Acta Math. Univ. Comenianae
Vol. LXXXII, 1 (2013), pp. 29–37

TOTAL VERTEX IRREGULARITY STRENGTH OF CONVEX POLYTOPE GRAPHS

O. AL-MUSHAYT, A. ARSHAD AND M. K. SIDDIQUI

Abstract. A total vertex irregular k-labeling ϕ of a graph G is a labeling of the vertices and edges of G with labels from the set $\{1, 2, \dots, k\}$ in such a way that for any two different vertices x and y their weights $wt(x)$ and $wt(y)$ are distinct. Here, the weight of a vertex x in G is the sum of the label of x and the labels of all edges incident with the vertex x. The minimum k for which the graph G has a vertex irregular total k-labeling is called the *total vertex irregularity strength* of G.

We have determined an exact value of the total vertex irregularity strength of some convex polytope graphs.

1. Introduction

Let us consider a simple (without loops and multiple edges) undirected graph $G = (V, E)$. For a graph G we define a labeling $\phi\colon V \cup E \to \{1, 2, \dots, k\}$ to be a total vertex irregular k-labeling of the graph G if for every two different vertices x and y of G one has $wt(x) \neq wt(y)$ where the weight of a vertex x in the labeling ϕ is

$$wt(x) = \phi(x) + \sum_{y \in N(x)} \phi(xy),$$

where $N(x)$ is the set of neighbors of x. In [**6**], Bača, Jendroľ, Miller and Ryan defined a new graph invariant $\mathrm{tvs}(G)$, called the *total vertex irregularity strength* of G and determined as the minimum k for which the graph G has a vertex irregular total k-labeling.

The original motivation for the definition of the total vertex irregularity strength came from irregular assignments and the irregularity strength of graphs introduced in [**9**] by Chartrand, Jacobson, Lehel, Oellermann, Ruiz and Saba, and studied by numerous authors [**8, 10, 11, 12, 14**].

An *irregular assignment* is a k-labeling of the edges

$$f\colon E \to \{1, 2, \dots, k\}$$

Received February 14, 2012.
2010 *Mathematics Subject Classification.* Primary 05C78.
Key words and phrases. Vertex irregular total k-labeling; total vertex irregularity strength; cycles, convex polytope graphs.

such that the vertex weights

$$w(x) = \sum_{y \in N(x)} f(xy)$$

are different for all vertices of G, and the smallest k for which there is an irregular assignment is the *irregularity strength*, $s(G)$.

The irregularity strength $s(G)$ can be interpreted as the smallest integer k for which G can be turned into a multigraph G' by replacing each edge by a set of at most k parallel edges, such that the degrees of the vertices in G' are all different.

It is easy to see that irregularity strength $s(G)$ of a graph G is defined only for graphs containing at most one isolated vertex and no connected component of order 2. On the other hand, the total vertex irregularity strength $\mathrm{tvs}(G)$ is defined for every graph G.

If an edge labeling $f\colon E \to \{1, 2, \dots, s(G)\}$ provides the irregularity strength $s(G)$, then we extend this labeling to total labeling ϕ in such a way

$$\begin{aligned} \phi(xy) &= f(xy) \quad \text{for every } xy \in E(G),\\ \phi(x) &= 1 \quad \text{for every } x \in V(G). \end{aligned}$$

Thus, the total labeling ϕ is a vertex irregular total labeling and for graphs with no component of order ≤ 2 is $\mathrm{tvs}(G) \le s(G)$.

Nierhoff [**15**] proved that for all (p, q)-graphs G with no component of order at most 2 and $G \neq K_3$, the irregularity strength $s(G) \le p - 1$. From this result, it follows that

$$\mathrm{tvs}(G) \le p - 1. \tag{1}$$

In [**6**], several bounds and exact values of $\mathrm{tvs}(G)$ were determined for different types of graphs (in particular for stars, cliques and prisms). Among others, the authors proved the following theorem

Theorem 1.1. *Let G be a (p, q)-graph with minimum degree $\delta = \delta(G)$ and maximum degree $\Delta = \Delta(G)$. Then*

$$\left\lceil \frac{p + \delta}{\Delta + 1} \right\rceil \le \mathrm{tvs}(G) \le p + \Delta - 2\delta + 1. \tag{2}$$

In the case of r-regular graphs we therefore obtain

$$\left\lceil \frac{p + r}{r + 1} \right\rceil \le \mathrm{tvs}(G) \le p - r + 1. \tag{3}$$

For graphs with no component of order ≤ 2, Bača *et al.* in [**6**] strengthened also these upper bounds proving that

$$\mathrm{tvs}(G) \le p - 1 - \left\lceil \frac{p - 2}{\Delta + 1} \right\rceil. \tag{4}$$

These results were then improved by Przybylo in [**16**] for sparse graphs and for graphs with large minimum degree. In the latter case the bounds

$$\mathrm{tvs}(G) < 32\, \frac{p}{\delta} + 8 \tag{5}$$

in general and

$$\text{tvs}(G) < 8\,\frac{p}{r} + 3 \tag{6}$$

for r-regular (p, q)-graphs were proved to hold.

In [**5**], Anholcer, Kalkowski and Przybylo established a new upper bound of the form

$$\text{tvs}(G) \leq 3\frac{p}{\delta} + 1. \tag{7}$$

Moreover, Ahmad et. al [**1**] determined the lower bound of total vertex irregularity strength of any graph and conjectured that the lower bound is tight.Wijaya and Slamin [**17**] found the exact values of the total vertex irregularity strength of wheels, fans, suns and friendship graphs. Wijaya, Slamin, Surahmat and Jendroľ [**18**] determined an exact value for complete bipartite graphs. Furthermore, Ahmad et. al [**2, 3, 4**] found an exact value of the total vertex irregularity strength for Jahangir graphs, circulant graphs, convex polytope and wheel related graphs.

The main aim of this paper is determined an exact value of the total vertex irregularity strength of some convex polytope graphs.

2. Main Results

The graph of convex polytope (double antiprism) $\mathbf{A_n}$ can be obtained from the graph of convex polytope R_n [**7**] by adding new edges $b_{i+1}c_i$. i.e., $V(\mathbf{A_n}) = V(R_n)$ and $E(\mathbf{A_n}) = E(R_n) \cup \{b_{i+1}c_i : 1 \leq i \leq n\}$ (Figure 1). Let $V(\mathbf{A_n}) = \{a_i, b_i, c_i :$

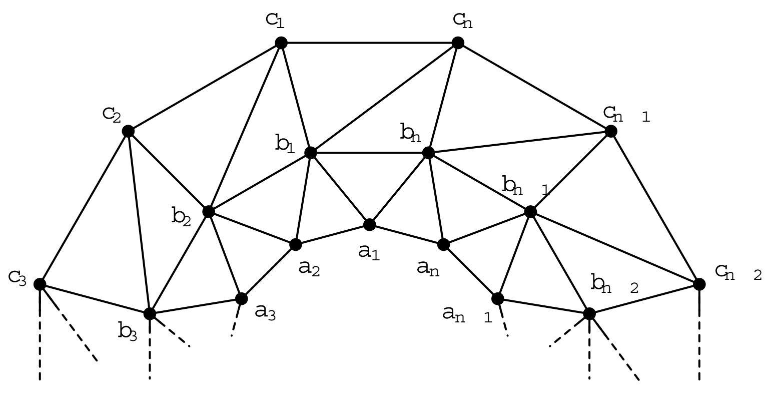

Figure 1. The graph of convex polytope (double antiprism) $\mathbf{A_n}$.

$1 \leq i \leq n\}$ and $E(\mathbf{A_n}) = \{a_ia_{i+1}, b_ib_{i+1}, c_ic_{i+1}, a_ib_i,\ b_ic_i, b_ia_{i+1}, c_ib_{i+1} : 1 \leq i \leq n\}$ be the vertex set and the edge set, respectively. The value of i is taken (mod n).

The first main result of this paper is the following

Theorem 2.1. *Let $n \geq 5$ and $\mathbf{A_n}$ be the convex polytope (double antiprism). Then*

$$\operatorname{tvs}(\mathbf{A_n}) = \left\lceil \frac{3n+4}{7} \right\rceil.$$

Proof. The convex polytope $\mathbf{A_n}$ has $2n$ vertices of degree 4 and n vertices of degree 6.

To prove the lower bound, we consider the weights of the vertices. The smallest weight among all vertices of $\mathbf{A_n}$ is at least 5, so the largest weight of a vertex of degree 4 is at least $2n+4$. Since the weight of any vertex of degree 4 is the sum of five positive integers, so at least one label is at least $\lceil \frac{2n+4}{5} \rceil$.

Moreover, the largest value among the weights of vertices of degrees 4 and 6, is at least $3n+4$, and this weight is the sum of at most 7 integers. Hence the largest label contributing to this weight must be at least $\lceil \frac{3n+4}{7} \rceil$.

Consequently, the largest label of a vertex or an edge of $\mathbf{A_n}$ is at least $\max\left\{\lceil \frac{2n+4}{5} \rceil, \lceil \frac{3n+4}{7} \rceil\right\} = \lceil \frac{3n+4}{7} \rceil$ for $n \geq 5$. Thus $\operatorname{tvs}(\mathbf{A_n}) \geq \lceil \frac{3n+4}{7} \rceil$. Let $k = \lceil \frac{3n+4}{7} \rceil$. It is enough to describe a suitable vertex irregular total k-labeling.

We will distinguish two cases and define a labeling $\phi\colon V(\mathbf{A_n}) \cup E(\mathbf{A_n}) \to \{1, 2, \ldots, k\}$ as follows:

<u>When $n > 2k-1$</u>

Let $x = \max\{2, n-2k+3\}$

$$\phi(a_i) = \begin{cases} x & \text{if } 1 \leq i \leq k \\ x+1 & \text{if } i = n, k+1 \leq i \leq 2k-1 \\ x+i+2-2k & \text{if } 2k \leq i \leq n-1 \end{cases}$$

$$\phi(b_i) = \begin{cases} \max\{x, 2n+3-4k\} & \text{if } i = 1 \\ \max\{x-1, x+n-2k-1\} & \text{if } 2 \leq i \leq k \\ \max\{x+1, 2n-4k+4\} & \text{if } k+1 \leq i \leq 2k \\ \max\{x+1+i-2k, 2n-6k+4+i\} & \text{if } 2k+1 \leq i \leq n \end{cases}$$

$$\phi(a_i a_{i+1}) = \begin{cases} \min\{\lceil \frac{i+1}{2} \rceil, k\} & \text{if } 1 \leq i \leq n-2 \\ k & \text{if } i = n-1 \\ 1 & \text{if } i = n \end{cases}$$

$$\phi(c_i b_{i+1}) = \begin{cases} \max\{1, i-k+1\} & \text{if } 1 \leq i \leq 2k-1 \\ k & \text{if } 2k \leq i \leq n \end{cases}$$

$$\phi(c_i) = \max\{1, i-2k+2\} \qquad \text{for } 1 \leq i \leq n$$

$$\phi(c_i c_{i+1}) = 1 \qquad \text{for } 1 \leq i \leq n$$

$$\phi(b_i b_{i+1}) = \phi(a_i b_i) = \phi(b_i a_{i+1}) = k \qquad \text{for } 1 \leq i \leq n$$

$$\phi(b_i c_i) = \min\{i, k\} \qquad \text{for } 1 \leq i \leq n$$

The weights of vertices of $\mathbf{A_n}$ are as follows:

$$wt(c_i) = \quad i+4 \qquad \text{for} \quad 1 \le i \le n$$

$$wt(a_i) = \quad n+4+i \quad \text{for} \quad 1 \le i \le n$$

$$wt(b_i) = \begin{cases} 2n+4+k & \text{for} \quad i = 1 \\ 2n+4+i-1 & \text{for} \quad 2 \le i \le k \\ 2n+4+i & \text{for} \quad k+1 \le i \le n \end{cases}$$

<u>When $n \le 2k-1$</u>

$$\phi(a_i) = \begin{cases} 2 & \text{if} \quad 1 \le i \le k \\ 3 & \text{if} \quad k+1 \le i \le n \end{cases}$$

$$\phi(b_i) = \begin{cases} 2 & \text{if} \quad i = 1 \\ 1 & \text{if} \quad 2 \le i \le k \\ 3 & \text{if} \quad k+1 \le i \le n-1 \\ k & \text{if} \quad i = n \end{cases}$$

$$\phi(c_i) = \max\{1, i-2k+2\} \qquad \text{for } 1 \le i \le n$$

$$\phi(c_i c_{i+1}) = 1 \qquad \text{for } 1 \le i \le n$$

$$\phi(b_i b_{i+1}) = \phi(a_i b_i) = \phi(b_i a_{i+1}) = k \qquad \text{for } 1 \le i \le n$$

$$\phi(c_i b_{i+1}) = \min\{i, k\} \qquad \text{for } 1 \le i \le n$$

$$\phi(b_i c_i) = \max\{1, i-k+1\} \qquad \text{for } 1 \le i \le n$$

$$\phi(a_i a_{i+1}) = \begin{cases} \min\{\lceil \frac{i+1}{2} \rceil, k\} & \text{if} \quad 1 \le i \le n-2 \\ k & \text{if} \quad i = n-1 \\ 1 & \text{if} \quad i = n \end{cases}$$

Thus, the weights of vertices a_i, c_i, $1 \le i \le n$, successively attain values $5, 6, \dots, 2n+4$ and the weights of vertices b_i, $1 \le i \le n$, receive distinct values from $2n+5$ up to $n+5k+1$.

The labeling ϕ is the desired vertex irregular total k-labeling and provides the upper bound on $\text{tvs}(\mathbf{A_n})$. Combining with the lower bound, we conclude that

$$\text{tvs}(\mathbf{A_n}) = \left\lceil \frac{3n+4}{7} \right\rceil.$$

□

The graph of convex polytope $\mathbf{B_n}$ consisting of 3-sided faces, 4-sided faces and n-sided faces was defined in [**13**] (Figure 2).

The following theorem gives the exact value of the total vertex irregularity strength for convex polytope $\mathbf{B_n}$.

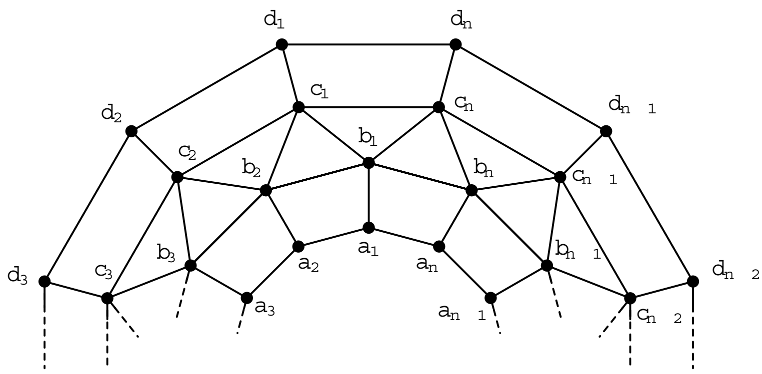

Figure 2. The graph of convex polytope $\mathbf{B_n}$.

Theorem 2.2. *The graph of convex polytope* $\mathbf{B_n}$ *with* $n > 7$ *satisfies*

$$\operatorname{tvs}(\mathbf{B_n}) = \left\lceil \frac{4n+3}{6} \right\rceil.$$

Proof. Let $V(\mathbf{B_n}) = \{a_i, b_i, c_i, d_i : 1 \le i \le n\}$ be the vertex set and $E(\mathbf{B_n}) = \{a_i a_{i+1}, b_i b_{i+1}, c_i c_{i+1}, d_i d_{i+1}, a_i b_i, b_i c_i, c_i d_i, c_i b_{i+1} : 1 \le i \le n\}$ be the edge set of of convex polytope $\mathbf{B_n}$. The value of i is taken $\pmod n$.

Thus the convex polytope $\mathbf{B_n}$ has $2n$ vertices of degree 3 and $2n$ vertices of degree 5. The smallest weight among all vertices of convex polytope $\mathbf{B_n}$ is at least 4. The largest weight of vertices of degree 3 is at least $2n+3$ and this weight is the sum of four integers. Hence the largest label contributing to this weight must be at least $\lceil \frac{2n+3}{4} \rceil$. Moreover, the largest value among the weights of vertices of degree 3 and 5 is at least $4n+3$ and this weight is the sum of at most six integers, so at least one label is at least $\lceil \frac{4n+3}{6} \rceil$. Consequently, the largest label of one of vertex or edge of convex polytope $\mathbf{B_n}$ is at least $\max\left\{\lceil \frac{2n+3}{4} \rceil, \lceil \frac{4n+3}{6} \rceil\right\} = \lceil \frac{4n+3}{6} \rceil$ for $n > 7$. Thus

$$\operatorname{tvs}(\mathbf{B_n}) \ge \left\lceil \frac{4n+3}{6} \right\rceil.$$

Put $k = \lceil \frac{4n+3}{6} \rceil$. To show that k is an upper bound for total vertex irregularity strength of convex polytope $\mathbf{B_n}$, we describe a total k-labeling $\phi : V(\mathbf{B_n}) \cup E(\mathbf{B_n}) \to \{1, 2, \ldots, k\}$ as follows:

$$\begin{aligned}
\phi(b_i) = \phi(d_i) &= \max\{1, i-k+1\} && \text{for} \quad 1 \le i \le n \\
\phi(d_i d_{i+1}) &= 1 && \text{for} \quad 1 \le i \le n \\
\phi(b_i b_{i+1}) &= \max\{1, n-k+1\} && \text{for} \quad 1 \le i \le n \\
\phi(a_i b_i) = \phi(c_i d_i) &= \min\{i, k\} && \text{for} \quad 1 \le i \le n \\
\phi(b_i c_i) = \phi(c_i c_{i+1}) = \phi(c_i b_{i+1}) &= k && \text{for} \quad 1 \le i \le n
\end{aligned}$$

$$\phi(c_i) = \max\{3n+3-4k, 3n+3-5k+i\} \qquad \text{for} \quad 1 \le i \le n$$

For $n \equiv 1 \pmod 2$,

$$\phi(a_i) = \begin{cases} \max\{1, i-k+1\} & \text{if } 1 \le i \le n-1 \\ k-1 & \text{if } i = n \end{cases}$$

$$\phi(a_i a_{i+1}) = \begin{cases} k & \text{if } 1 \le i \le n-1 \text{ odd} \\ \max\{2, n-k+2\} & \text{if } i = n \text{ and } 1 \le i \le n-1 \text{ even} \end{cases}$$

For $n \equiv 0 \pmod 2$,

$$\phi(a_i) = \max\{1, i-k+1\} \quad \text{if } 1 \le i \le n$$

$$\phi(a_i a_{i+1}) = \begin{cases} k & i \text{ odd} \\ \max\{2, n-k+2\} & i \text{ even} \end{cases}$$

Observe that under the labeling ϕ the weights of the vertices of convex polytope $\mathbf{B_n}$ are:

$$\begin{aligned} wt(d_i) &= i+3 && \text{for } 1 \le i \le n \\ wt(a_i) &= n+i+3 && \text{for } 1 \le i \le n \\ wt(b_i) &= 2n+i+3 && \text{for } 1 \le i \le n \\ wt(c_i) &= 3n+i+3 && \text{for } 1 \le i \le n \end{aligned}$$

Thus the labeling ϕ is the desired vertex irregular total k-labeling. □

The graph of convex polytope $\mathbf{D_n}$ consisting of 3-sided faces, 5-sided faces and n-sided faces can be obtained from the graph of convex polytope Q_n [**7**] by adding new edges $a_{i+1}b_i$. i.e., $V(\mathbf{D_n}) = V(Q_n)$ and $E(\mathbf{D_n}) = E(Q_n) \cup \{a_{i+1}b_i : 1 \le i \le n\}$. (Figure 3) Let $V(\mathbf{D_n}) = \{a_i, b_i, c_i, d_i : 1 \le i \le n\}$ and $E(\mathbf{D_n}) = \{a_i a_{i+1}, b_i b_{i+1}, d_i d_{i+1}, a_i b_i, b_i c_i, c_i d_i, c_i b_{i+1}, b_i a_{i+1} : 1 \le i \le n\}$ be the vertex set and the edge set, respectively. The value of i is taken $\pmod n$. The following

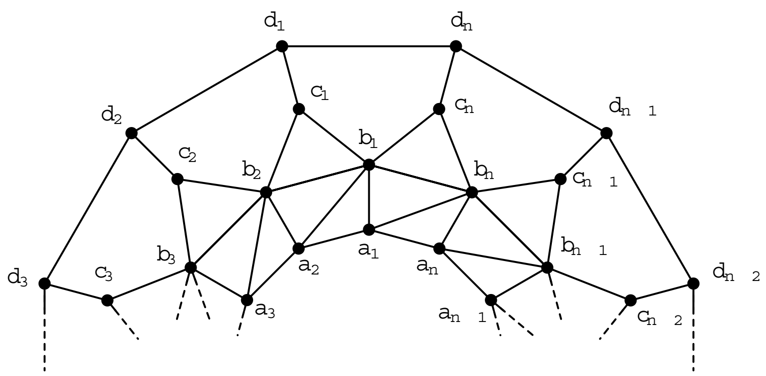

Figure 3. The graph of convex polytope $\mathbf{D_n}$.

lemma gives the lower and upper bound for total vertex irregularity strength of the graph of convex polytope $\mathbf{D_n}$.

Lemma 2.3. *Let $n \geq 4$ and $\mathbf{D_n}$ be the graph of convex polytope. Then*

$$\max\left\{\left\lceil\frac{2n+3}{4}\right\rceil, \left\lceil\frac{3n+3}{5}\right\rceil, \left\lceil\frac{4n+3}{7}\right\rceil\right\} \leq \mathrm{tvs}(\mathbf{D_n}) \leq 4n-1.$$

Proof. The convex polytope $\mathbf{D_n}$ contains $2n$ vertices of degree three, n vertices of degree four and n vertices of degree six. The upper bound of $\mathbf{D_n}$ follows from (1).

Now we consider the weights of the vertices. The smallest weight among all vertices of $\mathbf{D_n}$ is at least four, so the largest weight of vertex of degree three is at least $2n+3$. This weight is the sum of four labels, so at least one label is at least $\lceil\frac{2n+3}{4}\rceil$.

The largest value among the weights of vertices of degree three and four is at least $3n+3$ and this weight is the sum of at most five integers. Hence the largest label contributing to this weight must be at least $\lceil\frac{3n+3}{5}\rceil$.

If we consider all vertices of $\mathbf{D_n}$ then the lower bound $\lceil\frac{4n+3}{7}\rceil$ follows from (2). This gives

$$\max\left\{\left\lceil\frac{2n+3}{4}\right\rceil, \left\lceil\frac{3n+3}{5}\right\rceil, \left\lceil\frac{4n+3}{7}\right\rceil\right\} \leq \mathrm{tvs}(\mathbf{D_n})$$

and we are done. □

Next theorem gives an exact value of the total vertex irregularity strength of $\mathbf{D_n}$ for $n \geq 4$.

Theorem 2.4. *Let $n \geq 4$ and $\mathbf{D_n}$ be convex polytope. Then*

$$\mathrm{tvs}(\mathbf{D_n}) = \left\lceil\frac{3n+3}{5}\right\rceil.$$

Proof. The convex polytope $\mathbf{D_n}$ has $2n$ vertices of degree three, n vertices of degree four and n vertices of degree six. Let $k = \lceil\frac{3n+3}{5}\rceil$. Lemma 2.3 gives the lower bound of the total vertex irregularity strength, i.e., $\mathrm{tvs}(\mathbf{D_n}) \geq \lceil\frac{3n+3}{5}\rceil$.

We define a labeling $\phi : V(\mathbf{D_n}) \cup E(\mathbf{D_n}) \to \{1, 2, \dots, k\}$ in the following way

$$\phi(a_i) = \begin{cases} \max\{1, 2n-3k+3\} & \text{for } 1 \leq i \leq k \\ \max\{i+1-k, i-4k+2n+3\} & \text{for } k+1 \leq i \leq n \end{cases}$$

$$\begin{aligned} \phi(b_i) = \phi(d_i) = \;& \max\{1, i-k+1\} && \text{for } 1 \leq i \leq n \\ \phi(c_i) = \;& \max\{1, i-3k+n+3\} && \text{for } 1 \leq i \leq n \\ \phi(d_i d_{i+1}) = \;& 1 && \text{for } 1 \leq i \leq n \\ \phi(a_i b_i) = \phi(c_i d_i) = \;& \min\{i, k\} && \text{for } 1 \leq i \leq n \end{aligned}$$

$$\phi(a_i a_{i+1}) = \phi(b_i a_{i+1}) = \phi(b_i b_{i+1}) = \phi(c_i b_{i+1}) = k \quad \text{for } 1 \leq i \leq n$$

$$\phi(b_i c_i) = \begin{cases} n-k+2 & \text{for } 1 \leq i \leq k \\ \min\{n-2k+2+i, k\} & \text{for } k+1 \leq i \leq n \end{cases}$$

It is easy to see that the weights of the vertices are different. Thus the labeling ϕ is the desired vertex irregular total k-labeling. □

References

1. Ahmad A., Baskoro E. T., and Imran M., *Total vertex irregularity strength of disjoint union of Halm graphs*, Discuss. Math. Graph Theory, **32(3)** (2012), 427–434.
2. Ahmad A., tvs *of convex polytope graphs with pendent edges*, Sci. int.(Lahore) **23(2)** (2011), 91–95.
3. Ahmad A. and Bača M., *On vertex irregular total labelings*, Ars Combin., to appear.
4. Ahmad A., Awan K. M., Javaid I. and Slamin, *Total vertex strength of wheel related graphs*, Austrian Journal of Combinatorics, 51 (2011), 147–156.
5. Anholcer M., Kalkowski M. and Przybylo J., *A new upper bound for the total vertex irregularity strength of graphs*, Discrete Math. **309** (2009), 6316–6317.
6. Bača M., Jendroľ S., Miller M. and Ryan J., *On irregular total labellings*, Discrete Math. **307** (2007), 1378–1388.
7. Bača M., *On magic labellings of convex polytopes*, Annals Disc. Math. **51** (1992), 13–16.
8. Bohman T. and Kravitz D., *On the irregularity strength of trees*, J. Graph Theory **45** (2004), 241–254.
9. Chartrand G., Jacobson M. S., Lehel J., Oellermann O. R., Ruiz S. and Saba F., *Irregular networks*, Congr. Numer. **64** (1988), 187–192.
10. Faudree R. J., Jacobson M. S., Lehel J. and Schlep R. H., *Irregular networks, regular graphs and integer matrices with distinct row and column sums*, Discrete Math. **76** (1988), 223–240.
11. Frieze A., Gould R. J., Karonski M., and Pfender F., *On graph irregularity strength*, J. Graph Theory **41** (2002), 120–137.
12. Gyárfás A., *The irregularity strength of $K_{m,m}$ is 4 for odd m*, Discrete Math. **71** (1988), 273–274.
13. Imran M., Baig A. Q., Shafiq M. K. and Semaničová–Feňovčíková A., *Classes of convex polytopes with constant metric dimension*, Utilitas Math., to appear.
14. Jendroľ S., Tkáč M. and Tuza Z., *The irregularity strength and cost of the union of cliques*, Discrete Math. **150** (1996), 179–186.
15. Nierhoff T., *Tight bound on the irregularity strength of graphs*, SIAM J. Discrete Math. **13** (2000), 313–323.
16. Przybylo J., *Linear bound on the irregularity strength and the total vertex irregularity strength of graphs*, SIAM J. Discrete Math. **23** (2009), 511–516.
17. Wijaya K. and Slamin, *Total vertex irregular labeling of wheels, fans, suns and friendship graphs*, JCMCC **65** (2008), 103–112.
18. Wijaya K., Slamin, Surahmat, Jendrol S., *Total vertex irregular labeling of complete bipartite graphs*, JCMCC **55** (2005), 129–136.

O. Al-Mushayt, Faculty of Computer Science & Information Systems, Jazan University, Jazan, KSA, *e-mail*: `oalmushayt@yahoo.com`

A. Arshad, Faculty of Computer Science & Information Systems, Jazan University, Jazan, KSA, *e-mail*: `iffi_97@yahoo.com`

M. K. Siddiqui, Abdus Salam School of Mathematical Sciences, GC University, Lahore, Pakistan, *e-mail*: `kamransiddiqui75@gmail.com`