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Benders Decomposition for Local Access Network Design with Two Technologies

C. D. Randazzo and H. P. L. Luna† and P. Mahey

Departamento de Ciência da Computação - UFMG, Avenida Antônio Carlos, 6627 - 31270-010 - Belo Horizonte - MG, Brazil. `Email: {randazzo, pacca}@dcc.ufmg.br, mahey@isima.fr`

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We have worked with the local access network design problem with two cable technologies. This is an optimization problem in graphs that consists of linking an origin node to a set of terminal nodes which have a flow demand. There are also a set of Steiner or transsshipment nodes which do not have demand. Each arc of the graph has two associated costs: a variable cost depending on the flow through the arc and a fixed cost associated with the installation of the arc. Moreover, in each arc we can install one of two available technologies: optical fiber or copper (we can also use radio links with any other cable technology). Each one of these technologies has different variable and fixed costs. To be more precise, the fixed cost of the optical fiber is greater than that of the copper, but its variable cost is much smaller.

The problem was modeled using a multicommodity flow formulation in which we added some structural constraints. This model was used to apply the Benders decomposition method. The structural constraints have the objective of trying to guarantee that the master problem of the Benders decomposition will yield a tree. The Benders subproblems are trivial network flow problems. The dual variables have commodity meaningfull values and are evaluated in a systematic form. The algorithm was implemented in C++ with CPLEX 3.0 callable library. We have tested the algorithm with some test instances obtained by a generator of problems that we developed.

Keywords: network design, Benders decomposition

1 Introduction

This article addresses an extension of the local access network design problem (LAND). In the LAND problem we have to connect an origin node to a set of demand nodes minimizing the total cost. That cost is made of two parts: a variable cost which depends on the flow passing through the arc and a fixed cost to install the arc. There are also transshipment or Steiner nodes that do not have any demand and can be used or not in the optimal topology. In the problem that we work with in this paper, we can install one of two available technologies, for instance, optical fiber or copper cables, in each arc of the network. Note that it is also possible to think about radio links as an alternative to one of these cable technologies. We call this problem the local access network design problem with two technologies (LAND-2T). We

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will follow the notation given in [BMM94b] and will denote the two kinds of links by ”primary links” (optical fiber) and ”secondary links” (copper). The copper cable has a fixed cost smaller than that of the optical fiber, but its variable cost is greater than the variable cost of the optical fiber. We also work with primary connectivity constraints that require that primary links be connected to the origin node by a path consisting of primary links only. The reason for using such constraints is that a message which flows from one technology link to another technology link has to undergo some kind of data transformation which implies that a switching device has to be installed at every node where a change of technology takes place. In our problem, the primary connectivity constraints ensure that the number of such devices will be small and the cost of installing these devices is not considered. Another reason for requiring the primary connectivity constraints is that they imply that more paths can benefit from the higher quality of the primary links.

There are many possible variations of the LAND-2T problem that result in new problems. Balakrishnan *et al.* [BMM94a] have worked with a problem similar to the LAND-2T where a minimum cost spanning tree that contains an embedded primary subtree connecting all the primary nodes (and optionally including secondary nodes) has to be found. As in the LAND-2T, one can install one of two available cable technologies for each arc. Note that they divide the set of nodes into primary and secondary nodes. The primary nodes have to be connected to the origin node by primary arcs. The secondary nodes can be linked to the origin node by primary or secondary arcs. They have proposed a dual-ascent algorithm to find an approximate solution.

Gouveia and Janssen [GJ98] have worked with a similar problem that is an extension of the minimal spanning tree problem, but with two cable technologies too. The model that they explore has generalized hop constraints and primary connectivity constraints. Hop constraints limit the number of links (hops) between the root node and any terminal and measure in a certain way the reliability of the tree network. The primary connectivity constraints are the same that we described above. The problem is shown to be NP-hard and procedures to obtain lower and upper bounds are presented. They formulate the problem as a directed multicommodity flow model. To derive lower bounds they use Lagrangean relaxation with subgradient optimization. A Lagrangean heuristic is developed to construct feasible solutions. Moreover, they discuss several different ways of modeling the primary connectivity constraints. One outcome of their discussion is that an extended and compact representation of the convex hull of directed rooted subtrees when the underlying graph is series-parallel can be derived.

De Jongh *et al.* [dJGL99] have also worked with a closely related problem in which a pair of nodes has to be linked by two node disjoint paths with minimum total cost and two cable technologies. This problem is a little different from the LAND-2T since for each arc of the network there is only one available technology, that is, each arc has a specified technology and there are arcs of type 1 and arcs of type 2. In the LAND-2T problem, for each arc we have two available technologies. They also consider a transition cost that is associated with each node. This cost is incurred only when a flow enters and leaves the corresponding node on arcs of different types. Two heuristics were proposed in [dJGL99] and a lower bounding procedure based on Lagrangean relaxation is provided. These procedures are used in a branch-and-bound strategy to solve the problem.

In a previous paper, we worked with the local access network design problem [LRM98]. In this article, we studied two formulations of the problem: single commodity flow formulation; multicommodity flow formulation. The problem was solved exactly by CPLEX (with the two formulations), by a branch-and-bound algorithm, by a branch-and-cut algorithm and by a Benders decomposition. The success obtained by Benders decomposition to solve this problem has induced us to extend the algorithm developed to the local access network design problem to solve the LAND-2T. In Section 2 the mathematical programming formulation of the problem is presented. Section 3 presents the Benders decomposition applied to solve the problem. The implemented algorithm is showed in Section 4. Section 5 presents and discusses the computational results obtained. Section 6 closes this article with conclusions and final comments.

2 Formulation of the Problem

Consider a directed connected graph $G(V,E)$, where V denotes the set of nodes and E is a collection of directed arcs. Each arc of the graph represents a possible pair of nodes between which a directed transmission link can be placed. This transmission link can be a primary or a secondary link. Suppose we have an origin node o that must be linked to a number of $|K|$ demand nodes, each of them with a commodity flow requirement of d_k, where $k \in K$ and $K \subseteq V$.

With appropriate structural and operational costs, the problem is to find a minimal cost arborescence that links the origin node to all the terminal nodes and that has a connected set of primary links beginning from the origin node. Remark that all flows are originated at the origin node.

We define the variables:

$$x_{ij1} = \begin{cases} 1 & \text{if a direct primary transmission link is placed in arc } (i,j) \\ 0 & \text{otherwise;} \end{cases}$$

$$x_{ij2} = \begin{cases} 1 & \text{if a direct secondary transmission link is placed in arc } (i,j) \\ 0 & \text{otherwise;} \end{cases}$$

f_{ijk1}: flow passing through the primary arc (i,j) and destinated to the demand node k.
f_{ijk2}: flow passing through the secondary arc (i,j) and destinated to the demand node k.

And we also define the cost parameters:

b_{ij1}: fixed (structural) cost to install a direct primary transmission link in (i,j);
b_{ij2}: fixed (structural) cost to install a direct secondary transmission link in (i,j);
c_{ijk1}: variable (operational) cost to transmit one unit of commodity k through the primary arc (i,j);
c_{ijk2}: variable (operational) cost to transmit one unit of commodity k through the secondary arc (i,j);

The mathematical model, M, for the LAND-2T problem is:

$$\min \sum_{(i,j)\in E} \left(b_{ij1}x_{ij1} + b_{ij2}x_{ij2} + \sum_{k\in K}(c_{ijk1}f_{ijk1} + c_{ijk2}f_{ijk2})\right) \tag{1}$$

subject to:

$$-\sum_{(o,j)\in E}(f_{ojk1} + f_{ojk2}) = -d_k\,, \quad \text{for node } o \text{ and } \forall k \in K \tag{2}$$

$$\sum_{(i,k)\in E}(f_{ikk1} + f_{ikk2}) = d_k\,, \quad \forall k \in K \tag{3}$$

$$\sum_{(i,j)\in E}(f_{ijk1} + f_{ijk2}) - \sum_{(j,l)\in E}(f_{jlk1} + f_{jlk2}) = 0 \forall j \in V - \{o\} \text{ and } j \neq k \text{ and} \forall k \in K \tag{4}$$

$$f_{ijk1} \leq d_k x_{ij1}\,, \quad \forall (i,j) \in E \text{ and} \forall k \in K \tag{5}$$

$$f_{ijk2} \leq d_k x_{ij2}\,, \quad \forall (i,j) \in E \text{ and} \forall k \in K \tag{6}$$

$$f_{ijk1} \geq 0\,, \quad \forall (i,j) \in E \text{ and} \forall k \in K \tag{7}$$

$$f_{ijk2} \geq 0\,, \quad \forall (i,j) \in E \text{ and} \forall k \in K \tag{8}$$

$$\sum_{(i,k)\in E} (x_{ik1} + x_{ik2}) = 1\,, \quad \forall k \in K \tag{9}$$

$$\sum_{(l,j)\in E} (x_{lj1} + x_{lj2}) - \sum_{(i,l)\in E} (x_{il1} + x_{il2}) \geq 0\,, \quad \forall l \in V - K \tag{10}$$

$$\sum_{(i,l)\in E} (x_{il1} + x_{il2}) \geq \frac{\sum_{(l,j)\in E} (x_{lj1} + x_{lj2})}{\sum_{(l,j)\in E} 1}\,, \quad \forall l \in V - K - \{o\} \tag{11}$$

$$x_{ij1} + x_{ij2} + x_{ji1} + x_{ji2} \leq 1\,, \quad \forall (i,j) \in E \tag{12}$$

$$\sum_{(l,i)\in E} x_{li1} \geq x_{ij1}\,, \quad (i,j) \in E\,, \forall i \in V - \{o\} \tag{13}$$

$$x_{ij1}, x_{ij2} \in \{0,1\}\,, \quad \forall (i,j) \in E \tag{14}$$

The objective function minimizes the total cost associated with the fixed and variable costs. Constraints (2) ensure that the flow of commodity k that leaves the origin node is equal to the demand of the node k. Constraints (3) guarantee that the flow of commodity k that arrives at node k is equal to its demand. Constraints (4) are flow conservation constraints. The fact that a flow can pass through an arc only if this arc is selected to the design is expressed by constraints (5) (for primary arcs) and (6) (for secondary arcs). Constraints (7) and (8) ensure that the flow variables are greater than zero. Constraints from (9) to (13) are redundant, but they are important to try to guarantee that the solution of the master problem resulting of Benders decomposition is an arborescence (Benders decomposition method will be described in the next section). We call these constraints structural constraints. Constraints (9) ensure that at each demand node enters only one arc. Constraints (10) guarantee that at each Steiner node the number of nodes that leave the node is greater than or equal to the number of arcs that enter the node. Constraints (11) express the fact that if at least one arc leaves the node l, then at least one arc enters the node l. Between any nodes i and j the number of selected arcs, in any direction, has to be less than or equal to 1. Constraints (12) express this fact. Constraints (13) are the primary connectivity constraints and guarantee that the set of primary arcs constitutes a connected set from the origin. Finally, constraints (14) define the binary variables x_{ij}.

3 Benders Decomposition of the Problem

Benders partitioning method was published in 1962 [Ben62] and was initially developed to solve mixed integer programming problems. The computational success of the method to solve large scale multicommodity distribution system design models has been confirmed since the pioneering paper of Geoffrion and Graves [GG74]. Florian *et al.* [FGB76] have used Benders decomposition to schedule the movement of railways engines and Richardson [Ric76] has applied the algorithm to airline routing. Fisher and Jaikumar [FJ78] have discussed the advantages of using the algorithm for vehicle routing problems. Magnanti and Wong [MW81] have proposed methodology for improving the performance of Benders decomposition when applied to mixed integer programs. They have introduced a technique for accelerating the convergence of the algorithm and theory for distinguishing ”good” model formulations of a problem that has distinct, but equivalent mixed integer programming representations. In [MMW86], Magnanti *et al.* have applied Benders decomposition to solve the uncapacitated network design problem (with undirected edges) and have adapted this technique to be as efficient as possible. In [AB97], Benders decomposition is applied for solving network design problems with underlying tree structure. In [LRM98] Benders decomposition method was used to solve the local access network design problem and performed better than branch-and-cut and branch-and-bound algorithms.

3.1 Master Problem

The mathematical model of the master problem is constituted by the following objective function:

$$\min \sum_{(i,j)\in E} (b_{ij1}x_{ij1} + b_{ij2}x_{ij2}) + t \tag{15}$$

subject to the constraints (9), (10), (11), (12), (13), (14) and by the Benders cut constraint

$$t \geq \sum_{k\in K} d_k (p^h_{kk} - \sum_{(i,j)\in E} (\alpha^h_{ijk1}x_{ij1} + \alpha^h_{ijk2}x_{ij2})), \quad h = 1, 2, ..., H \tag{16}$$

The parameter h is a cycle counter and indicates the number of Benders cuts that must be taken into account. For given h and k, the corresponding value in the right-hand-side of constraints (16) provides a lower bound on the cost of the flow that leaves the origin node to the demand node k.

Consequently, there are three series of dual variables that can be interpreted as prices informations:

- p^h_{lk}: price of opening the communication k $(k \in K)$ at node l $(l \in V)$ in cycle h $(h = 1...H)$,
- α^h_{ijk1}: maximal reduction in the operational cost of commodity k if the primary link at arc (i, j) is selected to the design in cycle h,
- α^h_{ijk2}: maximal reduction in the operational cost of commodity k if the secondary link at arc (i, j) is selected to the design in cycle h.

The real variable t that appears in objective function (15) is a lower bound on the total operational cost.

3.2 Subproblems

For a fixed arborescence A^h, associated with the vectors x^h_1 and x^h_2, we have to solve separately a series of trivial network flow problems. Let C^h_{ok1} be the set of arcs of type 1 in the path from the source node to the demand node k and C^h_{ok2} be the set of arcs of type 2 in the path from the source node to the demand node k that have been defined by the master problem of cycle h. The primal-dual pair to be solved for commodity k is:

$$\min \sum_{(i,j)\in A} (c_{ijk1}f^h_{ijk1} + c_{ijk2}f^h_{ijk2}) \tag{17}$$

subject to:

$$-\sum_{(o,j)\in E}(f^h_{ojk1}+f^h_{ojk2}) = -d_k \quad \text{for the root } o \tag{18}$$

$$\sum_{(i,k)\in E}(f^h_{ikk1}+f^h_{ikk2}) = d_k \quad \text{for node } k \tag{19}$$

$$\sum_{(i,j)\in E}(f^h_{ijk1}+f^h_{ijk2}) - \sum_{(j,l)\in E}(f^h_{jlk1}+f^h_{jlk2}) = 0 \; \forall j\in V-\{o\} \text{ and } j\neq k \tag{20}$$

$$-f^h_{ijk1} \geq -d_k \quad \forall (i,j)\in A^h \tag{21}$$

$$-f^h_{ijk2} \geq -d_k \quad \forall (i,j)\in A^h \tag{22}$$

$$f^h_{ijk1} \geq 0 \quad \forall (i,j)\in A^h \tag{23}$$

$$f^h_{ijk2} \geq 0 \quad \forall (i,j)\in A^h \tag{24}$$

The trivial and unique solution of the problem is:

$$f^h_{ijk1} = \begin{cases} d_k & \text{if}\,(i,j)\in C^h_{ok1}\subseteq A^h \\ 0 & \text{otherwise} \end{cases}$$

$$f^h_{ijk2} = \begin{cases} d_k & \text{if}\,(i,j)\in C^h_{ok2}\subseteq A^h \\ 0 & \text{otherwise} \end{cases}$$

The dual subproblem for commodity k is:

$$\max_{p^h,\alpha^h\geq 0} d_k(-\sum_{(i,j)\in A^h}(\alpha^h_{ijk1}+\alpha^h_{ijk2}) + p^h_{kk} - p^h_{ok}) \tag{25}$$

subject to:

$$p^h_{jk} - p^h_{ik} - \alpha^h_{ijk1} \;\leq\; c_{ijk1} \quad \forall (i,j)\in E \tag{26}$$

$$p^h_{jk} - p^h_{ik} - \alpha^h_{ijk2} \;\leq\; c_{ijk2} \quad \forall (i,j)\in E \tag{27}$$

$$\alpha^h_{ijk1}, \alpha^h_{ijk2} \;\geq\; 0 \quad \forall (i,j)\in E \tag{28}$$

$$p^h_{ik} \text{ unrestricted} \quad \forall i\in V \tag{29}$$

From the complementary slackness condition we have:

$$p^h_{jk} - p^h_{ik} - \alpha^h_{ijk1} = c_{ijk1} \quad \forall (i,j)\in C^h_{ok1}\subset A^h$$

$$(f_{ijk1} = d_k)$$

$$p^h_{jk} - p^h_{ik} - \alpha^h_{ijk2} = c_{ijk2} \quad \forall (i,j)\in C^h_{ok2}\subset A^h$$

$$(f_{ijk2} = d_k)$$

in such a way that we can construct, associated with the primal solution x^h, the following dual feasible solution:

$$\begin{aligned}
p^h_{ok} &= 0 \quad \forall k \in K \text{ for the origin node } o && (30)\\
p^h_{jk} &= p^h_{ik} + c_{ijk1} \quad \forall (i,j) \in C^h_{ok1} \subset A^h && (31)\\
p^h_{jk} &= p^h_{ik} + c_{ijk2} \quad \forall (i,j) \in C^h_{ok2} \subset A^h && (32)\\
p^h_{ik} &= p^0_{ik} \quad \forall i \in V - V^h && (33)\\
\alpha^h_{ijk1} &= 0 \quad \forall (i,j) \in C^h_{ok1} \subset A^h && (34)\\
\alpha^h_{ijk1} &= p^h_{jk} - p^h_{ik} - c_{ijk1} \quad \forall (i,j) \in E - C^h_{ok1} \text{ such that} &&\\
& p^h_{jk} - p^h_{ik} > c_{ijk1} && (35)\\
\alpha^h_{ijk1} &= 0 \quad \forall (i,j) \in E - C^h_{ok1} \text{ such that } p^h_{jk} - p^h_{ik} \leq c_{ijk1} && (36)\\
\alpha^h_{ijk2} &= 0 \quad \forall (i,j) \in C^h_{ok2} \subset A^h && (37)\\
\alpha^h_{ijk2} &= p^h_{jk} - p^h_{ik} - c_{ijk2} \quad \forall (i,j) \in E - C^h_{ok2} \text{ such that} &&\\
& p^h_{jk} - p^h_{ik} > c_{ijk2} && (38)\\
\alpha^h_{ijk2} &= 0 \quad \forall (i,j) \in E - C^h_{ok2} \text{ such that } p^h_{jk} - p^h_{ik} \leq c_{ijk2} && (39)
\end{aligned}$$

The systematic evaluation of the dual variables with commodity meaningful values is a key factor for an efficient implementation. The dual variable α^h_{ijk1} evaluates for commodity k the maximal reduction in the operational cost that could be gained with the introduction of a primary link at the arc (i,j) in the solution. It can also be understood as a tax to be paid with the use of the primary arc (i,j) in order to maintain the distribution agents with no positive profit. Remark that the dual solution set represents spatial prices for which there is no positive profit for any distribution agent that pays the cost c_{ijk1} to flow commodity k across the primary arc (i,j). The same interpretation is valid for the dual variables α^h_{ijk2}.

4 Algorithm

In this section we present the implemented algorithm. Our algorithm is not simply a Benders decomposition algorithm. We work with a special feasible solution of the problem that minimizes the total variable cost. This solution is obtained applying the Dijkstra's algorithm [Dij59] to find the shortest path from the origin to all nodes, but only the variable cost to flow from origin to each demand node is computed. The main steps of the algorithm are the following:

1. Use Dijkstra's algorithm to find the shortest path from the origin o to every node of the network. Let E^0 be the arcs of the arborescence that contains the shortest paths to all the nodes and let $T(V^0, A^0)$ be the correspondent arborescence that links the origin o to all demand nodes $k \in K$ ($x^0_{ij1} = 1\,\forall (i,j) \in A^0$, $x^0_{ij1} = 0\,\forall (i,j) \in E - A^0$ and $x^0_{ij2} = 0\,\forall (i,j) \in E$). Make

$$p^0_{ok} = 0 \quad \forall k \in K, \text{ for the origin node } o$$

$$\begin{aligned} p^0_{jk} &= p^0_{ik} + c_{ijk1} \quad \forall (i,j) \in E^0, \forall k \in K \text{ across } E^0 \\ \alpha^0_{ijk1} &= 0 \quad \forall (i,j) \in E, \forall k \in K \\ \alpha^0_{ijk2} &= 0 \quad \forall (i,j) \in E, \forall k \in K \end{aligned}$$

Compute the cost associated with $T(V^0, A^0)$ (the sum of the fixed cost of the arcs in A^0 plus the sum of the variable costs of sending the flow requirement of each demand node k from the origin to the demand node). This value gives an initial upper bound, $UB = \sum_{(i,j)\in A^0}(b_{ij1}x^0_{ij1} + b_{ij2}x_{ij2}) + \sum_{k\in k} d_k p^0_{kk}$, and (x^0_1, x^0_2, f^0) is an incumbent solution. Also, the shortest paths solution provides the minimal total variable cost among all possible arborescences, and thus we can use it to initialize the lower bound, $LB = \sum_{k\in k} d_k p^0_{kk}$. This is also a lower bound on the variable t of the master problem.

2. Initialize the cycle counter with zero, *i. e.*, $h = 0$.

3. Solve the master problem. It provides a lower bound for the problem. If the lower bound is greater than (or equal to) the upper bound, then **stop**.

4. Increment the cycle counter, h.

5. Solve the subproblem. To solve it, verify first if the selected arcs in the master problem build an arborescence from the origin to all demand nodes.

   - If they do, let $T(V^h, A^h)$ be the arborescence that links the origin o to all demand nodes $k \in K$, contained in the original graph $G(V,E)$, let C^h_{ok1} be the set of primary arcs in the path from the origin to the demand node k across A^h, and let C^h_{ok2} be the set of secondary arcs in the path from the origin to the demand node k.
   - Else, the solution of the master problem turns the subproblem infeasible in the sense that it generates a cycle in the path from the origin to one or more demand nodes. In this case, the cycles are identified and constraints to avoid them are added to the master problem model and no new upper bound is generated. The master problem must again be solved, then go to step 3.

6. Compute the values of the dual variables shown by equations (30)-(39). A new value for the upper bound is calculated and if this value is less than the current upper bound then the current upper bound is updated. If the lower bound is greater than (or equal to) the upper bound, then **stop**.

7. Add a new Benders cut to the master problem using the dual variable calculated in the previous step. Go to step 3.

5 Computational Results

The tests were executed on a Sun Ultra Enterprise 3000 with two 250 MHz UltraSPARC processors and 512 Mbytes of RAM memory. The operational system is Solaris 2.5.1. The Benders decomposition algorithm was implemented in C++ with *CPLEX 3.0* callable library. The test problems from 1 to 5 are Euclidean graphs randomly generated using a procedure similar to that presented in [Ane80]. This procedure has extensively been applied for creating testbeds of the Steiner problem and we have used

Problem Number	$\|V\|$	$\|E\|$	$\|K\|$	CPLEX B&B Nodes	CPLEX Time(s)	Benders Decomposi tion Initial Gap	Benders Decomposi tion Cycles	Benders Decomposi tion T ime(s)
1	10	40	4	0	0.75	92%	7	0.70
2	10	50	4	0	0.27	58%	8	1.61
3	12	50	5	0	0.10	51%	5	0.95
4	15	50	8	0	0.11	48%	7	3.11
5	16	60	10	0	0.31	48%	9	2.37
6	21	112	4	0	2.40	95%	2	0.50
7	31	220	5	0	50.07	96%	2	49.27
8	31	240	6	0	29.79	2%	2	2.41
9	36	322	7	0	159.08	2%	4	30.21
10	41	417	8	0	269.69	2%	2	2.05

Table 1: Computational results for the problem LAND-2T.

this procedure to generate test instances of the local access network design problem [LRM98]. The test problems from 6 to 10 were generated by a different procedure which has been used to generate test instances for the local access network design problem in such a way that the linear relaxation of the multicommodity flow formulation of this problem does not find an optimal integer solution. This procedure was proposed by [HH98].

Table 1 shows the results obtained by solving the test problems by CPLEX and by Benders decomposition method. The number of cycles (number of master problems solved) was small, specially for the problems from 6 to 10 for which the greatest number of cycles was 4. Benders decomposition algorithm was better than CPLEX for 6 of the 10 problems. For the problems from 6 to 10, the execution time of Benders decomposition was smaller than the execution time of CPLEX for all problems (this difference between the execution times was more significative for problems 8, 9 and 10). Although the initial gap ((upper bound - lower bound)/upper bound) obtained from the first solved master problem may be high, the performance of Benders decomposition algorithm is encouraging.

It is important to note that for all test problems in Table 1 the linear relaxation of the model M produces an optimal integer solution. We have not tested the instances which do not have this property yet. From our previous experience with one cable technology, we know that Benders decomposition may be better than other methods as branch-and-bound and branch-and-cut. So, we believe that the performance of Benders decomposition can be better than this obtained with these preliminary experiments.

6 Conclusions

In this article we have extended a Benders decomposition algorithm that we have previously implemented to solve the local access network design problem. This algorithm performed very well on the local access network design problem and the obtained results have lead us to extend it to solve the LAND-2T.

We have presented a multicommodity flow formulation for the LAND-2T with primary connectivity constraints. Moreover, we have added some structural constraints to the model with the objective of getting feasible solutions from the master problem. Benders decomposition was applied to this model and the values of the dual variables were derived.

Some computational experiments were developed and the number of master problems solved was small, from 2 to 9. Benders decomposition has performed better than CPLEX for 6 of the 10 test problems. In some instances, Benders execution time was 10% of CPLEX execution time. We have not tested the algorithm with the instances for which the linear relaxation of model M does not find an optimal integer solution of the problem. From our previous experience, we expect that the best results of Benders decomposition algorithm will be reached for these problems.

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