On the Breaking of Mirror Symmetry in Homogeneous Isotropic Turbulence-Helicity Effect

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Dedicated to Prof.Dr. Constantin UDRIŞTE
on the occasion of his sixtieth birthday

Abstract

In homogeneous isotropic turbulence when reflexional (or mirror) symmetry breaks down, it becomes necessary to take accounts of the additional fundamental invariants in constructing forms for correlation tensors of different orders that characterize the flow. It is helicity which plays its role in such a situation. Attention is paid here, in particular to the construction of appropriate invariant forms for the several two-point second and third order correlation tensors that arise out of fluctuating pressure, velocity and external force fields of such a turbulent flow. Utilizing these forms of second and third order tensors, the two-point vorticity-vorticity and acceleration-acceleration correlations are determined, separately illustrating the effects of helicity on them.

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1 Introduction

The problems of turbulent flows are studied much in some simplifying situations as to make their mathematical representation possible and to verify the results experimentally. The turbulence is said to be simply homogeneous if the statistical parameters describing the flow, remain unchanged under the translation of the axes. The turbulence is said to be isotropic, or homogenous isotropic if the statistical parameters e.g., correlation functions of different orders remain invariant with respect to entire rotation group of transformations and as well as reflexion at an arbitrary point. The mean velocity is zero in homogeneous isotropic turbulence. It is well known that such turbulence can approximately be produced in the laboratory. Kitt et al. [1,2] however, demonstrated experimentally that grid generated homogeneous turbulence (invariant under the rotation group of transformations) may lack reflexional (or mirror) symmetry and possess significant mean helicity $< \vec{u} \cdot \vec{\varpi} >$, where $\vec{u}$ and $\vec{\varpi}$ are, respectively the velocity and vorticity fields and $< \cdot >$ indicates the ensemble average.

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Our aim in this paper, is to examine the constructions of tensorial forms for two-point correlation functions of different orders and their applications to fluctuating vorticity-vorticity and acceleration-acceleration correlations in homogeneous isotropic turbulence without possessing reflexional (or mirror) symmetry.

The fluid will be assumed incommpressible throughout the analysis (that is, density is constant).

2 Correlation tensors and their forms

Various quantities, namely the velocity, the vorticity, the acceleration, the pressure, the temperature or the density fluctuations etc., which characterize the turbulent flow are considered as random functions in the four dimensional space $(\vec{r}, t)$ (Lesieur [3]). In homogeneous isotropic turbulence possessing reflexional symmetry, the correlation functions of different orders formed from these quantities remain invariant, in geometrical sense, under arbitrary rigid rotations and reflexions of the configuration of the vector arguments. For instance, in respect of two arbitrary fixed vectors $\vec{a}$ and $\vec{b}$, the tensorial product $a_i U_{ij}(\vec{r}, t' t'') b_j$, where $U_{ij}(\vec{r}, t', t'') =< u'_i(\vec{r}\,', t') u'_j(\vec{r}\,'', t'') >$, $\vec{r}\,'' - \vec{r}\,' = \vec{r}$, is a scalar and accordingly, must be invariant under translation and rotation of the three vectors $(\vec{r}, \vec{a}, \vec{b}) \cdot u'_i, u''_j$ are the fluctuating velocity components. Robertson [4] (see also Batchelor [5], Hinze [6]) showed that such an invariant function can be expressed in terms of the fundamental invariants due to a rigorous result of group invariant theory. The invariants are in this case the scalar products $\vec{r} \cdot \vec{r}$, $\vec{r} \cdot \vec{a}$, $\vec{r} \cdot \vec{b}$, $\vec{a} \cdot \vec{a}$, $\vec{b} \cdot \vec{b}$ and $\vec{a} \cdot \vec{b}$. If the reflexional (or mirror) symmetry breaks down, an additional fundamental invariant $\vec{r} \cdot (\vec{a} \times \vec{b})$ is to be taken into account. In this last case helicity of the flow plays an important role, which will be illustrated in sections 3 and 4.

We now describe the forms for the correlation tensors of first, second and third orders pertaining to two points and at two different times in a non-stationary homogeneous isotropic turbulence possessing reflexional symmetry.

Homogeneous isotropic turbulence possessing reflexional symmetry

$$L_i(\vec{r}, t', t'') =< \theta'(\vec{r}\,', t') u_i(\vec{r}\,'', t'') >= L r_i, \tag{1}$$

$\vec{r}\,'' - \vec{r}\,' = \vec{r}, \theta'$ is a scalar, namely fluctuating pressure, temperature or density.

$$U_{ij}(\vec{r}, t', t'') =< u'_i(\vec{r}\,', t') u''_j(\vec{r}\,'', t'') >= A\delta_{ij} + B r_i r_j \tag{2}$$

$$\begin{aligned} T_{ij;k}(\vec{r}, t', t'') &= < u'_i(\vec{r}\,', t') u'_j(\vec{r}\,', t') u''_k(\vec{r}\,'', t'') >= \\ &= T_1 r_i r_j r_k + T_2 r_i \delta_{jk} + T_3 r_j \delta_{ki} + T_4 r_k \delta_{ij} \end{aligned} \tag{3}$$

$$A_{ij}(\vec{r}, t', t'') =< A'_i(\vec{r}\,', t') A''_j(\vec{r}\,'', t'') >= a\delta_{ij} + \beta r_i r_j, \tag{4}$$

where A'_i and A''_j are the fluctuating components of acceleration.

$$P_{ij}(\vec{r}\,, t', t'') =< u'_i(\vec{r}\,', t')u'_j(\vec{r}\,', t')\frac{p''}{\rho}(\vec{r}\,'', t'') >= P_1 r_i r_j + P_2\delta_{ij}, \tag{5}$$

where p'' is the pressure fluctuating and ρ is the density of the fluid.

$$\varpi_{ij}(\vec{r}\,, t', t'') =< \varpi'_i(\vec{r}\,', t')\varpi''_j(\vec{r}\,'', t'') >= a^*\delta_{ij} + \beta^* r_i r_j, \tag{6}$$

where ϖ'_i and ϖ''_j are the fluctuating components of vorticity. We can write forms similar to (1) for pressure-velocity correlation tensors like

$$B_i =< u'_i(\vec{r}\,', t')p''(\vec{r}\,'', t'') >$$

and

$$B_j =< p'_i(\vec{r}\,', t')u''_j(\vec{r}\,'', t'') > .$$

The defining scalars in (1)-(6), namely L, A, B, T_1, T_2, T_3, T_4, a, β, P_1, P_2, a^*, β^* are all functions of $r = |\vec{r}\,'' - \vec{r}\,'|, t'$ and t''. But in the case when the turbulence is homogeneous isotropic and stationary, all these defining become functions of r and $t = |t''-t'|$. It is to be mentioned that for a such a stationary turbulence a forcing term is appended in the dynamical equation (Navier-Stokes) for sustaining the turbulence.

Form for the fourth order correlation tensor of fluctuating velocity components, namely

$$U_{ij;kl} =< u'_i(\vec{r}\,', t')u'_j(\vec{r}\,', t')u''_k(\vec{r}\,'', t'')u''_l(\vec{r}\,'', t'') >$$

can be obtained easily by Robertson's invariant theory (see Batchelor [5]). The incompressibility condition $\dfrac{\partial u_i}{\partial r_i} = 0$, $\dfrac{\partial}{\partial r_i}U_{ij} = 0$ or the solenoidal condition $\dfrac{\partial}{\partial r_k}T_{ij;k} = 0$ will help in reducing the number of defining scalars. It can be shown easily that all first order tensors like correlation between a scalar (pressure, temperature etc.) and a vector, vanish in homogeneous isotropic turbulence whether reflexional symmetry holds or not (see Lesieur [3]).

Homogeneous isotropic turbulence without possessing reflexional symmetry

In this case,

$$U_{ij} = A\delta_{ij} + Br_i r_j + C\epsilon_{ijs} r_s, \tag{7}$$

where C is an additional defining scalar. Henceforth, the dependence of the defining scalars will be assumed on the absolute space separation and on absolute time separation or different times, accordingly as the turbulence is stationary or non-stationary.

We now choose the frame of reference as such $\vec{r}$ has components $(r, 0, 0)$. Therefore,

$$\left.\begin{array}{ll} U_{11} = A + r^2 B = F, & \text{say} \\ U_{22} = A = G, & \text{say} \\ U_{23} = rC = H, & \text{say} \end{array}\right\}. \tag{8}$$

Due to incompressibility condition $\dfrac{\partial}{\partial r_i}U_{ij} = 0$ and it can be easily shown that

$$G = F + \frac{r}{2}\frac{\partial F}{\partial r}. \tag{9}$$

In view of the relations (8) and (9), the relation (7) is transformed to

$$U_{ij} = \left(F + \frac{rF}{2}\right)\delta_{ij} + \left(-\frac{1}{2r}F\right) r_i r_j + \frac{H}{r}\epsilon_{ijs} r_s. \tag{10}$$

The mean helicity is defined by $H_e = \frac{1}{2} < \vec{u}(\vec{r}\,,t) \cdot [\nabla \times \vec{u}(\vec{r}\,,t)] >$ and it can be shown that $H_e = -3C(0,t)$ (see Lesieur [3]).

$$\begin{aligned} T_{ij;k} &= -\tfrac{T'}{r} r_i r_j r_k - \tfrac{1}{2}\left(rT' + 3T\right)\left(r_i\delta_{jk} + r_j\delta_{ki}\right) + T r_k \delta_{ij} + \\ &+ M\left(\epsilon_{jkm} r_m r_i - \epsilon_{kim} r_m r_j\right). \end{aligned} \tag{11}$$

It is to be noticed in the form (11) of the triple-velocity correlation that a single defining scalar T has appeared due to solenoidal condition at the index k and M has appeared due to lack of reflexional symmetry. Also, as u'_i and u'_j being both at the same point, $T_{ij;k}$ should not be changed if i and j are interchanged. The form (11) clearly satisfies this condition.

$$A_{ij} = a\delta_{ij} + \beta r_i r_j + \gamma\epsilon_{ijs} r_s. \tag{12}$$

As reflexional symmetry does not hold, the third term with defining scalar γ has appeared in the form (12) of correlation between acceleration fluctuations.

$$P_{ij} = P_1 r_i r_j + P_2\delta_{ij}. \tag{13}$$

P_{ij} being the second order correlation $< u'_i u'_j \frac{p''}{\rho} >$, it will remain unchanged if i and j are interchanged. So, lack of reflexional symmetry does not affect the form of P_{ij}.

The correlation between the vorticity fluctuations ϖ_{ij} will have the form

$$\varpi_{ij} = a^*\delta_{ij} + \beta^* r_i r_j + \gamma^*\epsilon_{ijs} r_s. \tag{14}$$

γ^* is the additional defining scalar appeared due to same reason as in (12).

In the case of homogeneous isotropic stationary turbulence we shall have to consider external forcing term f'_i, which is assumed to be divergence free i.e., $\frac{\partial f'_i}{\partial r'_i} = 0$. Accordingly, we need to consider the following correlation functions

$$M_{ij} =< u'_i f''_j >= \left(M_1 + \frac{rM'_1}{2}\right)\delta_{ij} + \left(-\frac{M'_1}{2r}\right) r_i r_j + \frac{M_2}{r}\epsilon_{ijs} r_s \tag{15}$$

$$G_{ij} =< f'_i u''_j >= \left(G_1 + \frac{rG'_1}{2}\right)\delta_{ij} + \left(-\frac{G'_1}{2r}\right) r_i r_j + \frac{G_2}{r}\epsilon_{ijs} r_s \tag{16}$$

$$S_{ij} =< f'_i f''_j >= \left(S_1 + \frac{rS'_1}{2}\right)\delta_{ij} + \left(-\frac{S'_1}{2r}\right) r_i r_j + \frac{S_2}{r}\epsilon_{ijs} r_s \tag{17}$$

$$I_{ij;k} = < u'_i u'_j f''_k >= -\frac{I'}{r} r_i r_j r_k - \tag{18}$$
$$- \frac{1}{2}\left(rI' + 3I\right)\left(r_i\delta_{jk} + r_j\delta_{ki}\right) + Ir_k\delta_{ij} + J\left(\epsilon_{jkm}r_m r_i - \epsilon_{kim}r_m r_j\right).$$

The forms (15), (16) and (17), respectively for M_{ij}, G_{ij} and S_{ij} are constructed in the same manner as in (10) for U_{ij}. M_2, G_2 and S_2 are the defining scalars due to lack of reflexional symmetry. The form for the triple order correlation $I_{ij;k}$ involves single defining scalar I similar to T in the form of $T_{ij;k}$ in (11). The defining scalar J has appeared in (18) due to the violation of reflexional symmetry.

It is to be noted that the defining scalars F in (10), T in (11), M_1 in (15), G_1 in (16) and S_1 in (17) with primes indicate their differentiations with respect to r.

3 Correlation between components of vorticity at two different points

The turbulent vorticity $\vec{\varpi}(\vec{r}\,', t)$ at a point $(\vec{r}\,', t)$, say is defined by

$$\overrightarrow{\varpi}\left(\vec{r}\,', t\right) = \nabla \times \vec{u}\left(\vec{r}\,', t\right). \tag{19}$$

The components of vorticity is given by

$$\varpi_i\left(\vec{r}\,', t\right) = \epsilon_{ijk}\frac{\partial u_k\left(\vec{r}\,', t\right)}{\partial r_j}; \tag{20}$$

ϖ_1, ϖ_2 and ϖ_3 are the components of vorticity.

The correlation tensor formed from the components of vorticity at two different points $\vec{r}\,'$ and $\vec{r}\,''$ and at the same instant of time t, say can be expressed from kinematical consideration as (Batchelor [5])

$$\begin{aligned} \varpi_{ij}(\vec{r}, t) &= < \varpi'_i(\vec{r}\,', t)\varpi''_j(\vec{r}\,'', t) >= \\ &= \epsilon_{ilm}\epsilon_{jpg}\frac{\partial u'_m(\vec{r}\,', t)}{\partial r'_l}\frac{\partial u''_q(\vec{r}\,'', t)}{\partial r''_p} - \\ &= -\delta_{ij}\nabla^2 U_{ll}\left(\vec{r}, t\right) + \frac{\partial^2 U_{ll}\left(\vec{r}, t\right)}{\partial r_i \partial r_j} + \nabla^2 U_{ji}\left(\vec{r}, t\right), \end{aligned} \tag{21}$$

where $\vec{r} = |\vec{r}\,'' - \vec{r}\,'|$; $r_i (i = 1, 2, 3)$ are the components of $\vec{r}$.

Now, we assume that the turbulence is homogeneous isotropic but lacks reflexional symmetry. The expression (10) for U_{ij} and the expression (14) for ϖ_{ij} are recalled here again, as

$$U_{ij}\left(\vec{r}, t\right) = \left(F + \frac{rF'}{2}\right)\delta_{ij} + \left(-\frac{1}{2r}F'\right)r_i r_j + \frac{H}{r}\epsilon_{ijs}r_s \tag{22}$$

$$\varpi_{ij}\left(\vec{r}, t\right) = a^*\delta_{ij} + \beta^* r_i r_j + \gamma^*\epsilon_{ijs}r_s; \tag{23}$$

F, H, a^*, β^* and γ^* are functions of $r\,(= |\vec{r}\,|)$ and t.

Taking (22) into account on the right hand side of (21) we obtain, after a long but straight forward calculations
(24)

$$\begin{aligned}\varpi_{ij}(\vec{r},t) &= \left[-\frac{r}{2}F''' - 3F'' - \frac{2}{r}F'\right]\delta_{ij} + \left[\frac{1}{2r}F''' + \frac{7}{2r^2}F'' - \frac{7}{2r^3}F'\right]r_i r_j + \\ &+ \left[-\frac{H''}{r} - \frac{2H'}{r^2} + \frac{2H}{r^3}\right]\epsilon_{ijs}r_s.\end{aligned}$$

Comparing (23) with (24), we obtain

$$\begin{aligned}a^* &= -\frac{r}{2}F''' - 3F'' - \frac{2}{r}F' \\ \beta^* &= -\frac{r}{2r}F''' + \frac{7}{2r^2}F'' - \frac{7}{2r^3}F' \\ \gamma^* &= -\frac{H''}{r} - \frac{2H'}{r^2} + \frac{2H}{r^3}.\end{aligned} \tag{25}$$

It is to be noted that F''', F'' and F' are, respectively the triple, double and single derivatives of F with respect to r. Similarly, H'' and H' are the double and single derivative of H with respect to r.

Since, the frame of reference be as such $\vec{r}$ has the components $(r, 0, 0)$. That is, $r_1 = r$, $r_2 = 0$, $r_3 = 0$. We therefore, obtain from (24) easily

$$\begin{aligned}&< \varpi_1'\varpi_1'' >= a^* + \beta^* r^2, \quad < \varpi_2'\varpi_2'' >=< \varpi_3'\varpi_3'' >= a^* \\ &< \varpi_1'\varpi_3'' >=< \varpi_1'\varpi_2'' >= 0 \\ &< \varpi_2'\varpi_3'' >= \gamma^* r.\end{aligned} \tag{26}$$

From (25) and (26), it follows that helicity $< u_2'u_3'' >$ $(= H)$ of the flow is related to cross correlation of vorticity fluctuations $< \varpi_2'\varpi_3'' >$ in homogeneous isotropic turbulence without possessing reflexional (mirror) symmetry.

4 Correlation between fluctuations of total acceleration at two different points

In this case, we assume the turbulence to be homogeneous isotropic, stationary and without possessing reflexional symmetry.

The Navier-Stokes equation for $u_i'(\vec{r}', t')$ with external force term $f_i'(\vec{r}', t)$ at point $(\vec{r}\,', t')$ is written as

$$\frac{\partial u_i'}{\partial t'} + \frac{\partial}{\partial r_j'}u_i'u_j' = -\frac{1}{\rho}\frac{\partial p'}{\partial r_i'} + \nu\nabla_{r'}^2 u_i' + f_i', \tag{27}$$

where $p'(\vec{r}\,', t')$ is the pressure fluctuation.

The total acceleration fluctuation $A_i'(\vec{r}\,', t')$ is given by

$$A_i'(\vec{r}\,', t) = \frac{Du_i'}{Dt'} = \frac{\partial u_i'}{\partial t'} + u_k\frac{\partial u_i'}{\partial r_k'}. \tag{28}$$

Taking (28) into account in (27), we obtain

$$A'_i = -\frac{1}{\rho}\frac{\partial p'}{\partial r'_i} + \nu\nabla^2_r u'_i + f'_i. \tag{29}$$

In a similar manner, we can write for acceleration fluctuation $A''_j(\vec{r}\,'', t'')$ at the point $(\vec{r}\,'', t'')$ as

$$A''_j = -\frac{1}{\rho}\frac{\partial p''}{\partial r''_j} + \nu\nabla^2_{r''} u''_j + f''_j. \tag{30}$$

Now, multiplying (30) with (29) and then averaging the resultant equation, we obtain

$$\begin{aligned} < A'_i A''_j > \;&=\; \frac{1}{\rho^2}\frac{\partial^2}{\partial r'_i \partial r''_j} < p'p'' > +\nu^2\nabla^2_{r''}\nabla^2_{r'} < u'_i u''_j > \\ &+\; \nu\nabla^2_{r''} < f'_i u''_j > +\nu\nabla^2_{r'} < u'_i f''_j > + < f'_i f''_j > . \end{aligned} \tag{31}$$

In view of the definitions of the correlations involved, we can rewrite equation (31) as

$$A_{ij} = -\frac{1}{\rho^2}\frac{\partial^2}{\partial r_i \partial r_j} < p'p'' > +\nu^2\nabla^4_r U_{ij} + \nu\nabla^2(G_{ij} + M_{ij}) + S_{ij}, \tag{32}$$

since

$$r''_i - r'_i = r_i,\ \frac{\partial}{\partial r''_i} = -\frac{\partial}{\partial r'_i} = \frac{\partial}{\partial r_i},\ \nabla^2_{r''} = \nabla^2_{r'} = \nabla^2_r = \frac{\partial^2}{\partial r_i \partial r_i} \text{ and } |\vec{r}\,| = r.$$

Applying (12), (15), (16) and (17) in equation (32), we obtain

$$\begin{aligned} &a\delta_{ij} + \beta r_i r_j + \gamma\epsilon_{ijs}r_s = -\frac{1}{\rho^2}\left\{\left[\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{P}{r}\right)\right] r_i r_j + \frac{P'}{r}\delta_{ij}\right\} - \\ &-\frac{\nu^2}{2}\left(\frac{F^{(v)}}{r} + \frac{8F^{(iv)}}{r^2} - \frac{24F''}{r^4} + \frac{24F'}{r^5}\right) r_i r_j - \\ &-\frac{\nu^2}{2}\left(-rF^{(v)} - 10F^{(iv)} - \frac{16F'''}{r} + \frac{8F''}{r^2} - \frac{8F'}{r^3}\right)\delta_{ij} + \\ &+\nu^2\left(\frac{H^{(iv)}}{r} + \frac{4H'''}{r^2} - \frac{4H''}{r^3}\right)\epsilon_{ijs}r_s + \\ &+\nu\left[-\frac{1}{2}\left(\frac{1}{r}(G'''_1 + M'''_1)\right) + \frac{4}{r^2}(G''_1 + M''_1) - \frac{4}{r^3}(G'_1 + M'_1)\right] r_i r_j + \\ &+\nu\left[-\frac{1}{2}(r((G'''_1 + M'''_1) + 2(G''_1 + M''_1) - \frac{12}{r}(G'_1 + M'_1)\right]\delta_{ij} + \\ &+\nu\left[D_5(G_3 + M_3)\right]\epsilon_{ijs}r_s - \frac{1}{2r}S'_1 r_i r_j + \left(S_1 + \frac{r}{2}S'_1\right)\delta_{ij} + S_3\epsilon_{ijs}r_s, \end{aligned} \tag{33}$$

where

$$P =< p'p'' >, \; G_3 = \frac{G_2}{r}, \; M_3 = \frac{M_2}{r}, \; S_3 = \frac{S_2}{r} \text{ and } D_5 = \frac{\partial^2}{\partial r^2} + \frac{4}{r}\frac{\partial}{\partial r}.$$

The quantities $F^{(v)}$, $F^{(iv)}$ represent respectively fifth and fourth order derivatives of F with respect to r. $H^{(iv)}$ and H''' are respectively the fourth order and third order derivatives of H with respect to r. The quantities Gs, Ms and Ss with single, double and triple primes represent, respectively their first, second and third order derivatives with respect to r.

It is to be mentioned here that all the defining scalars appeared in (33) are functions of r and $t = |t'' - t'|$, since the turbulence is homogeneous isotropic and stationary. Ghosh [7], after a long calculation derived the relation between $P =< p'p'' >$ and F as

$$\frac{1}{\rho^2}\frac{\partial}{\partial r}\left(\frac{P'}{r}\right) = \frac{4F'^2}{r}. \tag{34}$$

For convenience of the readers, we state, briefly the procedure of deriving equation (34).

Multiplying equation (27) by p'' at $(\vec{r}'', t'')$ and then averaging the resultant equation, one obtains

$$\frac{1}{\rho^2}\frac{\partial}{\partial r'_i} < p'p'' >= -\frac{\partial}{\partial r'_j} < u'_i u'_j \frac{p''}{\rho} > \tag{35}$$

since $< u'_i p'' >= 0$ and $< f'_i p'' >= 0$, as the first order solenoidal tensors are identically zero also in homogeneous isotropic turbulence without possessing reflexional symmetry.

From (35), with the help of (13), it can be easily derived

$$\frac{1}{\rho^2}\frac{\partial}{\partial r} < p'p'' >= -(r^2 P'_1 + 4rP_1 + P_2). \tag{36}$$

Considering the equation for u''_i at $(\vec{r}'', t'')$, namely

$$\frac{\partial u''_i}{\partial t''} + \frac{\partial}{\partial r''_k} u''_i u''_j = -\frac{1}{\rho}\frac{\partial p''}{\partial r''_i} + \nu \nabla^2_{r'} u''_i + f''_i \tag{37}$$

and multiplying it by $u'_j u'_l$ and then finally averaging, an equation connecting third order correlation tensor to fourth order correlation tensor is obtained as

$$\frac{\partial}{\partial t} t_{jl;i} + \nu \nabla^2_r T_{jl;i} + I_{jl;i} = \frac{\partial}{\partial r_k} Q_{jl;ik} + \frac{\partial}{\partial r_i} P_{jl}, \tag{38}$$

where $Q_{jl;ik} =< u'_j u'_l u''_i u''_k >$.

One may write

$$X_{jl;i} = \frac{\partial}{\partial r_k} Q_{jl;ik} + \frac{\partial}{\partial r_i} P_{jl}. \tag{39}$$

Now, applying the quasi-normally hypothesis that the fourth order moment is related to second order moments as in a normal distribution (Chandrasekhar [8]), namely

$$\begin{aligned} \text{(40)} \quad < u'_j u'_l u''_i u''_k > &= < u'_j u'_l >< u''_i u''_k > + < u''_i u'_l >< u'_j u''_k > + \\ &+ < u''_i u'_j >< u''_k u'_l >, \end{aligned}$$

the solenoidality of $X_{jl;i}$ in the index i and $X_{ll;i} = 0$ to equation (39), the following two relations can be obtained:

$$\text{(41)} \qquad P''_1 + \frac{6P'_1}{r} = -\frac{F'F''}{r} - \frac{6F'^2}{r^2} - \frac{HH'}{r^3} + \frac{4H^2}{r^4}$$

$$\text{(42)} \qquad P'_2 - \frac{P_2}{r} - rP'_1 = rF'F'' + 2F'^2 + \frac{4HH'}{r} - \frac{4H^2}{r^2}.$$

Eliminating P_1 and P_2 from (41) and (42), equation (34) is obtained.

It is to mentioned that forms for the third order tensors $T_{ij;k}$ and $I_{ij;k}$ given, respectively by (11) and (18) are used in deriving equation (34).

Taking (34) into consideration and comparing coefficients to both sides of (33), we finally obtain:

(43)

$$\begin{aligned} a^*(r,t) &= -\int \frac{4F^2}{r} dr - \frac{\nu^2}{2}\left(-rF^{(v)} - 10F^{(iv)} - \frac{16F'''}{r} + \frac{8F''}{r^2} - \frac{8F'}{r^3}\right) + \\ &+ \frac{\nu}{2}\left[r(G'''_1 + M'''_1) + 2(G''_1 + M''_1) - \frac{12}{r}(G'_1 + M'_1)\right] + \left(S_1 + \frac{r}{2}S'_1\right) \end{aligned}$$

$$\begin{aligned} \text{(44)} \quad \beta^*(r,t) &= -\frac{4F'^2}{r^2} - \frac{\nu^2}{2}\left(\frac{F^{(v)}}{r} + \frac{8F^{(iv)}}{r^2} - \frac{24F''}{r^4} + \frac{24F}{r^5}\right) - \\ &- \frac{\nu}{2}\left[\frac{1}{r}(G'''_1 + M'''_1)\right) + \frac{4}{r^2}(G''_1 + M''_1) - \frac{4}{r^3}(G'_1 + M'_1) - \frac{S'_1}{2r} \end{aligned}$$

$$\text{(45)} \qquad \gamma^*(r,t) = \nu^2\left(\frac{H^{(iv)}}{r} + \frac{4H'''}{r^2} - \frac{4H''}{r^3}\right) + \nu[D_5(G_3 + M - 3)] + S_3.$$

Thus to determine the defining scalars α^*, β^* and γ^* of the second order accelaration correlation, we need to know the defining scalars Fs, Ms, Gs, Ss of the respective second order correlations (10), (15), (16), (17) and H, involved in all these correlations.

Remark 1. Turbulent flow is essentially rotational in nature, so the result that the helicity $< u'_2 u''_3 >$ $(= H)$ is closely related to the second order cross correlation of vorticity fluctuations, namely $< \varpi'_2, \varpi''_3 >$ obtained in the present calculation is important.

Remark 2. Determination of second order correlation of acceleration fluctuations is more complicated that the determination of correlation of vorticity fluctuations in homogeneous isotropic turbulence without possessing reflexional (or mirror) symmetry.

Remark 3. The present calculation warrants extensive measurements of correlation functions in the turbulent flow under consideration.

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