Acta Universitatis Apulensis No. 39/2014
ISSN: 1582-5329 pp. 341-350
http://www.uab.ro/auajournal/ doi: 10.17114/j.aua.2014.39.29

NOTE ON CONTINUITIES IN MINIMAL SPACES

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Abstract. In this paper, we present and study some new class of sets in generalized grill spaces. We also discuss different types of continuities in terms of grill.

2000 *Mathematics Subject Classification*: 54A05, 54C10.

Keywords: grill minimal space, m-β-open set, m-semi-open set, m-β-continuous, m-semi-continuous.

1. Introduction

Generalization of topological space is not a new idea in literature but generalization of ideal topological space [12] is a new concept. In this regard minimal structure [2] is an important part. The study of this structure has been studied from 2005 in [2]. The authors Min [16] and Modak [17, 18, 19, 20] have further studied it in detail. The joint study of generalized topology and the mathematical structure had been started through ideal minimal space [21], grill topological space [23, 3, 4, 5, 7, 11], grill filter space [17, 18].

In this paper we have also considered one type of generalization that is grill minimal space. Here, we have defined and studied various types of set and different types of continuity. We have also related these continuities with the the generalized continuities which are already in literature.

2. Preliminaries

Definition 1. *[2] A family $\mathcal{M} \subseteq \wp(X)$ is said to be minimal structure on X if $\emptyset$, $X \in \mathcal{M}$. In this case $(X, \mathcal{M})$ is called a minimal space.*

Throughout this paper $(X, \mathcal{M})$ means minimal space.

Example 1. *Let (X, τ) be a topological space. Then $\mathcal{M} = \tau$, $SO(X)$ [13], $PO(X)$ [14] and $\alpha O(X)$ [22] are the examples of minimal structures on X.*

Example 2. *Let X be a nonempty set. Then the filter $\mathcal{F}$ [25] does not form a minimal structure on X.*

Definition 2. *[2] A set $A \in \wp(X)$ is said to be an m-open set if $A \in \mathcal{M}$. $B \in \wp(X)$ is a m-closed set if $X \setminus B \in \mathcal{M}$. We set $mInt(A) = \cup\{U : U \subseteq A, U \in \mathcal{M}\}$ and $mCl(A) = \cap\{F : A \subseteq F, X \setminus F \in \mathcal{M}\}$.*

[2] Let $(X, \mathcal{M})$ be a minimal space. Then for $A, B \in \wp(X)$,

1. $mInt(A) \subseteq A$ and $mInt(A) = A$ if A is an m-open set.
2. $A \subseteq mCl(A)$ and $A = mCl(A)$ if A is a m-closed set.
3. $mInt(A) \subseteq mInt(B)$ and $mCl(A) \subseteq mCl(B)$ if $A \subseteq B$.
4. $mInt(A \cap B) = (mInt(A)) \cap (mInt(B))$ and $(mInt(A)) \cup (mInt(B)) \subseteq mInt(A \cup B)$.
5. $mCl(A \cup B) = (mCl(A)) \cup (mCl(B))$ and $mCl(A \cap B) \subseteq (mCl(A)) \cap (mCl(B))$.
6. $mInt(mInt(A)) = mInt(A)$ and $mCl(mCl(B)) = mCl(B)$.
7. $x \in mCl(A)$ if and only if every m-open set U_x containing x, $U_x \cap A \neq \emptyset$.
8. $(X \setminus mCl(A)) = mInt(X \setminus A)$ and $(X \setminus mInt(A)) = mCl(X \setminus A)$.

Definition 3. *[2] A minimal space $(X, \mathcal{M})$ enjoys the property U if the arbitrary union of m-open sets is an m-open set.*

Example 3. *Let X be a nonempty set. Let μ be the supratopology [15] on X. Then supratopological space (X, μ) is an example of a minimal space with the property U.*

Definition 4. *[2] A minimal space $(X, \mathcal{M})$ enjoys the property I if the finite intersection of m-open sets is an m-open set.*

Definition 5. *Let X be a nonempty set. Let M be the m-structure [5] on X. Then the space (X, M) is an example of a minimal space with the property I.*

Definition 6. *[2] Let $(X, \mathcal{M})$ and $(Y, \mathcal{N})$ be two minimal spaces. We say that a function $f : (X, \mathcal{M}) \longrightarrow (Y, \mathcal{N})$ is a minimal continuous (briefly m-continuous) if $f^{-1}(B) \in \mathcal{M}$, for any $B \in \mathcal{N}$.*

Formal definition of grill [10] is:

A subcollection $\mathcal{G}$ (not containing the empty set) of $\wp(X)$ is called a grill on X if $\mathcal{G}$ satisfies the following conditions:

1. $A \in \mathcal{G}$ and $A \subseteq B$ implies $B \in \mathcal{G}$;

2. $A, B \subseteq X$ and $A \cup B \in \mathcal{G}$ implies that $A \in \mathcal{G}$ or $B \in \mathcal{G}$.

For example let $\Re$ be the set of all real numbers, consider a subset $\mathcal{M}= \{A \subseteq \Re : m(A) \neq 0\}$, where $m(A)$ is the Lebesgue measure of A, then $\mathcal{M}$ is a grill [4] on $\Re$.

Detail study of grill has been done through [8, 9, 25] and many others.

A minimal space $(X, \mathcal{M})$ with a grill $\mathcal{G}$ on X is called a grill minimal space and denoted as $(X, \mathcal{M}, \mathcal{G})$.

Definition 7. *Let$(X, \mathcal{M}, \mathcal{G})$ be a grill minimal space. A mapping $()^{*\mathcal{M}}$: $\wp(X) \to \wp(X)$ is defined as follows; $(A)^{*\mathcal{M}} = (A)^{*\mathcal{M}\mathcal{G}} = \{x \in X : A \cap U \in \mathcal{G}$ for all $U \in \mathcal{M}(x)\}$ for each $A \in \wp(X)$, where $\mathcal{M}(x) = \{U \in \mathcal{M}\text{: } x \in U\}$.*

Let $(X, \mathcal{M}, \mathcal{G})$ be a grill minimal space. Then

1. $(\emptyset)^{*\mathcal{M}} = \emptyset$.

2. $(A)^{*\mathcal{M}}= \emptyset$, if $A \notin \mathcal{G}$.

3. for A, $B \in \wp(X)$ and $A \subseteq B$, $(A)^{*\mathcal{M}}\subseteq (B)^{*\mathcal{M}}$.

4. for $A \subseteq X$, $(A)^{*\mathcal{M}}\subseteq mCl(A)$.

5. for $A \subseteq X$, $mCl[(A)^{*\mathcal{M}}] \subseteq (A)^{*\mathcal{M}}$.

6. for $A \subseteq X$, $(A)^{*\mathcal{M}}$ is a m-closed set.

7. for $A \subseteq X$, $[(A)^{*\mathcal{M}}]^{*\mathcal{M}}\subseteq (A)^{*\mathcal{M}}$.

8. $(A)^{*\mathcal{M}\mathcal{G}}\subseteq (A)^{*\mathcal{M}\mathcal{G}_1}$, where $\mathcal{G}_1$ is a grill on X with $\mathcal{G}\subseteq \mathcal{G}_1$.

Proof. The proof is obvious from definition.

Let $(X, \mathcal{M}, \mathcal{G})$ be a grill minimal space and $(X, \mathcal{M})$ enjoys the property I. Then

1. for $A, B \subseteq X$, $(A \cup B)^{*\mathcal{M}} = (A)^{*\mathcal{M}}\cup(B)^{*\mathcal{M}}$.

2. for $U \in \mathcal{M}$ and $A \subseteq X$, $U \cap (A)^{*\mathcal{M}} = U \cap (U \cap A)^{*\mathcal{M}}$.

3. for $A, B \subseteq X$, $[(A)^{*\mathcal{M}} \setminus(B)^{*\mathcal{M}}] = [(A \setminus B)^{*\mathcal{M}}\setminus(B)^{*\mathcal{M}}]$.

4. for $A, B \subseteq X$ with $B \notin \mathcal{G}$, $(A \cup B)^{*\mathcal{M}}= (A)^{*\mathcal{M}}= (A \setminus B)^{*\mathcal{M}}$.

Let $\mathcal{G}$ be a grill on a minimal space $(X, \mathcal{M})$ with enjoys the property I. Then we define a map $\psi_{\mathcal{M}}(A) = A \cup (A)^{*_{\mathcal{M}}}$ for all $A \in \wp(X)$. Then the map $\psi_{\mathcal{M}}$ is a Kuratowski closure axioms. Corresponding to a grill $\mathcal{G}$ on a minimal space $(X, \mathcal{M})$, there exists a unique topology $\tau_{\mathcal{MG}}$ on X given by $\tau_{\mathcal{MG}}= \{U \subseteq X : \psi_{\mathcal{M}}(X \setminus U) = X \setminus U\}$, where for any $A \subseteq X$, $\psi_{\mathcal{M}}(A)= A \cup (A)^{*_{\mathcal{M}}}$. For any grill $\mathcal{G}$ on a minimal space $(X, \mathcal{M})$, $\mathcal{M} \subseteq \tau_{\mathcal{MG}}$.

Lemma 1. *For any grill $\mathcal{G}$ on a minimal space $(X, \mathcal{M})$, $\mathcal{M} \subseteq \beta(\mathcal{G}, \mathcal{M}) \subseteq \tau_{\mathcal{MG}}$, where $\beta(\mathcal{G}, \mathcal{M})= \{V \setminus A : V \in \mathcal{M}$ and $A \notin \mathcal{G}\}$ is an open base for $\tau_{\mathcal{MG}}$.*

Example 4. *Let $(X, \mathcal{M})$ be a minimal space. If $\mathcal{G}= \wp(X) \setminus \{\emptyset\}$, then $\tau_{\mathcal{MG}}= \mathcal{M}$. Since for any $\tau_{\mathcal{MG}}$-basic open set $V = U \setminus A$ with $U \in \tau_{\mathcal{MG}}$ and $A \notin \mathcal{G}$, we have $A = \emptyset$, so that $V = U \in \tau_{\mathcal{MG}}$. Hence by Lemma 1 we have in this case $\mathcal{M} = \beta(\mathcal{G}, \mathcal{M}) = \tau_{\mathcal{MG}}$.*

3. Sets on minimal space and grill minimal space

In this section, we define and study some generalized sets on minimal space and grill minimal space.

Definition 8. *Let $(X, \mathcal{M})$ be a minimal space. A subset A of X is said to be*
(1) αm-open [15] if $A \subseteq mInt(mCl(mInt(A)))$,
(2) m-semi-open if $A \subseteq mCl(mInt(A))$,
(3) m-preopen if $A \subseteq mInt(mCl(A))$,
(4) m-β-open if $A \subseteq mCl(mInt(mCl(A)))$,

The family of all αm-open (resp. m-semi-open, m-preopen, m-β-open) sets in a minimal space $(X, \mathcal{M})$ is denoted by $\mathcal{M}\alpha(X)$ (resp. $\mathcal{M}SO(X)$, $\mathcal{M}PO(X)$, $\mathcal{M}\beta(X)$).

From above definition and Proposition 2, we get following results:

Theorem 2. *Let $(X, \mathcal{M})$ be a minimal space then following hold:*
(1) $\mathcal{M} \subseteq \mathcal{M}\alpha(X) \subseteq \mathcal{M}SO(X)$.
(2) $\mathcal{M} \subseteq \mathcal{M}PO(X) \subseteq \mathcal{M}\beta(X)$).
(3) $A \in \mathcal{M}\alpha(X)$ if and only if $A \in \mathcal{M}SO(X)$ and $A \in \mathcal{M}PO(X)$.

Proof. Necessity. This is obvious.

Sufficiency. Given that $A \in \mathcal{M}SO(X)$ and $A \in \mathcal{M}PO(X)$. Then $A \subseteq mInt(mCl(A)) \subseteq mInt(mCl(mInt(A)))$.

Reverse inclusion of (1) and (2) does not hold in general:

Example 5. *Let* $X = \{a, b, c, d\}, \mathcal{M}= \{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$ *and* $C(\mathcal{M})$*(m-closed sets)* $= \{\emptyset, X, \{b, c, d\}, \{a, b, d\}, \{b, d\}\}$. *Consider* $A = \{b, c, d\}$, *then* $A \in \mathcal{M}\beta(X)$ *and* $A \in \mathcal{M}SO(X)$. *But* $A \notin \mathcal{M}\alpha(X)$ *and* $A \notin \mathcal{M}PO(X)$.

Remark 1. *If minimal structure* $\mathcal{M}$ *is a topology, then an* αm*-open set(resp. m-semi-open set, m-preopen set, m-*β*-open set) is* α*-open [22](resp. semi-open[13], preopen [14],* β*-open [1]).*

Definition 9. *A subset* F *of a minimal space* $(X, \mathcal{M})$ *is said to be* αm*-closed [16] (resp. m-semi-closed, m-preclosed , m-*β*-closed) if its complement is* αm*-open (resp. m-semi-open, m-preopen, m-*β*-open).*

Application of Proposition 2(8) are:

Theorem 3. *Let* $(X, \mathcal{M})$ *be a minimal space and* $A \subseteq X$. *Then*
(1) A *is a* αm*-closed set if and only if* $mCl(mInt(mCl(A))) \subseteq A$ *[15].*
(2) A *is a m-semi-closed set if and only if* $mInt(mCl(A)) \subseteq A$.
(3) A *is a m-preclosed set if and only if* $mCl(mInt(A)) \subseteq A$.
(4) A *is a m-*β*-closed set if and only if* $mInt(mCl(mInt(A))) \subseteq A$.

Theorem 4. *Let* $(X, \mathcal{M}, \mathcal{G})$ *be a grill minimal space and* $(X, \mathcal{M})$ *enjoys the property* I. *A subset* A *of* X *is said to be*
(1) ${}^*\mathcal{M}$ *- open if* $A \subseteq mInt((A)^{*\mathcal{M}})$,
(2) $\mathcal{GM}$*-*α*-open if* $A \subseteq mInt(\psi_{\mathcal{M}}(mInt(A)))$,
(3) $\mathcal{GM}$*-preopen if* $A \subseteq mInt(\psi_{\mathcal{M}}(A))$,
(4) $\mathcal{GM}$*-semi-open if* $A \subseteq \psi_{\mathcal{M}}(mInt(A))$,
(5) $\mathcal{GM}$*-*β*-open if* $A \subseteq mCl(mInt(\psi_{\mathcal{M}}(A)))$.

The family of all ${}^*\mathcal{M}$ - open (resp.$\mathcal{GM}$-α-open, $\mathcal{GM}$-preopen, $\mathcal{GM}$-semi-open, $\mathcal{GM}$-β-open) sets in a grill minimal space $(X, \mathcal{M}, \mathcal{G})$ is denoted by ${}^*\mathcal{M}O(X)$ (resp. $\mathcal{GM}\alpha O(X)$, $\mathcal{GM}PO(X)$, $\mathcal{GM}SO(X)$, $\mathcal{GM}\beta O(X)$).

Properties of above sets:

Theorem 5. *Let* $(X, \mathcal{M}, \mathcal{G})$ *be a grill minimal space and* $(X, \mathcal{M})$ *enjoys the property* I. *Then*
(1) $\mathcal{M} \subseteq \mathcal{GM}\alpha O(X) \subseteq \mathcal{GM}SO(X) \subseteq \mathcal{M}SO(X) \subseteq \mathcal{M}\beta(X)$.
(2) ${}^*\mathcal{M}O(X) \subseteq \mathcal{GM}PO(X) \subseteq \mathcal{GM}\beta O(X) \subseteq \mathcal{M}\beta(X)$.
(3) $\mathcal{M} \subseteq \mathcal{GM}\alpha O(X) \subseteq \mathcal{GM}PO(X) \subseteq \mathcal{M}PO(X) \subseteq \mathcal{M}\beta(X)$.
(4) $\mathcal{M} \subseteq \mathcal{GM}SO(X) \subseteq \mathcal{GM}\beta O(X) \subseteq \mathcal{M}\beta(X)$.

Proof. (1). Let $A \in \mathcal{M}$. Then $A \subseteq mInt(A)$, so $A \subseteq [mInt(A) \cup (mInt(A))^{*\mathcal{M}}]$. Hence $A \subseteq \psi_{\mathcal{M}}(mInt(A))$. This implies that $A \subseteq mInt(\psi_{\mathcal{M}}(mInt(A)))$. Again it is obvious that $A \subseteq \psi_{\mathcal{M}}(mInt(A))$. Now from $A \subseteq \psi_{\mathcal{M}}(mInt(A))$, $A \subseteq [mInt(A) \cup$

$(mInt(A))^{*\mathcal{M}}]$. Therefore $A \subseteq mCl(mInt(A))$ (using Proposition 2(4). Finally from $A \subseteq mCl(mInt(A))$, $A \subseteq mCl(A) \subseteq mCl(mInt(mCl(A)))$, from above.

Hence (1) is proved.

(2). Let $A \in {}^{*}\mathcal{M}O(X)$. Then $A \subseteq mInt((A)^{*\mathcal{M}})$, so $A \subseteq mInt[A \cup (A)^{*\mathcal{M}}]$. Hence $A \subseteq mInt(\psi_{\mathcal{M}}(A))$. Again it is obvious that $A \subseteq mCl(mInt(\psi_{\mathcal{M}}(A)))$. At last we get $A \subseteq mCl(mInt(A \cup (A)^{*\mathcal{M}}))$, and hence $A \subseteq mCl(mInt(mCl(A)))$(using Proposition 2(4).

Therefore proof of (2) is completed.

(3). Let $A \in \mathcal{M}$. Then $A \subseteq mInt(A)$, and hence $A \subseteq \psi_{\mathcal{M}}(mInt(A))$. So $A \subseteq mInt(\psi_{\mathcal{M}}(mInt(A)))$. Again it is obvious that $A \subseteq mInt(\psi_{\mathcal{M}}(A))$. Now from $A \subseteq mInt(\psi_{\mathcal{M}}(A))$, $A \subseteq mInt(mCl(A))$. Finally we have $A \subseteq mCl(mInt(mCl(A)))$.

Hence (3) is proved.

(4). Let $A \in \mathcal{M}$. Then it is obvious that $A \subseteq \psi_{\mathcal{M}}(mInt(A))$. Therefore $A \subseteq (mInt(A)) \cup (mInt(A))^{*\mathcal{M}}$. So $A \subseteq (mInt(A)) \cup (mCl(mInt(A)))$ implies $A \subseteq mCl(mInt(A))$. Hence $A \subseteq mCl(mInt(A \cup (A)^{*\mathcal{M}}))$. Then we have $A \subseteq mCl(mInt(\psi_{\mathcal{M}}(A)))$.

From above we get $A \subseteq mCl(mInt(A \cup (A)^{*\mathcal{M}}))$, so $A \subseteq mCl(mInt(mCl(A)))$ (using Proposition 2(4). For reverse inclusion, we are intimating following examples:

Example 6. *Let* $X = \{a, b, c, d\}$, $\mathcal{M} = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$ *and* $\mathcal{G} = \{\{a\}, \{b\}, \{a, c\}, \{a, b\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, b, d\}, \{a, c, d\}, \{b, c\}, \{b, d\}, \{b, c, d\}, X\}$. *Then* $C(\mathcal{M})$*(m-closed sets)* $= \{\emptyset, X, \{b, c, d\}, \{a, b, d\}, \{b, d\}\}$.

(1) $A = \{b, c, d\}$ *is a m-semi-open set which is not* $\mathcal{GM}$*-semi-open, because* $(\{c\})^{*\mathcal{M}} = \emptyset$.

(2) $A = \{b, c, d\}$ *is a* $\mathcal{GM}$*-*β*-open set which is not* $\mathcal{GM}$*-semi-open, because* $(\{c\})^{*\mathcal{M}} = \emptyset$.

(3) $B = \{a, b\}$*, then* $\psi_{\mathcal{M}}(mInt(\{a, b\})) = \psi_{\mathcal{M}}(\{a\}) = \{a\} \cup (\{a\})^{*\mathcal{M}} = \{a, b, d\}$. *Again* $\psi_{\mathcal{M}}(\{a, b\}) = \{a, b\} \cup (\{a, b\})^{*\mathcal{M}} = \{a, b, d\}$. *Therefore* B *is a* $\mathcal{GM}$*-semi-open set which is not a* $\mathcal{GM}$*-preopen set.*

(4) $C = \{a, b, c\}$. *Now For* C, $\psi_{\mathcal{M}}(mInt(C)) = \psi_{\mathcal{M}}(\{a, c\}) = \{a, c\} \cup (\{a, c\})^{*\mathcal{M}} = \{a, c\} \cup \{a, b, d\} = \{a, b, c, d\}$. *Therefore* C *is a* $\mathcal{GM}$*-*α*-open set which is not* $\mathcal{M}$*-open.*

Example 7. *Let* $X = \{a, b, c, d\}$, $\mathcal{M} = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$ *and* $\mathcal{G} = \{\{b\}, \{a, b\}, \{a, b, c\}, \{c, b, d\}, \{b, c\}, \{b, d\}, \{a, b, d\}, X\}$. *Then* $C(\mathcal{M}) = \{\emptyset, X, \{b, c, d\}, \{a, b, d\}, \{b, d\}\}$. *Consider* $A = \{a, c, d\}$. *Here* $(A)^{*\mathcal{M}} = \emptyset$. *Then* A *is* $m\alpha$*-open set and* $\mathcal{GM}$*-*β*-open set which is not* $\mathcal{GM}$*-preopen set.*

Example 8. *Let* $X = \{a, b, c\}$, $\mathcal{M} = \{\emptyset, X, \{a\}, \{b, c\}\}$ *and* $\mathcal{G} = \{\{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$. *So* $C(\mathcal{M}) = \{\emptyset, X, \{a\}, \{b, c\}\}$. *Then*

(1) $A = \{a, c\}$ *is a m-*β*-open set which is not* $\mathcal{GM}$*-*β*-open, since* $(A)^{*\mathcal{M}} = \{a\}$.

(2) $B = \{a, b\}$ is a $\mathcal{GM}$-preopen set which is not $\mathcal{GM}$-semi-open, since $(B)^{*_\mathcal{M}} = X$.

Theorem 6. *Let A be a subset of a grill minimal space $(X, \mathcal{M}, \mathcal{G})$ and $(X, \mathcal{M})$ enjoys the property I. Then following properties hold:*

(1) A is $\mathcal{GM}$-α-open if and only if it is $\mathcal{GM}$-semi-open and $\mathcal{GM}$-preopen.

(2) If A is $\mathcal{GM}$-semi-open, then A is $\mathcal{GM}$-β-open.

(3) If A is $\mathcal{GM}$-preopen, then A is $\mathcal{GM}$-β-open.

Proof. (1) Necessity. this is obvious.

Sufficiency. Given that $A \subseteq mInt(\psi_\mathcal{M}(A)) \subseteq mInt(\psi_\mathcal{M}(\psi_\mathcal{M}(mInt(A)))) \subseteq mInt(\psi_\mathcal{M}(mInt(A)))$. This shows that A is $\mathcal{GM}$-α-open.

(2) Given that $A \subseteq \psi_\mathcal{M}(mInt(A)) = (mInt(A)) \cup (mInt(A))^{*_\mathcal{M}} \subseteq mCl(mInt(A))$ (using Proposition 2.12(4))$\subseteq mCl(mInt(A \cup (A)^{*_\mathcal{M}})) = mCl(mInt(\psi_\mathcal{M}(A)))$. This shows that A is $\mathcal{GM}$-β-open.

(3) Proof is obvious.

Theorem 7. *Let $(X, \mathcal{M}, \mathcal{G})$ be a grill minimal space and $(X, \mathcal{M})$ enjoys the property I. Then for $A \subseteq X$, $A \in \mathcal{GM}SO(X)$ if and only if $\psi_\mathcal{M}(A) = \psi_\mathcal{M}(mInt(A))$.*

Proof. It is obvious that $\psi_\mathcal{M}(mInt(A)) \subseteq \psi_\mathcal{M}(A)$. Reverse inclusion is obvious from $A \in \mathcal{GM}SO(X)$.

Converse part is obvious from given the condition.

Remark 2. *If the minimal structure $\mathcal{M}$ of the grill minimal space $(X, \mathcal{M}, \mathcal{G})$ is a topology, then $^*\mathcal{M}$-open set(resp. $\mathcal{GM}$-α-open set, $\mathcal{GM}$-preopen set, $\mathcal{GM}$-semi-open set, $\mathcal{GM}$-β-open set) represents Al-Omari and Noiri's ϕ-open set(resp. $\mathcal{G}$-α-open set, $\mathcal{G}$-preopen set, $\mathcal{G}$-semi-open set, $\mathcal{G}$-β-open set).*

At the end of this section, we define following:

Definition 10. *Let $(X, \mathcal{M}, \mathcal{G})$ be a grill minimal space and $(X, \mathcal{M})$ enjoys the property I. A subset F of X is said to be $^*\mathcal{M}$-closed (resp. $\mathcal{GM}$-α-closed, $\mathcal{GM}$-preclosed, $\mathcal{GM}$-semi-closed, $\mathcal{GM}$-β-closed) if its complement is $^*\mathcal{M}$-open (resp. $\mathcal{GM}$-α-open, $\mathcal{GM}$-open, $\mathcal{GM}$-semi-open, $\mathcal{GM}$-β-open).*

Theorem 8. *If a subset A of a grill minimal space $(X, \mathcal{M}, \mathcal{G})$ is $\mathcal{GM}$-semi-closed, then $mInt(\psi_\mathcal{M}(A)) \subseteq A$.*

Theorem 9. *If a subset A of a grill minimal space $(X, \mathcal{M}, \mathcal{G})$ is $\mathcal{GM}$-preclosed, then $\psi_\mathcal{M}$
$(mInt(A)) \subseteq A$.*

4. Continuities on minimal space and grill minimal space

In this section we shall define some new types of continuity with the help of generalized set which have already been defined at earlier section. Further we have studied these types of continuity in detail.

Definition 11. *Let $(X, \mathcal{M})$ and $(Y, \mathcal{N})$ be two minimal spaces. We say that a function $f : (X, \mathcal{M}) \longrightarrow (Y, \mathcal{N})$ is said to αm-continuous [16](resp. m-semi-continuous, m-precontinuous, m-β-continuous) if $f^{-1}(B)$ is αm-open [16](resp. m-semi-open, m-preopen, m-β-open) for each m-open subset B of $(Y, \mathcal{N})$.*

As the application of Theorem 2, we get following results:

Theorem 10. *Every αm-continuous map is a m-semi-continuous map.*

(b). Every m-precontinuous continuous map is a m-β-continuous map.

(c). A map is αm-continuous if and only if it is m-semi-continuous as well as m-precontinuous.

Theorem 11. *Let $(X, \mathcal{M})$ and $(Y, \mathcal{N})$ be two minimal spaces. Then the function $f : (X, \mathcal{M}) \longrightarrow (Y, \mathcal{N})$ is αm-continuous [16] (resp. m-semi-continuous, m-precontinuous, m-β-continuous) if and only if $f^{-1}(B)$ is αm-closed [15] (resp. m-semi-closed, m-preclosed, m-β-closed) for each m-closed subset B of $(Y, \mathcal{N})$.*

Proof. Proof. Proof is obvious.

Continuity on grill minimal spaces:

Definition 12. *A function $f : (X, \mathcal{M}, \mathcal{G}) \longrightarrow (Y, \mathcal{N})$ is said to be $^*\mathcal{M}$-continuous (resp. $\mathcal{GM}$-α-continuous, $\mathcal{GM}$-precontinuous, $\mathcal{GM}$-semi-continuous, $\mathcal{GM}$-β-continuous) if the inverse image of each m-open set of Y is $^*\mathcal{M}$ open(resp. $\mathcal{GM}$-α-open, $\mathcal{GM}$-α-continuous, $\mathcal{GM}$-preopen, $\mathcal{GM}$-β-open).*

From Theorem 5 we get following remark:

Remark 3. *(a). m-Continuity $\Rightarrow$ $\mathcal{GM}\alpha$-continuity $\Rightarrow$ $\mathcal{GM}$-semi-continuity $\Rightarrow$ m-semi-continuity $\Rightarrow$ m-β-continuity.*

(b). $^*\mathcal{M}$-continuity $\Rightarrow$ $\mathcal{GM}$-precontinuity $\Rightarrow$ $\mathcal{GM}$-β - continuity $\Rightarrow$ m-β-continuity.

(c). m-continuity $\Rightarrow$$\mathcal{GM}\alpha$-continuity $\Rightarrow$ $\mathcal{GM}$-precontinuity $\Rightarrow$ m-precontinuity $\Rightarrow$ m-β-continuity.

(d). m-continuity $\Rightarrow$ $\mathcal{GM}$-semi-continuity $\Rightarrow$ $\mathcal{GM}$-β-continuity $\Rightarrow$ m-β-continuity.

From Theorem 6 we get following results:

Theorem 12. *A function* $f : (X, \mathcal{M}, \mathcal{G}) \longrightarrow (Y, \mathcal{N})$ *is*
(1) $\mathcal{GM}$*-*α*-continuous if and only if* $\mathcal{GM}$*-semi-continuous and* $\mathcal{GM}$*-precontinuous.*
(2) $\mathcal{GM}$*-semi-continuous, then it is* $\mathcal{GM}$*-*β*-continuous.*
(3) $\mathcal{GM}$*-precontinuous, then it is* $\mathcal{GM}$*-*β*-continuous.*

Above continuities have also been redefined with the help of corresponding closed sets.

Theorem 13. *A map* $f : (X, \mathcal{M}, \mathcal{G}) \longrightarrow (Y, \mathcal{N})$ *is* $^*\mathcal{M}$*-continuous (resp.* $\mathcal{GM}$*-*α*-continuous,* $\mathcal{GM}$*-precontinuous,* $\mathcal{GM}$*-semi-continuous,* $\mathcal{GM}$*-*β*-continuous) if and only if* $f^{-1}(B)$ *is* $^*\mathcal{M}$*-closed (resp.* $\mathcal{GM}$*-*α*-closed,* $\mathcal{GM}$*-preclosed,* $\mathcal{GM}$*-semiclosed,* $\mathcal{GM}$*-*β*-closed) for each m-closed subset* B *of* $(Y, \mathcal{N})$*.*

As the consequence of the Theorem 7 we get following theorem:

Theorem 14. *A function* $f : (X, \mathcal{M}, \mathcal{G}) \longrightarrow (Y, \mathcal{N})$ *is* $\mathcal{GM}$*-semi-contionuos if and only* $\psi_{\mathcal{M}}(f^{-1}(B)) = \psi_{\mathcal{M}}(mInt(f^{-1}(B)))$ *for each* $\mathcal{M}$*-open subset* B *of* $(Y, \mathcal{N})$*.*

Corollary 15. *A function* $f : (X, \mathcal{M}, \mathcal{G}) \longrightarrow (Y, \mathcal{N})$ *is* $\mathcal{GM}$*-semi-contionuos if and only* $\psi_{\mathcal{M}}(f^{-1}(C)) = \psi_{\mathcal{M}}(mInt(f^{-1}(C)))$ *for each* $\mathcal{M}$*-closed subset* C *of* $(Y, \mathcal{N})$*.*

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