INTEGRAL GEOMETRY AND REAL ZEROS OF THUE–MORSE POLYNOMIALS

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Abstract. We study the average number of intersecting points of a given curve with random hyperplanes in an n-dimensional Euclidean space. As noticed by A. Edelman and E. Kostlan, this problem is closely linked to finding the average number of real zeros of random polynomials. They showed that a real polynomial of degree n has on average $\frac{2}{\pi}\log n + O(1)$ real zeros (M. Kac's theorem).

This result leads us to the following problem: given a real sequence $(\alpha_k)_{k\in\mathbb{N}}$, study the average

$$\frac{1}{N}\sum_{n=0}^{N-1}\rho(f_n),$$

where $\rho(f_n)$ is the number of real zeros of $f_n(X) = \alpha_0+\alpha_1 X+\cdots+\alpha_n X^n$. We give theoretical results for the Thue–Morse polynomials and numerical evidence for other polynomials.