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TETRACYCLIC HARMONIC GRAPHS

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A b s t r a c t. A graph G on n vertices $v_1, v_2, \ldots, v_n$ is said to be harmonic if $(d(v_1), d(v_2), \ldots, d(v_n))^t$ is an eigenvector of its $(0,1)$-adjacency matrix, where $d(v_i)$ is the degree (= number of first neighbors) of the vertex v_i , $i = 1, 2, \ldots, n$. Earlier all acyclic, unicyclic, bicyclic and tricyclic harmonic graphs were characterized. We now show that there are 2 regular and 18 non-regular connected tetracyclic harmonic graphs and determine their structures.

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1. *Introduction*

Let $G = (V(G), E(G))$ be a graph with $|V(G)| = n$ vertices and $|E(G)| = m$ edges, whose vertices are labeled by $v_1, v_2, \ldots, v_n$. A *walk of length k* in G is an ordered $(k+1)$-tuple of vertices, $(v_{i_0}, v_{i_1}, \ldots, v_{i_k})$, such that for all $j = 1, \ldots, k$, $(v_{i_{j-1}}, v_{i_j}) \in E(G)$. The number of all walks of length k in the graph G is denoted by $W_k(G)$. It is both consistent and convenient to set $W_0(G) = n$; note also that $W_1(G) = 2m$.

In a recent work [7] the way in which $W_k(G)$ increases with k was studied. For this an auxiliary quantity $\Delta_k(G)$ was introduced [4, 5], defined as

$$\Delta_k(G) = W_{k+1}(G)\, W_{k-1}(G) - W_k(G)^2 \ .$$

It is easy to show that the equality $\Delta_k(G) = 0$ holds for all $k \geq 1$ if and only if G is a regular graph. There exist graphs for which the equality $\Delta_k(G) = 0$ holds for all $k > 1$. These were named *harmonic graphs* [4, 5] and may be viewed as a peculiar generalization of regular graphs. Grünewald [6] determined all harmonic trees (for details see below) and the present authors together with Grünewald determined all unicyclic, bicyclic and tricyclic harmonic graphs [1]. In this work we go a step further and find all tetracyclic harmonic graphs. In order to do this we need some preparation.

If the graph G has p components, then $c = m - n + p$ is the cyclomatic number of G and this graph is said to be c-cyclic. In particular, if $c = 4$ we speak of tetracyclic graphs. If the graph G is connected ($p = 1$) and $c = 0$ then G is a tree.

The number of first neighbors of the vertex v_i is the degree of this vertex and is denoted by $d(v_i)$. A vertex of degree k will be referred to as a k-vertex. The column–vector $(d(v_1), d(v_2), \ldots, d(v_n))^t$ is denoted by $d(G)$.

The number of k-vertices is denoted by n_k. Then

$$\sum_{k \geq 0} n_k \quad = \quad n \ , \tag{1}$$

$$\sum_{k \geq 0} k\, n_k \quad = \quad 2m \ . \tag{2}$$

A graph G is said to be *harmonic* [4, 5] if there exists a constant λ, such that the equality

$$\lambda\, d(v_i) = \sum_{(v_i, v_j) \in E(G)} d(v_j) \tag{3}$$

holds for all $i = 1, 2, \ldots, n$. The fact that the property $W_k(G) = 0$ for all $k > 0$ is a consequence of Eq. (3) has been demonstrated elsewhere [4, 5].

In [1] the following connection to graph spectral theory [2] was pointed out. If $A(G)$ is the adjacency matrix of G then the system of equations (3) is equivalent to

$$A(G)\, d(G) = \lambda\, d(G) \ . \tag{4}$$

Consequently, the graph G is harmonic if and only if $d(G)$ is one of its eigenvectors. A graph satisfying Eqs. (3) and (4) will be referred to as a λ-

harmonic graph. Clearly, λ is the eigenvalue associated with the eigenvector $d(G)$. It is easy to show [1] that λ must be a non-negative integer.

Summing the expressions (3) over all $i = 1, 2, \dots, n$ we obtain

$$\sum_{k \geq 0} k(k-\lambda)\, n_k = 0 \ , \tag{5}$$

which is a necessary, but not sufficient, condition that harmonic graphs must obey.

2. *Some Auxiliary Results*

In our previous work [1] a number of results were obtained, applicable either to all harmonic graphs or to harmonic graphs with small number of cycles. Here we re-state (without proof) some of these results, needed for the proof of our main result, i. e., of Theorem 10. These are the Lemmas 1, 2, 4, 5, 6, 7 and 8. The result stated here as Theorem 3 is due to Grünewald [6]. We also state (with proof) a novel Lemma 9.

Lemma 1. *(a) Let the graph G' be obtained from the graph G by adding to it an arbitrary number of 0-vertices. Then G' is harmonic if and only if G is harmonic.*
(b) If G is a graph without 0-vertices, then G is λ-harmonic if and only if all its components are λ-harmonic.
(c) Every regular graph is harmonic. Every regular graph of degree k is k-harmonic.

Lemma 2. *Let G be a connected λ-harmonic graph. Then*
(a) λ is the greatest eigenvalue of G and its multiplicity is one;
(b) if $m > 0$ then $\lambda \geq 1$;
(c) $\lambda = 1$ if and only if $n = 2$ and $m = 1$.

From Lemma 1 we conclude that it is reasonable to restrict our considerations to connected non-regular graphs. The fact that such harmonic graphs do exist and that their structure is non-trivial became evident after the discovery of Theorem 3 [6].

Let λ be a positive integer. Construct the *Grünewald tree* T_λ in the following manner. T_λ has a total of $\lambda^3 - \lambda^2 + \lambda + 1$ vertices, of which one vertex is a $(\lambda^2 - \lambda + 1)$-vertex, $\lambda^2 - \lambda + 1$ vertices are λ-vertices and

$(\lambda - 1)(\lambda^2 - \lambda + 1)$ vertices are 1-vertices. Each λ-vertex is connected to $\lambda - 1$ 1-vertices and to the $(\lambda^2 - \lambda + 1)$-vertex.

Theorem 3 [6]. *For any positive integer λ there exists a unique λ-harmonic tree, isomorphic to T_λ .*

Lemma 4. *The Grünewald tree T_2 is the unique connected non-regular 2-harmonic graph.*

Bearing in mind Lemmas 2 and 4, in the following we may assume that $\lambda \geq 3$.

Lemma 5. *(a) In a λ-harmonic graph every 1-vertex is adjacent to a vertex of degree λ .*
(b) If a λ-harmonic graph is not regular, then it has a vertex of degree greater than λ .
(c) In a harmonic graph (with $n > 2$) no 1-vertex is attached to any vertex of greatest degree.

Lemma 6. *If x is a vertex of a λ-harmonic graph, then $d(x) \leq \lambda^2 - \lambda + 1$. If $d(x) = \lambda^2 - \lambda + 1$ then x belongs to a Grünewald tree T_λ . Otherwise, $d(x) < \lambda^2 - \lambda + 1$.*

Lemma 7. *Let $G \neq T_\lambda$ be a connected c-cyclic λ-harmonic graph with $\lambda \geq 3$. Then $c \geq \frac{1}{2}(\lambda^2 - 2\lambda + 2)$.*

Lemma 8. *For the λ-harmonic tree, $n_1 = (\lambda - 1)\, n_\lambda$. For any other connected λ-harmonic graph, $n_1 \leq (\lambda - 2)\, n_\lambda$.*

For the below considerations is of importance the relation [1]

$$\sum_{k \geq 0} (k - 2)\, n_k = 2\,c - 2 \tag{6}$$

obtained by combining the equalities (1) and (2) and using the fact that the respective graphs are connected ($m = n + c - 1$).

Before we formulate our main result – Theorem 10 – we demonstrate the validity of another auxiliary result that will be often used in the proof of Theorem 10.

Lemma 9. *Let v be a vertex of a λ-harmonic graph, such that $d(v) > \lambda^2 - 3\lambda + 4$, and let u be a vertex adjacent to v . Then $d(u) = \lambda$.*

P r o o f. Let $v \in V(G)$, $d(v) > \lambda^2 - 3\,\lambda + 4$ and $u, u_2, \dots, u_{d(v)}$ be the vertices of G adjacent to the vertex v. Assume first that $d(u) = \lambda - 1$. Then, because of (3),

$$\lambda\, d(u) = \lambda(\lambda - 1) = d(v) + d(x_1) + \cdots + d(x_{\lambda-2})$$

where $v, x_1, \dots, x_{d(u)-1}$ are the vertices adjacent to the vertex u. This yields

$$\begin{aligned} d(x_1) + \cdots + d(x_{\lambda-2}) &= \lambda^2 - \lambda - d(v) \\ &< \lambda^2 - \lambda - (\lambda^2 - 3\,\lambda + 4) \\ &= 2(\lambda - 2)\ . \end{aligned}$$

If follows that there must exist at least one i $(i = 1, 2, \dots, \lambda - 2)$, such that $d(x_i) = 1$, which because of Lemma 5 (a) is impossible.

Therefore, it cannot be $d(u) = \lambda - 1$.

Consider now the case $d(u) = \lambda - t$ for some $t \geq 2$. Then from Eq. (3),

$$\lambda\, d(u) = \lambda(\lambda - t) = d(v) + d(x_1) + \cdots + d(x_{\lambda-t-1}) > \lambda^2 - 3\,\lambda + 4 + \lambda - t - 1$$

i. e.,

$$\lambda\,(\lambda - t) > \lambda^2 - 2\,\lambda + 3 - t$$

i. e.,

$$\lambda\,(t - 2) < t - 3\ . \tag{7}$$

This again is a contradiction: for $t = 2$ inequality (7) becomes $0 < -1$. For $t > 2$ inequality (7) implies $\lambda < (t-3)/(t-2) < 1$ which is impossible in view of the assumption $\lambda \geq 3$.

Thus, it cannot be $d(u) < \lambda - 1$.

Consequently, if $d(v) > \lambda^2 - 3\,\lambda + 4$ and $(u, v) \in E(G)$ then it must be $d(u) \geq \lambda$.

If, however, the degree of any neighbor of the vertex v is greater or equal to λ then from

$$\lambda\, d(v) = d(u) + d(u_2) + \cdots + d(u_{d(v)})$$

there follows that it must be $d(u) = d(u_2) = \cdots = d(u_{d(v)}) = \lambda$. This implies Lemma 9. $\square$

3. *The Main Result*

Theorem 10. *There are exactly 18 non-regular connected tetracyclic harmonic graphs, depicted in Fig. 1.*

Fig 1. The connected non-regular tetracyclic harmonical graphs

P r o o f. Because of Lemma 7, if $c = 4$ then λ cannot be greater than 3. Then, in view of Lemmas 2 and 4, we conclude that it must be $\lambda = 3$. By Lemma 6, if D is the maximal vertex degree in a tetracyclic harmonic graph, then $D \leq 6$. From Lemma 5 (b) we then conclude that only the following three cases need to be examined:

Case 1: $\lambda = 3$, $D = 6$,
Case 2: $\lambda = 3$, $D = 5$,
Case 3: $\lambda = 3$, $D = 4$.

C a s e 1. From Lemma 8 follows that $n_3 - n_1 \geq 0$. By means of relation (6), for $c = 4$ we get

$$-n_1 + n_3 + 2\,n_4 + 3\,n_5 + 4\,n_6 = 6$$

from which

$$2\,n_4 + 3\,n_5 + 4\,n_6 - 6 = n_1 - n_3 \leq 0$$

and we conclude that

$$n_4 \leq 1 \quad ; \quad n_5 = 0 \quad ; \quad n_6 = 1 \ . \tag{8}$$

From Eq. (5) we get

$$-2\,n_1 - 2\,n_2 + 4\,n_4 + 10\,n_5 + 18\,n_6 = 0$$

which, by taking into account (8), implies

$$n_1 + n_2 = 9 + 2\,n_4 \ .$$

According to Lemma 9, a 6-vertex (i. e., a vertex of degree 6) is adjacent only to 3-vertices. The two neighbors of every 3-vertex, adjacent to a 6-vertex, must be a 1 and a 2-vertex. Therefore, $n_1 \geq 6$, $n_2 \geq 3$ and, consequently, $n_1 + n_2 \geq 9$. In what follows we distinguish between two subcases.

S u b c a s e 1.1.

$$n_4 = 0 \quad ; \quad n_5 = 0 \quad ; \quad n_6 = 1 \quad ; \quad n_3 = n_1 + 2 \quad ; \quad n_1 + n_2 = 9 \tag{9}$$

In this case it is easy to see that $n_1 = 6$, $n_2 = 3$, $n_3 = 8$, $n_4 = 0$, $n_5 = 0$, $n_6 = 1$. Each of the three 2-vertices must be adjacent to two 3-vertices (which, in turn, are adjacent to the 6-vertex), and an excess of two 3-vertices remains. Therefore there cannot exist a 3-harmonic graph satisfying the conditions (9).

S u b c a s e 1.2.

$$n_4 = 1 \quad ; \quad n_5 = 0 \quad ; \quad n_6 = 1 \quad ; \quad n_3 = n_1 \quad ; \quad n_1 + n_2 = 11 \tag{10}$$

The 4 and 6-vertices are adjacent only to 3-vertices, and therefore the number of 3-vertices is greater than or equal to 10. Because of $n_2 \geq 3$ we now have $n_1 = n_3 \geq 10$ and $n_1 + n_2 \geq 13$, which contradicts to the last equality in (10). Therefore, there cannot exist a 3-harmonic graph satisfying the conditions (10).

C a s e 2. Equations (5) and (6) now become

$$\begin{aligned} -2\,n_1 - 2\,n_2 + 4\,n_4 + 10\,n_5 &= 0 \\ -n_1 + n_3 + 2\,n_4 + 3\,n_5 &= 6 \end{aligned}$$

which together with the relation $n_3 - n_1 \geq 0$ imply that either $n_4 = 0$, $n_5 = 2$ or $n_4 = 1$, $n_5 = 1$ or $n_4 = 0$, $n_5 = 1$. We distinguish between three subcases.

S u b c a s e 2.1.

$$n_4 = 0 \quad ; \quad n_5 = 2 \quad ; \quad n_3 = n_1 \quad ; \quad n_1 + n_2 = 10 \tag{11}$$

Because of Lemma 9, every 5-vertex is adjacent only with 3-vertices. Therefore $n_3 \geq 10$ and $n_1 \geq 10$ and in view of the last equality in (11), $n_1 = 10$, $n_2 = 0$, $n_3 = 10$, $n_4 = 0$, $n_5 = 2$. Denote the two 5-vertices by u and v. Denote the 3-vertices adjacent to u and v by $x_1, x_2, \ldots, x_5$ and $y_1, y_2, \ldots, y_5$, respectively. The three neighbors of any 3-vertex are a 5–, a 3– and a 1-vertex. The graph induced by the 3-vertices is $5\,K_2$, and there are either one or three or five edges connecting the vertices $\{x_1, x_2, \ldots, x_5\}$ and $\{y_1, y_2, \ldots, y_5\}$. In view of this, G_1, G_2 and G_3 (depicted in Fig. 1) are the only 3-harmonic graphs satisfying the conditions (11).

S u b c a s e 2.2.

$$n_4 = 1 \quad ; \quad n_5 = 1 \quad ; \quad n_3 = n_1 + 1 \quad ; \quad n_1 + n_2 = 7 \tag{12}$$

By Lemma 9, the 5– and 4-vertices are adjacent only with 3-vertices. Furthermore, no 3-vertex can be simultaneously adjacent to a 5– and a 4-vertex. Consequently, $n_3 \geq 9$, which implies $n_1 = n_3 - 1 \geq 8$ and $n_1 + n_2 \geq 8$. This violates the last equality in (12), and we conclude that there cannot exist a 3-harmonic graph satisfying the conditions (12).

S u b c a s e 2.3.

$$n_4 = 0 \quad ; \quad n_5 = 1 \quad ; \quad n_3 = n_1 + 3 \quad ; \quad n_1 + n_2 = 5 \tag{13}$$

If there is a 3-harmonic graph obeying conditions (13), then its vertices have the following properties:
(i) The 5-vertex is adjacent only to 3-vertices (by Lemma 9), and thus $n_3 \geq 5$ and $n_1 = n_3 - 3 \geq 2$.
(ii) The 1-vertices are adjacent to only those 3-vertices which are adjacent to the 5-vertex.
(iii) Every 2-vertex is adjacent to two 3-vertices. Furthermore, $n_2 \neq 1$, because if it were $n_2 = 1$ then the 3-vertex adjacent to this 2-vertex would be adjacent either to another 2-vertex or to a 4-vertex, which both are impossible.
(iv) Exactly n_1 3-vertices, which are adjacent to the 5-vertex, are adjacent to one 3– and one 1–vertex. Each of the remaining $5-n_1$ 3-vertices, adjacent to the 5-vertex, are adjacent to a pair of 2-vertices.

Bearing in mind the above, the parameters n_1, n_2, n_3, n_4, n_5 may assume the following values:

	n_1	n_2	n_3	n_4	n_5
(a)	2	3	5	0	1
(b)	3	2	6	0	1
(c)	5	0	8	0	1

(a) In view of the properties *(i)–(iv)*, we conclude that the graph G_4 (depicted in Fig. 1) is the only 3-harmonic graph satisfying the values of the parameters n_i , $i = 1, 2, \ldots, 5$, given under (a).

(b) The only 3-vertex not adjacent to the 5-vertex, is adjacent to three other 3-vertices, implying that G_5 is the only 3-harmonic graph with the vertex degree distribution (b).

(c) Any of the three 3-vertices, which are not adjacent to the 5-vertex, are adjacent only to 3-vertices. The graph induced by these vertices is either K_3 or P_3. Therefore G_6 and G_7 are the only 3-harmonic graphs obeying the choice (c) of the parameters n_i.

C a s e 3. Equalities (5) and (6) now become

$$\begin{aligned} n_1 + n_2 &= 2\,n_4 \\ -n_1 + n_3 + 2\,n_4 &= 6 \end{aligned}$$

from which, in view of $n_3 - n_1 \geq 0$, there follows that n_4 may assume the value 1 or 2 or 3. We thus distinguish three subcases.

S u b c a s e 3.1.

$$n_4 = 1 \quad ; \quad n_3 = n_1 + 4 \quad ; \quad n_1 + n_2 = 2 \tag{14}$$

In this case $n_1 = 0$. Indeed, if it were $n_1 > 0$ then the 3-vertex adjacent to a 1-vertex would be adjacent also to two 4-vertices, which is impossible. Therefore, $n_1 = 0$, $n_2 = 2$, $n_3 = 4$, $n_4 = 1$. The 4-vertex must be adjacent to four 3-vertices. Every 3-vertex is adjacent to a 2–, a 3– and a 4-vertex. From this we conclude that G_8 and G_9 (depicted in Fig. 1) are the only graphs with the required properties.

S u b c a s e 3.2.

$$n_4 = 2 \quad ; \quad n_3 = n_1 + 2 \quad ; \quad n_1 + n_2 = 4 \tag{15}$$

The vertices of 3-harmonic graphs obeying conditions (15) have the following properties:
(i) Every 1-vertex is adjacent to a 3-vertex.
(ii) n_1 3-vertices are adjacent to one 1– and two 4-vertices. The remaining two 3-vertices are adjacent to a 2–, a 3– and a 4-vertex.
(iii) Exactly one 2-vertex is adjacent to two 3-vertices. All other 2-vertices are adjacent to a 2– and a 4-vertex. Therefore the number of 2-vertices is odd.
(iv) Every 4-vertex is adjacent to an even number of vertices of odd degree (i. e., to an even number of 3-vertices).

From the above follows that the parameters n_1, n_2, n_3, n_4 may assume two sets of values:

	n_1	n_2	n_3	n_4
(a)	3	1	5	2
(b)	1	3	3	2

The graphs G_{10} and G_{11} are the only 3-harmonic graphs satisfying conditions (a) and (b), respectively.

S u b c a s e 3.3.

$$n_4 = 3 \quad ; \quad n_3 = n_1 \quad ; \quad n_1 + n_2 = 6 \tag{16}$$

This time the vertices have the following properties:
(i) Every 1-vertex is adjacent to a 3-vertex.

(ii) Every 3-vertex is adjacent to one 1– and two 4-vertices.
(iii) Every 2-vertex is adjacent to a 4– and a 2-vertex. Therefore the number of 2-vertices is even.
(iv) Every 4-vertex is adjacent to an even number of vertices of odd degree (i. e., to an even number of 3-vertices).

In this subcase the following vertex degree distributions may occur:

	n_1	n_2	n_3	n_4
(a)	6	0	6	3
(b)	4	2	4	3
(c)	2	4	2	3
(d)	0	6	0	3

(a) Taking into account properties *(ii)* and *(iv)* we conclude that the 3– and 4-vertices are connected by exactly 12 edges, and that every 4-vertex is adjacent to four 3-vertices. This results in the graph G_{12} .

(b) Taking into account properties *(ii)* and *(iv)* we see that the 3– and 4-vertices are connected by exactly 8 edges. Further, one 4-vertex is adjacent to four 3-vertices whereas each of the other two 4-vertices is adjacent to one 2-, one 4– and two 3-vertices. This results in the graph G_{13} .

(c) Because of *(ii)* and *(iv)* this time two 3-vertices must be adjacent to the same pair of 4-vertices. These two 4-vertices are not adjacent, because otherwise the third 4-vertex would be adjacent to 2-vertices only, which is impossible. Further, every 4-vertex adjacent to 3-vertices is adjacent also to a 2– and a 4-vertex. Taking into account the mutual connectedness of the 2-vertices we arrive at the graphs G_{14} and G_{15} , which are the only 3-harmonic species with vertex degree distribution (c) .

(d) Every 4-vertex is adjacent to two 2-vertices and to two 4-vertices. Thus, the 4-vertices must be mutually adjacent. In view of this, and bearing in mind *(iii)*, the only possible solutions are the graphs G_{16} , G_{17} and G_{18} , depicted in Fig. 1.

By this all possible cases have been examined. The proof of Theorem 10 is complete. □

4. *The Regular Case*

In order to complete the list of connected tetracyclic harmonic graphs we prove the following elementary result.

Theorem 11. *There are exactly 2 regular connected tetracyclic harmonic graphs, depicted in Fig. 2.*

Fig 2. The connected regular tetracyclic harmonic graphs

P r o o f. A regular graph of degree D has $\frac{1}{2}\,n\,D$ edges. If it is connected and tetracyclic, then

$$\frac{1}{2}n\,D - n + 1 = 4 \quad \text{i. e.,} \quad n = \frac{6}{D-2}$$

For $D = 3, 4, 5$ and $D \geq 6$ we obtain $n = 6, 3, 2$ and $n < 2$, respectively. Thus only the case $D = 3$, $n = 6$ is possible.

It is well known [3] (and easy to show) that there are exactly two cubic graphs on 6 vertices, the graphs G_{19} and G_{20} depicted in Fig. 2. □

5. *Summary*

Together with the results reported elsewhere [1, 6] we may now summarize the achievements of the search for harmonic graphs with small number of cycles. For a fixed value of the cyclomatic number c the number of connected c-cyclic regular and non-regular harmonic graphs is denoted by $\#\mathrm{r}(c)$ and $\#\mathrm{nr}(c)$, respectively. The following are the known values of $\#\mathrm{r}(c)$ and $\#\mathrm{nr}(c)$:

c	$\#\mathrm{r}(c)$	$\#\mathrm{nr}(c)$	remark
0	1	∞	one for each $\lambda \geq 1$
1	∞	0	one for each $n \geq 3$; $\lambda = 2$
2	0	0	
3	1	4	all with $\lambda = 3$
4	2	18	all with $\lambda = 3$
≥ 5	finite	finite	

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