Acta Universitatis Apulensis No. 30/2012
ISSN: 1582-5329 pp. 293-303

ON PRE-I-OPEN SETS, SEMI-I-OPEN SETS AND b-I-OPEN SETS IN IDEAL TOPOLOGICAL SPACES[1]

Erdal Ekici

Abstract. The aim of this paper is to investigate some properties of pre-I-open sets, semi-I-open sets and b-I-open sets in ideal topological spaces. Some relationships of pre-I-open sets, semi-I-open sets and b-I-open sets in ideal topological spaces are discussed. Moreover, decompositions of continuity are provided.

2000 *Mathematics Subject Classification*: 54A05, 54A10, 54C08, 54C10.

1. Introduction

Pre-I-open sets, semi-I-open sets and b-I-open sets in ideal topological spaces were studied by [3], [9] and [8], respectively. In this paper, some properties of pre-I-open sets, semi-I-open sets and b-I-open sets in ideal topological spaces are investigated. Some relationships of pre-I-open sets, semi-I-open sets and b-I-open sets in ideal topological spaces are discussed. Furthermore, decompositions of continuous functions are established.

Throughout this paper, (X, τ) or (Y, σ) will denote a topological space with no separation properties assumed. $Cl(V)$ and $Int(V)$ will denote the closure and the interior of V in X, respectively for a subset V of a topological space (X, τ). An ideal I on a topological space (X, τ) is a nonempty collection of subsets of X which satisfies

(1) $V \in I$ and $U \subset V$ implies $U \in I$,

(2) $V \in I$ and $U \in I$ implies $V \cup U \in I$ [13].

For an ideal I on (X, τ), (X, τ, I) is called an ideal topological space or simply an ideal space. Given a topological space (X, τ) with an ideal I on X and if $P(X)$ is the set of all subsets of X, a set operator $(.)^* : P(X) \to P(X)$, called a local function

[1]This paper is supported by Canakkale Onsekiz Mart University, BAP: 2010/179.

[13] of K with respect to τ and I is defined as follows: for $K \subset X$, $K^*(I,\tau) = \{x \in X : U \cap K \notin I$ for every $U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau : x \in U\}$. A Kuratowski closure operator $Cl^*(.)$ for a topology $\tau^*(I,\tau)$, called the $\star$-topology, finer than τ, is defined by $Cl^*(K) = K \cup K^*(I,\tau)$ [11]. We will simply write K^* for $K^*(I,\tau)$ and τ^* for $\tau^*(I,\tau)$.

Definition 1. *A subset V of an ideal topological space (X,τ,I) is said to be*
(1) *pre-I-open [3] if $V \subset Int(Cl^*(V))$.*
(2) *semi-I-open [9] if $V \subset Cl^*(Int(V))$.*
(3) *α-I-open [9] if $V \subset Int(Cl^*(Int(V)))$.*
(4) *b-I-open [8] if $V \subset Int(Cl^*(V)) \cup Cl^*(Int(V))$.*
(5) *weakly I-local closed [12] if $V = U \cap K$, where U is an open set and K is a $\star$-closed set in X.*
(6) l*ocally closed [2] if $V = U \cap K$, where U is an open set and K is a closed set in X.*

The complement of a pre-I-open (resp. semi-I-open, b-I-open, α-I-open) set is called pre-I-closed (resp. semi-I-closed, b-I-closed, α-I-closed). A subset V of an ideal topological space (X,τ,I) is said to be a $\mathcal{BC}_I$-set [5] if $V = U \cap K$, where U is an open set and K is a b-I-closed set in X. The b-I-interior of V, denoted by $b_I Int(V)$, is defined by the union of all b-I-open sets contained in V [1]. For a subset V of an ideal topological space (X,τ,I), the intersection of all b-I-closed (resp. pre-I-closed, semi-I-closed) sets containing V is called the b-I-closure [1] (resp. pre-I-closure [4], semi-I-closure [4]) of V and is denoted by $b_I Cl(V)$ (resp. $p_I Cl(V)$, $s_I Cl(V)$). For a subset V of an ideal topological space (X,τ,I), $p_I Cl(V) = V \cup Cl(Int^*(V))$ [4] and $s_I Cl(V) = V \cup Int^*(Cl(V))$ [4]. For a subset V of an ideal topological space (X,τ,I), the pre-I-interior (resp. semi-I-interior [4]) of V, denoted by $p_I Int(V)$ (resp. $s_I Int(V)$), is defined by the union of all pre-I-open (resp. semi-I-open) sets contained in V.

Corollary 2. *Let (X,τ,I) be an ideal topological space and $V \subset X$. Then, $p_I Int(V) = V \cap Int(Cl^*(V))$ and $s_I Int(V) = V \cap Cl^*(Int(V))$.*

Lemma 3. ([10]) *Let V be a subset of an ideal topological space (X,τ,I). If $G \in \tau$, then $G \cap Cl^*(V) \subset Cl^*(G \cap V)$.*

Lemma 4. ([14]) *A subset V of an ideal space (X,τ,I) is a weakly I-local closed set if and only if there exists $K \in \tau$ such that $V = K \cap Cl^*(V)$.*

Theorem 5. ([5]) *For a subset V of an ideal topological space (X,τ,I), V is a $\mathcal{BC}_I$-set if and only if $V = K \cap b_I Cl(V)$ for an open set K in X.*

Definition 6. ([6]) *An ideal topological space (X, τ, I) is said to be $\star$-extremally disconnected if the $\star$-closure of every open subset V of X is open.*

Theorem 7. ([6]) *For an ideal topological space (X, τ, I), the following properties are equivalent:*
(1) X is $\star$-extremally disconnected,
(2) $Cl^*(Int(V)) \subset Int(Cl^*(V))$ for every subset V of X.

2. Pre-I-open sets, semi-I-open sets and b-I-open sets in ideal topological spaces

Theorem 8. *Let (X, τ, I) be a $\star$-extremally disconnected ideal space and $V \subset X$, the following properties are equivalent:*
(1) V is an open set,
(2) V is α-I-open and weakly I-local closed,
(3) V is pre-I-open and weakly I-local closed,
(4) V is semi-I-open and weakly I-local closed,
(5) V is b-I-open and weakly I-local closed.

Proof. (1) $\Rightarrow$ (2) : *It follows from the fact that every open set is α-I-open and weakly I-local closed.*

(2) $\Rightarrow$ (3), (2) $\Rightarrow$ (4), (3) $\Rightarrow$ (5) *and* (4) $\Rightarrow$ (5) : *Obvious.*

(5) $\Rightarrow$ (1) : *Suppose that V is a b-I-open set and a weakly I-local closed set in X. It follows that $V \subset Cl^*(Int(V)) \cup Int(Cl^*(V))$. Since V is a weakly I-local closed set, then there exists an open set G such that $V = G \cap Cl^*(V)$. It follows from Theorem 7 that*

$$
\begin{aligned}
& V \subset G \cap (Cl^*(Int(V)) \cup Int(Cl^*(V))) \\
& = (G \cap Cl^*(Int(V))) \cup (G \cap Int(Cl^*(V))) \\
& \subset (G \cap Int(Cl^*(V)) \cup (G \cap Int(Cl^*(V))) \\
& = Int(G \cap Cl^*(V)) \cup Int(G \cap Cl^*(V)) \\
& = Int(V) \cup Int(V) \\
& = Int(V).
\end{aligned}
$$

Thus, $V \subset Int(V)$ and hence V is an open set in X.

Theorem 9. *Let (X, τ, I) be a $\star$-extremally disconnected ideal space and $V \subset X$, the following properties are equivalent:*
(1) V is an open set,
(2) V is α-I-open and a locally closed set.
(3) V is pre-I-open and a locally closed set.

(4) V is semi-I-open and a locally closed set.
(5) V is b-I-open and a locally closed set.

Proof. *By Theorem 8, It follows from the fact that every open set is locally closed and every locally closed set is weakly I-local closed.*

Theorem 10. *The following properties hold for a subset V of an ideal topological space (X, τ, I):*
(1) If V is a pre-I-open set, then $s_I Cl(V) = Int^*(Cl(V))$.
(2) If V is a semi-I-open set, then $p_I Cl(V) = Cl(Int^*(V))$.

Proof. (1) : *Suppose that V is a pre-I-open set in X. Then we have $V \subset Int(Cl^*(V)) \subset Int^*(Cl(V))$. This implies*

$$s_I Cl(V) = V \cup Int^*(Cl(V)) = Int^*(Cl(V)).$$

(2) : *Let V be a semi-I-open set in X. It follows that $V \subset Cl^*(Int(V)) \subset Cl(Int^*(V))$. Thus, we have*

$$p_I Cl(V) = V \cup Cl(Int^*(V)) = Cl(Int^*(V)).$$

Remark 11. The reverse implications of Theorem 10 are not true in general as shown in the following example:

Example 12. Let $X = \{a, b, c, d\}$, $\tau = \{X, \varnothing, \{a\}, \{b, c\}, \{a, b, c\}\}$ and $I = \{\varnothing, \{a\}, \{d\}, \{a, d\}\}$. Then $s_I Cl(A) = Int^*(Cl(A))$ for the subset $A = \{b, d\}$ but A is not pre-I-open. Moreover, $p_I Cl(B) = Cl(Int^*(B))$ for the subset $B = \{a, d\}$ but B is not semi-I-open.

Theorem 13. *Let (X, τ, I) be an ideal topological space and $V \subset X$, the following properties hold:*
(1) If V is a pre-I-closed set, then $s_I Int(V) = Cl^*(Int(V))$.
(2) If V is a semi-I-closed set, then $p_I Int(V) = Int(Cl^*(V))$.

Proof. (1) : *Let V be a pre-I-closed set. Then $Cl^*(Int(V)) \subset Cl(Int^*(V)) \subset V$. This implies that $s_I Int(V) = V \cap Cl^*(Int(V)) = Cl^*(Int(V))$.*
(2) : *Suppose that V is a semi-I-closed set. We have $Int(Cl^*(V)) \subset Int^*(Cl(V)) \subset V$. Hence, $p_I Int(V) = V \cap Int(Cl^*(V)) = Int(Cl^*(V))$.*

Theorem 14. *For a subset K of an ideal topological space (X, τ, I), K is a b-I-closed set if and only if $K = p_I Cl(K) \cap s_I Cl(K)$.*

Proof. ($\Rightarrow$) : *Suppose that K is a b-I-closed set in X. This implies $Int^*(Cl(K)) \cap Cl(Int^*(K)) \subset K$. We have*

$$\begin{aligned} p_I Cl(K) \cap s_I Cl(K) &= (K \cup Cl(Int^*(K))) \cap (K \cup Int^*(Cl(K))) \\ &= K \cup (Cl(Int^*(K)) \cap Int^*(Cl(K))) \\ &= K. \end{aligned}$$

Thus, $K = p_I Cl(K) \cap s_I Cl(K)$.

($\Leftarrow$) : *Let $K = p_I Cl(K) \cap s_I Cl(K)$. Then we have*

$$\begin{aligned} K &= p_I Cl(K) \cap s_I Cl(K) \\ &= (K \cup Cl(Int^*(K))) \cap (K \cup Int^*(Cl(K))) \\ &\supset Cl(Int^*(K)) \cap Int^*(Cl(K)). \end{aligned}$$

This implies $Cl(Int^*(K)) \cap Int^*(Cl(K)) \subset K$. Thus, K is a b-I-closed set in X.

Theorem 15. *Let (X, τ, I) be an ideal topological space and $V \subset X$. If V is pre-I-open and semi-I-open, then $b_I Cl(V) = Cl(Int^*(V)) \cap Int^*(Cl(V))$.*

Proof. Suppose that V is a pre-I-open set and a semi-I-open set in X. By Theorem 10, we have $p_I Cl(V) = Cl(Int^*(V))$ and $s_I Cl(V) = Int^*(Cl(V))$.

Since $b_I Cl(V) \subset p_I Cl(V) \cap s_I Cl(V)$ and $b_I Cl(V)$ is b-I-closed, then we have

$$\begin{aligned} b_I Cl(V) &\supset Cl(Int^*(b_I Cl(V))) \cap Int^*(Cl(b_I Cl(V))) \\ &\supset Cl(Int^*(V)) \cap Int^*(Cl(V)). \end{aligned}$$

It follows that

$$\begin{aligned} p_I Cl(V) \cap s_I Cl(V) &= (V \cup Cl(Int^*(V))) \cap (V \cup Int^*(Cl(V))) \\ &\subset b_I Cl(V). \end{aligned}$$

Consequently, we have, $b_I Cl(V) = p_I Cl(V) \cap s_I Cl(V)$. This implies that $b_I Cl(V) = p_I Cl(V) \cap s_I Cl(V) = Cl(Int^*(V)) \cap Int^*(Cl(V))$.

Remark 16. The reverse implication of Theorem 15 is not true in general as shown in the following example:

Example 17. Let $X = \{a, b, c, d\}$, $\tau = \{X, \varnothing, \{a\}, \{b, c\}, \{a, b, c\}\}$ and $I = \{\varnothing, \{a\}, \{d\}, \{a, d\}\}$. Take $A = \{b, c, d\}$. Then $b_I Cl(A) = Cl(Int^*(A)) \cap Int^*(Cl(A))$ but A is not pre-I-open.

Theorem 18. *Let (X, τ, I) be an ideal topological space and $V \subset X$. If V is pre-I-closed and semi-I-closed, then $b_I Int(V) = Cl^*(Int(V)) \cup Int(Cl^*(V))$.*

Proof. *Suppose that V is a pre-I-closed set and a semi-I-closed set. By Theorem 13, we have $s_IInt(V) = Cl^*(Int(V))$ and $p_IInt(V) = Int(Cl^*(V))$. Thus, $b_IInt(V) = p_IInt(V) \cup s_IInt(V) = Int(Cl^*(V)) \cup Cl^*(Int(V))$.*

Theorem 19. *For a subset V of an ideal topological space (X, τ, I), the following properties hold:*
(1) $b_ICl(Int(V)) = Int^*(Cl(Int(V)))$.
(2) $Int(s_ICl(V)) = Int(Cl(V))$.
(3) $Cl(p_IInt(V)) = Cl(Int(Cl^*(V)))$.

Proof. (1) : *We have*

$$
\begin{aligned}
&b_ICl(Int(V))\\
&= p_ICl(Int(V)) \cap s_ICl(Int(V))\\
&= (Int(V) \cup Cl(Int^*(Int(V)))) \cap (Int(V) \cup Int^*(Cl(Int(V))))\\
&= Cl(Int^*(Int(V))) \cap Int^*(Cl(Int(V)))\\
&= Cl(Int(V)) \cap Int^*(Cl(Int(V)))\\
&= Int^*(Cl(Int(V))).
\end{aligned}
$$

Hence, $b_ICl(Int(V)) = Int^*(Cl(Int(V)))$.
(2) : *We have*

$$
\begin{aligned}
&Int(s_ICl(V))\\
&= Int(V \cup Int^*(Cl(V)))\\
&\supset Int(V) \cup Int(Int^*(Cl(V)))\\
&\supset Int(V) \cup Int(Int(Cl(V)))\\
&= Int(V) \cup Int(Cl(V))\\
&= Int(Cl(V)).
\end{aligned}
$$

Conversely,

$$
\begin{aligned}
&Int(s_ICl(V))\\
&= Int(V \cup Int^*(Cl(V)))\\
&\subset Int(Cl(V) \cup Int^*(Cl(V)))\\
&= Int(Cl(V)).
\end{aligned}
$$

This implies $Int(s_ICl(V)) = Int(Cl(V))$.
(3) : *We have*

$$
\begin{aligned}
&Cl(p_IInt(V))\\
&= Cl(V \cap Int(Cl^*(V)))\\
&\supset Cl(V) \cap Int(Cl^*(V))\\
&= Int(Cl^*(V)).
\end{aligned}
$$

Thus, we have $Cl(p_IInt(V)) \supset Cl(Int(Cl^*(V)))$.

Conversely, we have

$$\begin{aligned}&Cl(p_IInt(V))\\&= Cl(V \cap Int(Cl^*(V)))\\&\subset Cl(V) \cap Cl(Int(Cl^*(V)))\\&= Cl(Int(Cl^*(V))).\end{aligned}$$

Hence, $Cl(p_IInt(V)) = Cl(Int(Cl^*(V)))$.

Corollary 20. *For a subset* V *of an ideal topological space* (X, τ, I)*, the following properties hold:*
(1) $b_IInt(Cl(V)) = Cl^*(Int(Cl(V)))$.
(2) $Cl(s_IInt(V)) = Cl(Int(V))$.
(3) $Int(p_ICl(V)) = Int(Cl(Int^*(V)))$.

Proof. *It follows from Theorem 19.*

Theorem 21. *For a subset* V *of an ideal topological space* (X, τ, I)*, the following properties hold:*
(1) $Int(b_ICl(V)) = Int(Cl(Int^*(V)))$.
(2) $Cl(b_IInt(V)) = Cl(Int(Cl^*(V)))$.

Proof. (1) : *We have*

$$\begin{aligned}&Int(b_ICl(V))\\&= Int(p_ICl(V) \cap s_ICl(V))\\&= Int(p_ICl(V)) \cap Int(s_ICl(V))\\&= Int(p_ICl(V)) \cap Int(Cl(V))\\&= Int(p_ICl(V))\\&= Int(Cl(Int^*(V))).\end{aligned}$$

by Theorem 19. Thus, $Int(b_ICl(V)) = Int(Cl(Int^*(V)))$.
(2) : *It follows from (1).*

Theorem 22. *For a subset* V *of an ideal topological space* (X, τ, I)*, the following properties hold:*
(1) $p_ICl(s_IInt(V)) \subset Cl(Int^*(V))$.
(2) $s_IInt(s_ICl(V)) = s_ICl(V) \cap Cl^*(Int(Cl(V)))$.
(3) $p_IInt(s_ICl(V)) \supset Int(Cl^*(V))$.
(4) $s_ICl(s_IInt(V)) = s_IInt(V) \cup Int^*(Cl(Int(V)))$.

Proof. (1) : *By Theorem 10, we have*

$$p_ICl(s_IInt(V)) = Cl(Int^*(s_IInt(V))) \subset Cl(Int^*(V)).$$

This implies $p_ICl(s_IInt(V)) \subset Cl(Int^*(V))$.

(2) : *By Theorem 19, we have*

$$\begin{aligned} &s_IInt(s_ICl(V)) \\ &= s_ICl(V) \cap Cl^*(Int(s_ICl(V))) \\ &= s_ICl(V) \cap Cl^*(Int(Cl(V))). \end{aligned}$$

Hence, $s_IInt(s_ICl(V)) = s_ICl(V) \cap Cl^*(Int(Cl(V)))$.

(3) *and* (4) *follow from* (1) *and* (2), *respectively.*

Theorem 23. *For a subset V of an ideal topological space (X, τ, I), the following properties hold:*

(1) $b_ICl(s_IInt(V)) \subset s_IInt(V) \cup Int^*(Cl(Int(V)))$.

(2) $p_IInt(b_ICl(V)) \supset p_ICl(V) \cap Int(Cl^*(V))$.

(3) $s_IInt(b_ICl(V)) \supset s_ICl(V) \cap Cl^*(Int(V))$.

Proof. (1) : *By Theorem 22 and Corollary 20, we have*

$$\begin{aligned} &b_ICl(s_IInt(V)) = p_ICl(s_IInt(V)) \cap s_ICl(s_IInt(V)) \\ &\subset Cl(Int^*(V)) \cap (s_IInt(V) \cup Int^*(Cl(s_IInt(V)))) \\ &= Cl(Int^*(V)) \cap (s_IInt(V) \cup Int^*(Cl(Int(V)))) \\ &= s_IInt(V) \cup (Cl(Int^*(V)) \cap Int^*(Cl(Int(V)))) \\ &= s_IInt(V) \cup Int^*(Cl(Int(V))). \end{aligned}$$

Thus, $b_ICl(s_IInt(V)) \subset s_IInt(V) \cup Int^*(Cl(Int(V)))$.

(2) : *We have*

$$\begin{aligned} &p_IInt(b_ICl(V)) = p_IInt(p_ICl(V) \cap s_ICl(V)) \\ &= p_ICl(V) \cap s_ICl(V) \cap Int(Cl^*(p_ICl(V) \cap s_ICl(V))) \\ &\supset p_ICl(V) \cap Int^*(Cl(V)) \cap s_ICl(V) \cap Int(Cl^*(p_ICl(V) \cap s_ICl(V))) \\ &= p_ICl(V) \cap Int^*(Cl(V)) \cap Int(Cl^*(p_ICl(V) \cap s_ICl(V))) \\ &= p_ICl(V) \cap Int^*(Cl(V)) \cap Int(Cl^*(b_ICl(V))) \\ &\supset p_ICl(V) \cap Int(Cl^*(V)) \cap Int(Cl^*(b_ICl(V))) \\ &= p_ICl(V) \cap Int(Cl^*(V)). \end{aligned}$$

This implies $p_IInt(b_ICl(V)) \supset p_ICl(V) \cap Int(Cl^*(V))$.

(3) : *We have*

$$s_I Int(b_I Cl(V)) = s_I Int(p_I Cl(V) \cap s_I Cl(V))$$
$$= p_I Cl(V) \cap s_I Cl(V) \cap Cl^*(Int(p_I Cl(V) \cap s_I Cl(V)))$$
$$\supset p_I Cl(V) \cap Cl(Int^*(V)) \cap s_I Cl(V) \cap Cl^*(Int(p_I Cl(V) \cap s_I Cl(V)))$$
$$= Cl(Int^*(V)) \cap s_I Cl(V) \cap Cl^*(Int(p_I Cl(V) \cap s_I Cl(V)))$$
$$\supset Cl^*(Int(V)) \cap s_I Cl(V) \cap Cl^*(Int(p_I Cl(V) \cap s_I Cl(V)))$$
$$= s_I Cl(V) \cap Cl^*(Int(V)).$$

Thus, $s_I Int(b_I Cl(V)) \supset s_I Cl(V) \cap Cl^*(Int(V))$.

Corollary 24. *For a subset V of an ideal topological space (X, τ, I), the following properties hold:*
(1) $b_I Int(s_I Cl(V)) \supset s_I Cl(V) \cap Cl^*(Int(Cl(V)))$.
(2) $p_I Cl(b_I Int(V)) \subset p_I Int(V) \cup Cl(Int^*(V))$.
(3) $s_I Cl(b_I Int(V)) \subset s_I Int(V) \cup Int^*(Cl(V))$.

Proof. It follows from Theorem 23.

3. Decompositions of continuous functions and further properties

Definition 25. *A function $f : (X, \tau, I) \to (Y, \sigma)$ is called α-I-continuous [9] (rep. pre-I-continuous [3], semi-I-continuous [9], b-I-continuous [8], $W_I LC$-continuous [12], LC-continuous [7]) if $f^{-1}(K)$ is α-I-open (rep. pre-I-open, semi-I-open, b-I-open, weakly I-local closed, locally closed) for each open set K in Y.*

Theorem 26. *For a function $f : (X, \tau, I) \to (Y, \sigma)$, where (X, τ, I) is a $\star$-extremally disconnected ideal space, the following properties are equivalent:*
(1) f is continuous,
(2) f is α-I-continuous and $W_I LC$-continuous,
(3) f is pre-I-continuous and $W_I LC$-continuous,
(4) f is semi-I-continuous and $W_I LC$-continuous,
(5) f is b-I-continuous and $W_I LC$-continuous.

Proof. It follows from Theorem 8.

Theorem 27. *For a function $f : (X, \tau, I) \to (Y, \sigma)$, where (X, τ, I) is a $\star$-extremally disconnected ideal space, the following properties are equivalent:*
(1) f is continuous,
(2) f is α-I-continuous and LC-continuous,
(3) f is pre-I-continuous and LC-continuous,

(4) f is semi-I-continuous and LC-continuous,
(5) f is b-I-continuous and LC-continuous.

Proof. It follows from Theorem 9.

Definition 28. *A subset V of an ideal topological space (X, τ, I) is said to be*
(1) *generalized b-I-open (gb_I-open) if $K \subset b_I Int(V)$ whenever $K \subset V$ and K is a closed set in X.*
(2) *generalized b-I-closed (gb_I-closed) if $X \backslash V$ is a gb_I-open in X.*

Theorem 29. *Let (X, τ, I) be an ideal topological space and $V \subset X$. Then V is a gb_I-closed set if and only if $b_I Cl(V) \subset G$ whenever $V \subset G$ and G is an open set in X.*

Proof. Let V be a gb_I-closed set in X. Suppose that $V \subset G$ and G is an open set in X. This implies that $X \backslash V$ is a gb_I-open set and $X \backslash G \subset X \backslash V$ where $X \backslash G$ is a closed set. Since $X \backslash V$ is a gb_I-open set, then $X \backslash G \subset b_I Int(X \backslash V)$. Since $b_I Int(X \backslash V) = X \backslash b_I Cl(V)$, then we have $b_I Cl(V) = X \backslash b_I Int(X \backslash V) \subset G$. Thus, $b_I Cl(V) \subset G$. The converse is similar.

Theorem 30. *Let (X, τ, I) be an ideal topological space and $V \subset X$. Then V is a b-I-closed set if and only if V is a $\mathcal{BC}_I$-set and a gb_I-closed set in X.*

Proof. It follows from the fact that any b-I-closed set is a $\mathcal{BC}_I$-set and a gb_I-closed.

Conversely, let V be a $\mathcal{BC}_I$-set and a gb_I-closed set in X. By Theorem 5, $V = G \cap b_I Cl(V)$ for an open set G in X. Since $V \subset G$ and V is gb_I-closed, then we have $b_I Cl(V) \subset G$. Thus, $b_I Cl(V) \subset G \cap b_I Cl(V) = V$ and hence V is b-I-closed.

Theorem 31. *For a subset V of an ideal topological space (X, τ, I), if V is a $\mathcal{BC}_I$-set in X, then $b_I Cl(V) \backslash V$ is a b-I-closed set and $V \cup (X \backslash b_I Cl(V))$ is a b-I-open set in X.*

Proof. Suppose that V is a $\mathcal{BC}_I$-set in X. By Theorem 5, we have $V = G \cap b_I Cl(V)$ for an open set G. This implies

$$
\begin{aligned}
b_I Cl(V) \backslash V &= b_I Cl(V) \backslash (G \cap b_I Cl(V)) \\
&= b_I Cl(V) \cap (X \backslash (G \cap b_I Cl(V))) \\
&= b_I Cl(V) \cap ((X \backslash G) \cup (X \backslash b_I Cl(V))) \\
&= (b_I Cl(V) \cap (X \backslash G)) \cup (b_I Cl(V) \cap (X \backslash b_I Cl(V))) \\
&= b_I Cl(V) \cap (X \backslash G).
\end{aligned}
$$

Consequently, $b_ICl(V)\backslash V$ is b-I-closed. On the other hand, since $b_ICl(V)\backslash V$ is a b-I-closed set, then $X\backslash(b_ICl(V)\backslash V)$ is a b-I-open set. Since $X\backslash(b_ICl(V)\backslash V) = X\backslash(b_ICl(V) \cap (X\backslash V)) = (X\backslash b_ICl(V)) \cup V$, then $V \cup (X\backslash b_ICl(V))$ is a b-I-open set.

References

[1] M. Akdag, *On b-I-open sets and b-I-continuous functions*, Int. J. Math. Math. Sci, Volume 2007, ID 75721, 1-13.

[2] N. Bourbaki, *General Topology, Part I*, Addison Wesley, Reading, Mass 1966.

[3] J. Dontchev, *Idealization of Ganster-Reilly decomposition theorems*, arxiv:math. GN/9901017v1 (1999).

[4] E. Ekici and T. Noiri, *$\star$-hyperconnected ideal topological spaces*, Analele Stiin. Ale Univ. A. I. Cuza Din Iasi-Serie Noua-Mat., in press.

[5] E. Ekici, *On $\mathcal{AC}_I$-sets, $\mathcal{BC}_I$-sets, β_I^*-open sets and decompositions of continuity in ideal topological spaces*, Creative Mathematics and Informatics, 20 (1) (2011), 47-54.

[6] E. Ekici and T. Noiri, *$\star$-extremally disconnected ideal topological spaces*, Acta Math. Hungar., 122 (1-2) (2009), 81-90.

[7] M. Ganster and I. L. Reilly, *Locally closed sets and LC-continuous functions*, Internat. J. Math. Math. Sci., 12 (3) (1989), 417-424.

[8] A. C. Guler and G. Aslim, *b-I-open sets and decomposition of continuity via idealization*, Proc. Inst. Math. Mech. Natl. Acad. Sci. Azerb., 22 (2005), 27-32.

[9] E. Hatir and T. Noiri, *On decompositions of continuity via idealization*, Acta Math. Hungar., 96 (2002), 341-349.

[10] E. Hatir, A. Keskin, and T. Noiri, *A note on strong β-I-sets and strongly β-I-continuous functions*, Acta Math. Hungar., 108 (2005), 87-94.

[11] D. Janković and T. R. Hamlett, *New topologies from old via ideals*, Amer. Math. Monthly, 97 (1990), 295-310.

[12] A. Keskin, T. Noiri and S. Yuksel, *Decompositions of I-continuity and continuity*, Commun. Fac. Sci. Univ. Ankara Series A1, 53 (2004), 67-75.

[13] K. Kuratowski, *Topology, Vol. I*, Academic Press , NewYork, 1966.

[14] E. G. Yang, *Around decompositions of continuity via idealization*, Q. and A. in General Topology, 26 (2008), 131-138.

Erdal Ekici
Department of Mathematics,
Canakkale Onsekiz Mart University,
Terzioglu Campus,
17020 Canakkale, TURKEY
E-mail: *eekici@comu.edu.tr*