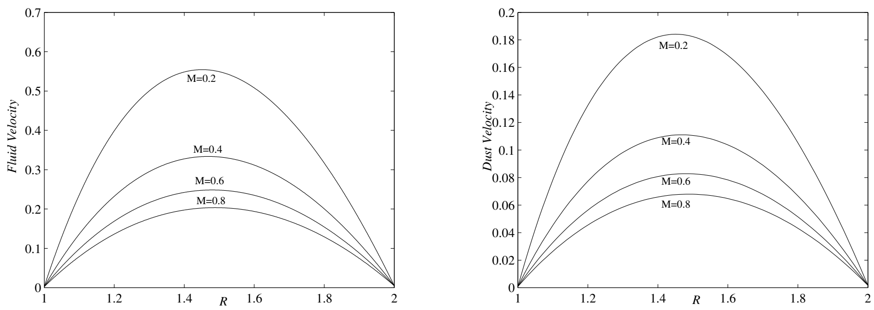

Figure 8. Variation of fluid and dust velocities with R for Case 4.

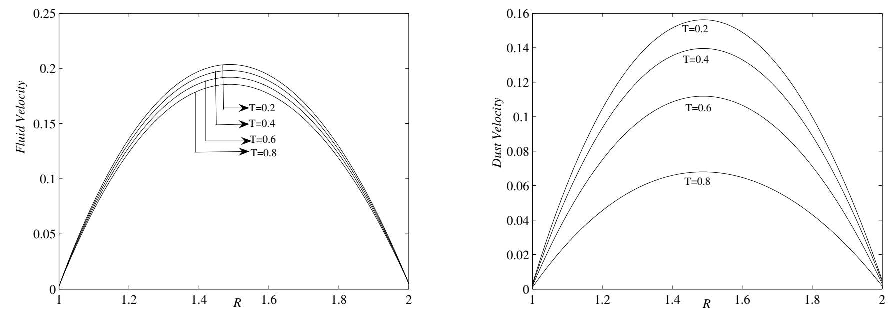

Figure 9. Variation of fluid and dust velocities with R for Case 4.

Shearing Stress (Skin Friction).

The Shear stress, i.e. the skin friction at the boundaries $R = 1$ and $R = b$, respectively, is given by

$$D_1 = \frac{\mu c}{M}\left[\frac{T_1 J_0'(M) + T_2 K_0'(M)}{T_0}\right] + \frac{\mu\lambda_2\,\mathrm{e}^{\mathrm{i}\,\alpha t}}{\lambda_1}\left[\frac{Q_1 J_0'(\lambda_1) + Q_3 K_0'(\lambda_1)}{Q_0}\right] + \mu\lambda_1[H(T) - H(T-t)]\left[\frac{Q_2 J_0'(\lambda_1) + Q_4 K_0'(\lambda_1)}{Q_0}\right],$$

$$D_b = \frac{\mu c}{M}\left[\frac{T_1 J_0'(Mb) + T_2 K_0'(Mb)}{T_0}\right] + \frac{\mu\lambda_2\,\mathrm{e}^{\mathrm{i}\,\alpha t}}{\lambda_1}\left[\frac{Q_1 J_0'(\lambda_1 b) + Q_3 K_0'(\lambda_1 b)}{Q_0}\right] + \mu\lambda_1[H(T) - H(T-t)]\left[\frac{Q_2 J_0'(\lambda_1 b) + Q_4 K_0'(\lambda_1 b)}{Q_0}\right],$$

where

$T_0 = J_0(M)K_0(Mb) - J_0(Mb)K_0(M)$, $T_1 = K_0(Mb) - K_0(M)$,
$T_2 = J_0(M) - J_0(Mb)$,
$Q_0 = J_0(\lambda_1)K_0(\lambda_1 b) - J_0(\lambda_1 b)K_0(\lambda_1)$, $Q_1 = K_0(\lambda_1 b) - K_0(\lambda_1)$,
$Q_2 = u_1 K_0(\lambda_1 b) - u_2 K_0(\lambda_1)$, $Q_3 = J_0(\lambda_1) - J_0(\lambda_1 b)$,
$Q_4 = u_2 J_0(\lambda_1) - u_1 J_0(\lambda_1 b)$.

5. Conclusion

In the present paper, we have studied the laminar flow of a conducting dusty fluid between two circular cylinders. The four different cases based on the time dependent pressure gradient are discussed. Variable separable and Eigen expansion methods are employed to solve the governing equations. The graphs for velocity profiles are shown as in Figures from 2 to 9 for different values of parameters like Hartmann number (M) and Time (T) showing that they are parabolic in nature. From Figures 2, 4, 6 and 8 one can observed the appreciable effect of Hartmann number on the flow of both fluid and dust phases, i.e. the magnetic field has retarding influence. Also, it is evident from the Figures 3, 5, 7 and 9 that as time increases the velocities of both phases decrease, which is desirable in physical situations. Further, we can see that if $\gamma \to 0$, i.e. if the dust is very fine, then the velocities of both fluid and dust particles will be the same.

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