General Mathematics Vol. 19, No. 1 (2011), 27–43

Meromorphic functions concerning their differential polynomials sharing the fixed-points with finite weight [1]

Hong-Yan Xu, Ting-Bin Cao and Tang-Sen Zhan

Abstract

This paper deals with some uniqueness problems of meromorphic functions concerning their differential polynomials sharing the fixed-points or a small function with finite weight. These results in this paper greatly improve the recent results given by X.-Y. Zhang& J.-F. Chen and W.C. Lin[X.-Y. Zhang, J.-F. Chen, W.C. Lin , Entire or meromorphic functions sharing one value, Comput. Math. Appl. 56(2008), 1876-1883.].

2000 Mathematics Subject Classification: 30D30, 30D35.
Key words and phrases: Meromorphic function; Weighted sharing; Uniqueness.

1 Introduction and Main Results

Let f be a non-constant meromorphic function in the whole complex plane. We shall use following standard notations of the value distribution theory:

$$T(r,f), m(r,f), N(r,f), \overline{N}(r,f), \ldots$$

(see Hayman [6],Yang [17] and Yi and Yang[14]). We denote by $S(r,f)$ any quantity satisfying $S(r,f) = o(T(r,f))$, as $r \to +\infty$, possibly outside of a set with finite measure. A meromorphic function α is called a small function with respect to f if $T(r,\alpha) = S(r,f)$. Let $S(f)$ be the set of meromorphic functions

[1] *Received 24 March, 2009*
Accepted for publication (in revised form) 14 April, 2009

in the complex plane $\mathbb{C}$ which are small functions with respect to f.For some complex number $a \in \mathbb{C} \cup \infty$, we define $\Theta(a,f) = 1 - \overline{\lim}_{r\to\infty}\frac{\overline{N}(r,a;f)}{T(r,f)}$.

Let f and g be two nonconstant meromorphic functions defined in the open complex plane $\mathbb{C}$.If for some $\alpha \in S(f) \cap S(g)$ the roots of $f-\alpha$ and $g-\alpha$ coincide in locations and multiplicities we say that f and g share the value α CM (counting multiplicities) and if coincide in locations only we say that f and g share α IM (ignoring multiplicities).

For $a \in \mathbb{C} \cup \infty$ and k a positive integer. We denote by $N(r,a;f| = 1)$ the counting function of simple a-points of f, denote by $N(r,a;f| \le k)$ ($N(r,a;f| \ge k)$) the counting functions of those a-points of f whose multiplicities are not greater (less) than k where each a-point is counted according to its multiplicity(see [6]). $\overline{N}(r,a;f| \le k)(\overline{N}(r,a;f| \ge k))$ are defined similarly, where in counting the a-points of f we ignore the multiplicities. Set $N_k(r,a;f) = \overline{N}(r,a;f) + \overline{N}(r,a;f| \ge 2) + \cdots + \overline{N}(r,a;f \ge k)$.

In 1997, Yang and Hua [13] proved the following result.

Theorem A[13] *Let f and g be two nonconstant meromorphic functions, $n \ge 11$ an integer, and $a \in \mathbb{C} - \{0\}$. If f^nf' and g^ng' share the value a CM, then either $f = dg$ for some $(n+1)th$ root of unity d or $g = c_1e^{cz}$ and $f = c_2e^{-cz}$ where c, c_1, and c_2 are constants satisfying $(c_1c_2)^{n+1}c^2 = -a^2$.*

W.C.Lin and H.X.Yi [10] obtained some unicity theorems corresponding to Theorem A.

Theorem B[10] *Let f and g be two nonconstant meromorphic functions satisfying $\Theta(\infty,f) > \frac{2}{n+1}$, $n \ge 12$. If $[f^n(f-1)]f'$ and $[g^n(g-1)]g'$ share 1 CM, then $f \equiv g$.*

W.C.Lin and H.X.Yi [11] extended Theorem B by replacing the value 1 with the function z and obtained the following result.

Theorem C[11] *Let f and g be two transcendental meromorphic functions, $n \ge 12$ an integer. If $f^n(f-1)f'$ and $g^n(g-1)g'$ share z CM, then either $f \equiv g$ or $g = \frac{(n+2)(1-h^{n+1})}{(n+1)(1-h^{n+2})}$ and $f = \frac{(n+2)h(1-h^{n+1})}{(n+1)(1-h^{n+2})}$, where h is a nonconstant meromorphic function.*

Recently, Xiao-Yu Zhang, Jun-Fan Chen and Wei-Chuan Lin[18] extended Theorem F and G and obtained the following result.

Theorem D[18]*Let f and g be two nonconstant meromorphic functions, and let n and m be two positive integers with $n > \max\{m+10, 3m+3\}$, and let $P(z) = a_mz^m + a_{m-1}z^{m-1} + \cdots + a_1z + a_0$, where $a_0 \ne 0, a_1, \ldots, a_{m-1}, a_m \ne 0$*

are complex constants. If $f^nP(f)f'$ and $g^nP(g)g'$ share 1 CM, then either $f \equiv tg$ for a constant t such that $t^d = 1$, where $d = (n+m+1, \ldots, n+m+1-i, \ldots, n+1), a_{m-i} \neq 0$ for some $i = 0, 1, \ldots, m$, or f and g satisfy the algebraic equation $R(f, g) \equiv 0$, where $R(\omega_1, \omega_2) = \omega_1^n(\frac{a_m\omega_1^m}{n+m+1} + \frac{a_{m-1}\omega_1^{m-1}}{n+m} + \cdots + \frac{a_0}{n+1}) - \omega_2^n(\frac{a_m\omega_2^m}{n+m+1} + \frac{a_{m-1}\omega_2^{m-1}}{n+m} + \cdots + \frac{a_0}{n+1})$.

Recent years, I. Lahiri[7,8] and A.Banerjee[1,2] employ the idea of weighted sharing of values which measures how close a shared value is to being shared IM or to being shared CM. Many interesting results[3,4,7,8,12,14,15] were obtained by many mathematicians such as H.X. Yi, I. Lahiri, A.Banerjee, W.C. Lin, X.M.Li and so on.

In 2008, A.Banerjee [2] employed the idea of weighted sharing of values and obtained the following result which improved Theorem A.

Theorem E[2] *Let f and g be two nonconstant meromorphic functions and $n > 22 - [5\Theta(\infty; f) + 5\Theta(\infty; g) + \min\{\Theta(\infty; f), \Theta(\infty; g)\}]$, is an integer. If for any $a \in \mathbb{C} - 0$, $\overline{E}_{2)}(a; f^nf') = \overline{E}_{2)}(a; g^ng')$ the conclusion of Theorem A holds.*

Regarding Theorem C,D and E, it is natural to ask the following questions.

Question 1.1 *Is it possible that the value 1 can be replaced by a function z or a small function in Theorems D and E?*

Question 1.2 *Is it possible to relax the nature of sharing z or a small function in Theorem C and D and if possible, how far?*

In this paper we shall investigate the possible solutions of the above questions. We now state the following theorems which are the main results of the paper.

Theorem 1.1 *Let f and g be two transcendental meromorphic functions, and let n and m be two positive integers with $n > \max\{m + 10 - (2\Theta(\infty; f) + 2\Theta(\infty; g)), 3m + 1\}$, and let $P(z) = a_mz^m + a_{m-1}z^{m-1} + \cdots + a_1z + a_0$, where $a_0 \neq 0, a_1, \ldots, a_{m-1}, a_m \neq 0$ are complex constants. If $f^nP(f)f'$ and $g^nP(g)g'$ share z CM, then either $f \equiv tg$ for a constant t such that $t^d = 1$, where $d = (n+m+1, \ldots, n+m+1-i, \ldots, n+1), a_{m-i} \neq 0$ for some $i = 0, 1, \ldots, m$, or f and g satisfy the algebraic equation $R(f, g) \equiv 0$, where $R(\omega_1, \omega_2) = \omega_1^n(\frac{a_m\omega_1^m}{n+m+1} + \frac{a_{m-1}\omega_1^{m-1}}{n+m} + \cdots + \frac{a_0}{n+1}) - \omega_2^n(\frac{a_m\omega_2^m}{n+m+1} + \frac{a_{m-1}\omega_2^{m-1}}{n+m} + \cdots + \frac{a_0}{n+1})$.*

Theorem 1.2 *Let f and g be two transcendental meromorphic functions, and let n, l and m be three positive integers with $n > \max\{\frac{5}{3}m + \frac{38}{3} - 2(\Theta(\infty; g) +$*

$\Theta(\infty; f) - \frac{2}{3}\min\{\Theta(\infty; g), \Theta(\infty; f)\}, 3m+1\}$. If $\overline{E}_{l)}(z, f^nP(f)f') = \overline{E}_{l)}(z, g^nP(g)g')$ and $E_{1)}(z, f^nP(f)f') = E_{1)}(z, g^nP(g)g')$, where $l \geq 3$, then the conclusion of Theorem 1.1 holds.

Theorem 1.3 *Let f and g be two transcendental meromorphic functions, and let n, l and m be three positive integers with $n > \max\{m + 10 - (2\Theta(\infty; f) + 2\Theta(\infty; g)), 3m+1\}$. If $\overline{E}_{l)}(z, f^nP(f)f') = \overline{E}_{l)}(z, g^nP(g)g')$ and $E_{2)}(z, f^nP(f)f') = E_{2)}(z, g^nP(g)g')$, where $l \geq 4$, then the conclusion of Theorem 1.1 holds.*

Remark 1.1 *From Theorem 1.1 and 1.3, we can get the same conclusion under the condition of F_1, G_1 sharing z CM or $\overline{E}_{4)}(z, F_1) = \overline{E}_{4)}(z, G_1)$ and $E_{2)}(z, F_1) = E_{2)}(z, G_1)$, where $F_1 = f^nP(f)f', G_1 = g^nP(g)g'$.*

Though the standard definitions and notations of the value distribution theory are available in[6], we explain some definitions and notations which are used in the paper.

Definition 1.1 [2] *Let k and r be two positive integers such that $1 \leq r < k-1$ and for $a \in \mathbb{C}$, $\overline{E}_{k)}(a; f) = \overline{E}_{k)}(a; g), E_{r)}(a; f) = E_{r)}(a; g)$. Let z_0 be a zero of $f - a$ of multiplicity p and a zero of $g - a$ of multiplicity q. We denote by $\overline{N}_L(r, a; f)(\overline{N}_L(r, a; g))$ the reduced counting function of those a-points of f and g for which $p > q \geq r + 1 (q > p \geq r + 1)$, by $\overline{N}_E^{(r+1}(r, a; f)$ the reduced counting function of those a-points of f and g for which $p = q \geq r + 1$, by $\overline{N}_{f \geq k+1}(r, a; f|g \neq a)(\overline{N}_{g \geq k+1}(r, a; g|f \neq a))$ the reduced counting functions of those a-points of f and g for which $p \geq k+1$ and $q = 0$($q \geq k+1$ and $p = 0$).*

Definition 1.2 [2] *If $r = 0$ in definition 1.1 then we use the same notations as in definition 1.5 except by $\overline{N}_E^{1)}(r, a; f)$ we mean the common simple a-points of f and g and by $\overline{N}_E^{(2}(r, a; f)$ we mean the reduced counting functions of those a-points of f and g for which $p = q \geq 2$.*

Definition 1.3 [8] *Let $a, b \in \mathbb{C} \cup \{\infty\}$, We denote by $N(r, a; f|g = b)$ the counting function of those $a-$points of f, counted according to multiplicity, which are b-points of g; by $N(r, a; f|g \neq b)$ the counting function of those a-points of f, counted according to multiplicity, which are not the b-points of g.*

2 Some Lemmas

For the proof of our results we need the following lemmas.

Lemma 2.1 [16] *Let f be a nonconstant meromorphic function and $P(f) = a_0+a_1f+a_2f^2+\cdots+a_nf^n$, where $a_0, a_1, a_2, \cdots, a_n$ are constants and $a_n \neq 0$. Then*

$$T(r, P(f)) = nT(r, f) + S(r, f).$$

Lemma 2.2 [9] *If $N(r, 0; f^{(k)}|f \neq 0)$ denotes the counting function of those zeros of $f^{(k)}$ which are not the zeros of f, where a zero of $f^{(k)}$ is counted according to its multiplicity then*

$$N(r, 0; f^{(k)}|f \neq 0) \leq k\overline{N}(r, \infty; f)+N(r, 0; f| < k)+k\overline{N}(r, 0; f| \geq k)+S(r, f).$$

Lemma 2.3 [5] *Let F and G be two meromorphic functions. If F and G share 1 CM, one of the following three cases holds:*

(i) $T(r, F) \leq N_2(r, \infty, F)+N_2(r, \infty, G)+N_2(r, 0, F)+N_2(r, 0, G)+S(r, F)+S(r, G)$, the same inequality holding for $T(r, G)$;

(ii) $F \equiv G$;

(iii) $F \cdot G \equiv 1$.

Lemma 2.4 [2] *Let F, G be two nonconstant meromorphic functions such that $E_{1)}(1; F) = E_{1)}(1; F)$ and $H \not\equiv 0$. Then*

$$N_E^{1)}(r, 1; F) \leq N(r, \infty; H) + S(r, F) + S(r, G),$$

where $H = (\frac{F''}{F'} - \frac{2F'}{F-1}) - (\frac{G''}{G'} - \frac{2G'}{G-1})$.

Lemma 2.5 [2] *Let $\overline{E}_{l)}(1; F) = \overline{E}_{l)}(1; G), E_{1)}(1; F) = E_{1)}(1; G)$ and $H \not\equiv 0$, where $l \geq 3$. Then*

$$\begin{aligned} & N(r, \infty; H) \\ \leq\ & \overline{N}(r, 0; F| \geq 2) + \overline{N}(r, 0; G| \geq 2) + \overline{N}(r, \infty; F| \geq 2) + \overline{N}(r, \infty; G| \geq 2) \\ & +\overline{N}_L(r, 1; F) + \overline{N}_L(r, 1; G) + \overline{N}_{F\geq l+1}(r, 1; F|G \neq 1) \\ & +\overline{N}_{G\geq l+1}(r, 1; G|F \neq 1) + \overline{N}_0(r, 0; F') + \overline{N}_0(r, 0; G'), \end{aligned}$$

where $\overline{N}_0(r, 0; F')$ is the reduced counting function of those zeros of F' which are not the zeros of $F(F-1)$ and $\overline{N}_0(r, 0; G')$ is similarly defined.

Lemma 2.6 [2] *Let $\overline{E}_{2)}(1; F) = \overline{E}_{2)}(1; G)$ and $H \not\equiv 0$. Then*

$$\begin{aligned} & N(r, \infty; H) \\ \leq\ & \overline{N}(r, 0; F| \geq 2) + \overline{N}(r, 0; G| \geq 2) + \overline{N}(r, \infty; F| \geq 2) + \overline{N}(r, \infty; G| \geq 2) \\ & +\overline{N}_L(r, 1; F) + \overline{N}_L(r, 1; G) + \overline{N}_{F\geq 3}(r, 1; F|G \neq 1) \\ & +\overline{N}_{G\geq 3}(r, 1; G|F \neq 1) + \overline{N}_0(r, 0; F') + \overline{N}_0(r, 0; G'). \end{aligned}$$

Lemma 2.7 [2] *Let* $\overline{E}_{l)}(1;F)=\overline{E}_{l)}(1;G)$ *and* $E_{1)}(1;F)=E_{1)}(1;G)$ *and* $H\not\equiv 0$*, where* $l\geq 3$*. Then*

$$\begin{array}{rl} & 2\overline{N}_L(r,1;F)+2\overline{N}_L(r,1;G)+\overline{N}_E^{(2}(r,1;F) \\ & +l\overline{N}_{G\geq l+1}(r,1;G|F\neq 1)-\overline{N}_{F>2}(r,1;G) \\ \leq & N(r,1;G)-\overline{N}(r,1;G). \end{array}$$

Lemma 2.8 *Let* $\overline{E}_{l)}(1;F)=\overline{E}_{l)}(1;G), E_{1)}(1;F)=E_{1)}(1;G)$*, where* $l\geq 3$*. Then*

$$\begin{array}{rl} & \overline{N}_{F>2}(r,1;G)+2\overline{N}_{F\geq l+1}(r,1;F|G\neq 1) \\ \leq & \frac{2}{3}\overline{N}(r,0;F)+\frac{2}{3}\overline{N}(r,\infty;F)-\frac{2}{3}\overline{N}_0(r,0;F')+S(r,F). \end{array}$$

Proof: We note that any 1-point of F with multiplicity ≥ 3 is counted at most twice. Hence by using Lemma 2.2 we see that

$$\begin{array}{rl} & \overline{N}_{F>2}(r,1;G)+2\overline{N}_{F\geq l+1}(r,1;F|G\neq 1) \\ \leq & \overline{N}(r,1;F|\geq 3;G|=2)+2\overline{N}(r,1;F|G\neq 1) \\ \leq & \frac{2}{3}N(r,0;F'|F=1) \\ \leq & \frac{2}{3}N(r,0;F'|F\neq 0)-\frac{2}{3}\overline{N}_0(r,0;F') \\ \leq & \frac{2}{3}\overline{N}(r,0;F)+\frac{2}{3}\overline{N}(r,\infty;F)-\frac{2}{3}\overline{N}_0(r,0;F')+S(r,F), \end{array}$$

where by $\overline{N}(r,1;F|\geq 3;G|=2)$ we mean the reduced counting function of 1 points of F with multiplicity not less than 3 which are the 1-points of G with multiplicity 2. This completes the proof of the lemma.

Lemma 2.9 *Let* $\overline{E}_{l)}(1;F)=\overline{E}_{l)}(1;G), E_{1)}(1;F)=E_{1)}(1;G)$ *and* $H\not\equiv 0$*, where* $l\geq 3$*. Then*

$$\begin{array}{rl} & T(r,F) \\ \leq & N_2(r,0;F)+N_2(r,\infty;F)+N_2(r,0;G)+N_2(r,\infty;G) \\ & +\frac{2}{3}\overline{N}(r,0;F)+\frac{2}{3}\overline{N}(r,\infty;F)+S(r,F)+S(r,G). \end{array}$$

Proof: Using Lemmas 2.4, 2.5 and 2.7 , we get

$$\begin{aligned}
& \overline{N}(r,1;F)+\overline{N}(r,1;G) \\
\le\ & N(r,1;F|=1)+\overline{N}_L(r,1;F)+\overline{N}_L(r,1;G)+\overline{N}_E^{(2}(r,1;F) \\
& +\overline{N}_{F\ge l+1}(r,1;F|G\ne 1)+\overline{N}(r,1;G) \\
\le\ & \overline{N}(r,0;F|\ge 2)+\overline{N}(r,\infty;F|\ge 2)+\overline{N}(r,0;G|\ge 2) \\
& +\overline{N}(r,\infty;G|\ge 2)+\overline{N}_L(r,1;F)+\overline{N}_L(r,1;G) \\
& +\overline{N}_{F\ge l+1}(r,1;F|G\ne 1)+\overline{N}_{G\ge l+1}(r,1;G|F\ne 1) \\
& +\overline{N}_L(r,1;F)+\overline{N}_L(r,1;G)+\overline{N}_E^{(2}(r,1;F) \\
& +\overline{N}_{F\ge l+1}(r,1;F|G\ne 1)+T(r,G)-m(r,1;G) \\
& +O(1)-2\overline{N}_L(r,1;F)-2\overline{N}_L(r,1;G)-\overline{N}_E^{(2}(r,1;F) \\
& -l\overline{N}_{G\ge l+1}(r,1;G|F\ne 1)+\overline{N}_{F>2}(r,1;G)+\overline{N}_0(r,0;F') \\
& +\overline{N}_0(r,0;G')+S(r,F)+S(r,G) \\
\le\ & \overline{N}(r,0;F|\ge 2)+\overline{N}(r,\infty;F|\ge 2)+\overline{N}(r,0;G|\ge 2) \\
& +\overline{N}(r,\infty;G|\ge 2)+T(r,G)-m(r,1;G) \\
& +2\overline{N}_{F\ge l+1}(r,1;F|G\ne 1)+\overline{N}_{F>2}(r,1;G) \\
& -(l-1)\overline{N}_{G\ge l+1}(r,1;G|F\ne 1)+\overline{N}_0(r,0;F') \\
& +\overline{N}_0(r,0;G')+S(r,F)+S(r,G).
\end{aligned}$$

From Lemma 2.8, we can get

$$\begin{aligned}
& \overline{N}(r,1;F)+\overline{N}(r,1;G) \\
\le\ & \overline{N}(r,0;F|\ge 2)+\overline{N}(r,\infty;F|\ge 2)+\overline{N}(r,0;G|\ge 2) \\
& +\overline{N}(r,\infty;G|\ge 2)+T(r,G)-m(r,1;G)+\tfrac{2}{3}\overline{N}(r,0;F) \\
& +\tfrac{2}{3}\overline{N}(r,\infty;F)-(l-1)\overline{N}_{G\ge l+1}(r,1;G|F\ne 1) \\
& +\overline{N}_0(r,0;F')+\overline{N}_0(r,0;G')+S(r,F)+S(r,G).
\end{aligned}\tag{1}$$

By the second fundamental theorem, we have

$$T(r,F)\le \overline{N}(r,\infty;F)+\overline{N}(r,0;F)+\overline{N}(r,1;F)-\overline{N}_0(r,0;F')+S(r,F),\tag{2}$$

$$T(r,G)\le \overline{N}(r,\infty;G)+\overline{N}(r,0;G)+\overline{N}(r,1;G)-\overline{N}_0(r,0;G')+S(r,G).\tag{3}$$

Adding (2) and (3) and from (1), we get

$$\begin{aligned}
& T(r,F)+T(r,G) \\
\le\ & \overline{N}(r,\infty;F)+\overline{N}(r,0;F)+\overline{N}(r,\infty;G)+\overline{N}(r,0;G) \\
& +\overline{N}(r,1;F)+\overline{N}(r,1;G)-\overline{N}_0(r,0;F')-\overline{N}_0(r,0;G') \\
& +S(r,F)+S(r,G) \\
\le\ & N_2(r,0;F)+N_2(r,\infty;F)+N_2(r,0;G)+N_2(r,\infty;G) \\
& +T(r,G)-m(r,1;G)+\tfrac{2}{3}\overline{N}(r,0;F)+\tfrac{2}{3}\overline{N}(r,\infty;F) \\
& -(l-1)\overline{N}_{G\ge l+1}(r,1;G|F\ne 1)+S(r,F)+S(r,G).
\end{aligned}\tag{4}$$

Thus, we can get

$$\begin{aligned}
& T(r,F) \\
\leq\ & N_2(r,0;F)+N_2(r,\infty;F)+N_2(r,0;G)+N_2(r,\infty;G) \\
& +\tfrac{2}{3}\overline{N}(r,0;F)+\tfrac{2}{3}\overline{N}(r,\infty;F)+S(r,F)+S(r,G).
\end{aligned}$$

Therefore, we complete the proof of Lemma 2.9.

Lemma 2.10 *Let* $\overline{E}_{l)}(1;F)=\overline{E}_{l)}(1;G), E_{2)}(1;F)=E_{2)}(1;G)$ *and* $H\not\equiv 0$, *where* $l\geq 4$. *Then*

$$\begin{aligned}
T(r,F)+T(r,G) \leq\ & 2N_2(r,\infty;F)+2N_2(r,\infty;G)+2N_2(r,0;F) \\
& +2N_2(r,0;G)+S(r,F)+S(r,G).
\end{aligned}$$

Proof: By Lemma 2.6, we can get

$$\begin{aligned}
& T(r,F)+T(r,G) \\
\leq\ & \overline{N}(r,\infty;F)+\overline{N}(r,0;F)+\overline{N}(r,\infty;G)+\overline{N}(r,0;G) \\
& +\overline{N}(r,1;F)+\overline{N}(r,1;G)-\overline{N}_0(r,0;F') \\
& -\overline{N}_0(r,0;G')+S(r,F)+S(r,G) \\
\leq\ & \overline{N}(r,\infty;F)+\overline{N}(r,0;F)+\overline{N}(r,\infty;G)+\overline{N}(r,0;G) \\
& +N(r,1;F|=1)+\overline{N}(r,1;F|\geq 2)+\overline{N}(r,1;G) \qquad (5)\\
& -\overline{N}_0(r,0;F')-\overline{N}_0(r,0;G')+S(r,F)+S(r,G) \\
\leq\ & N_2(r,\infty;F)+N_2(r,0;F)+N_2(r,\infty;G)+N_2(r,0;G) \\
& +\overline{N}_L(r,1;F)+\overline{N}_L(r,1;G)+\overline{N}_{F\geq l+1}(r,1;F|G\neq 1) \\
& +\overline{N}(r,1;G)+\overline{N}_{G\geq l+1}(r,1;G|F\neq 1)+\overline{N}(r,1;F|\geq 2) \\
& +S(r,F)+S(r,G).
\end{aligned}$$

Since

$$\begin{aligned}
\overline{N}(r,1;F|=l;G|=l-1)+\cdots\ & +\overline{N}(r,1;F|=l;G|=3) \\
& \leq \overline{N}(r,1;F|=l);
\end{aligned}$$

and

$$\begin{aligned}
\overline{N}(r,1;G|=l;F|=l-1)+\cdots\ & +\overline{N}(r,1;G|=l;F|=3) \\
& \leq \overline{N}(r,1;G|=l),
\end{aligned}$$

we see that

$$
\begin{array}{rl}
 & \overline{N}_L(r,1;F)+\overline{N}_L(r,1;G)+\overline{N}_{F\geq l+1}(r,1;F|G\neq 1) \\
 & +\overline{N}_{G\geq l+1}(r,1;G|F\neq 1)+\overline{N}(r,1;F|\geq 2)+\overline{N}(r,1;(G) \\
\leq & \overline{N}(r,1;F|=l;G|=l-1)+\cdots+\overline{N}(r,1;F|=l;G|=3) \\
 & +\overline{N}(r,1;F|\geq l+2)+\overline{N}(r,1;G|=l;F|=l-1)+\cdots \\
 & +\overline{N}(r,1;G|=l;F|=3)+\overline{N}(r,1;G|\geq l+2) \\
 & +\overline{N}(r,1;G|\geq l+2)+\overline{N}(r,1;F|\geq l+1) \\
 & +\overline{N}(r,1;G|\geq l+1)+\overline{N}(r,1;F|=2)+\cdots \\
 & +\overline{N}(r,1;F|=l)+\overline{N}(r,1;F|\geq l+1)+\overline{N}(r,1;G|=1) \\
 & +\cdots+\overline{N}(r,1;G|=l)+\overline{N}(r,1;G|\geq l+1) \\
\leq & \frac{1}{2}N(r,1;F|=1)+\overline{N}(r,1;F|=2)+\cdots+2\overline{N}(r,1;F|=l) \\
 & +2\overline{N}(r,1;F|\geq l+1)+\overline{N}(r,1;F|\geq l+2)+\frac{1}{2}N(r,1;G|=1) \\
 & +\overline{N}(r,1;G|=2)+\cdots+2\overline{N}(r,1;G|=l)+2\overline{N}(r,1;G|\geq l+1) \\
 & +\overline{N}(r,1;G|\geq l+2) \\
\leq & \frac{1}{2}[N(r,1;F)+N(r,1;G)] \\
\leq & \frac{1}{2}[T(r,F)+T(r,G)].
\end{array} \tag{6}
$$

From (5) and (6), we can get

$$
\begin{array}{rcl}
T(r,F)+T(r,G) & \leq & 2N_2(r,\infty;F)+2N_2(r,\infty;G)+2N_2(r,0;F) \\
 & & +2N_2(r,0;G)+S(r,F)+S(r,G).
\end{array}
$$

Thus, we complete the proof of Lemma 2.10.

Lemma 2.11 *Let f and g be two transcendental meromorphic functions, and let n and m be three positive integers with $n\geq 7$, and let $P(z)=a_mz^m+a_{m-1}z^{m-1}+\cdots+a_1z+a_0$, where $a_0\neq 0, a_1,\ldots,a_{m-1}, a_m\neq 0$ are complex constants. If $f^nP(f)f'$ and $g^nP(g)g'$ share z IM, then $S(r,f)=S(r,g)$.*

Proof Let $F_1=f^nP(f)f'$ and $G_1=g^nP(g)g'$, by Lemma 2.1, we have

$$(n+m)T(r,f)=T(r,\frac{F_1}{f'})+O(1)\leq T(r,F_1)+T(r,f')+S(r,f).$$

Hence,

$$(n+m-2)T(r,f)+S(r,f)\leq T(r,F_1). \tag{7}$$

Since

$$
\begin{array}{rl}
T(r,F_1) & \leq T(r,f^nP(f))+T(r,f')+S(r,f) \\
 & \leq (n+m+2)T(r,f)+S(r,f).
\end{array} \tag{8}
$$

From (7) and (8), we have $S(r,F_1)=S(r,f)$. From the condition of Lemma 2.11 and the second fundamental theory, we have

$$\begin{aligned}T(r,F_1) &\le \overline{N}(r,\infty;F_1)+\overline{N}(r,0;F_1)+\overline{N}(r,z;F_1)+S(r,F_1)\\ &\le \overline{N}(r,\infty;f)+\overline{N}(r,0;f^nP(f))+\overline{N}(r,0;f')+\overline{N}(r,z;G_1)+S(r,f)\\ &\le (m+4)T(r,f)+T(r,G_1)+S(r,f)\\ &\le (m+4)T(r,f)+(n+m+2)T(r,g)+S(r,f)+S(r,g).\end{aligned}$$

Thus, we have

$$(n-6)T(r,f)\le (n+m+2)T(r,g)+S(r,g)+S(r,f).$$

Since $n\ge 7$, we can get the conclusion of Lemma 2.11.

Lemma 2.12 *Let f and g be two transcendental meromorphic functions. Then*

$$f^nP(f)f'g^nP(g)g'\not\equiv z^2,$$

where $n>3m+1$ is a positive integer.

Proof: Let

$$f^nP(f)f'g^nP(g)g'\equiv z^2. \tag{9}$$

Now we rewrite $P(z)=a_mz^m+a_{m-1}z^{m-1}+\cdots+a_1z+a_0$ as

$$P(z)=a_m(z-\beta_1)^{\gamma_1}\cdots(z-\beta_i)^{\gamma_i}\cdots(z-\beta_s)^{\gamma_s},$$

where $\gamma_1+\cdots+\gamma_i+\cdots+\gamma_s=m$, $1\le s\le m,\beta_i\ne\beta_j,i\ne j,1\le i,j\le s$ and $\beta_1,\ldots,\beta_i,\ldots,\beta_s$ are nonzero constants and $\gamma_1,\ldots,\gamma_i,\ldots,\gamma_s$ are positive integers.

Let $z_0(\ne 0,\infty)$ be a zero of f of order $p(\ge 1)$ and it be a pole of g.Suppose that z_0 is an order $q(\ge 1)$. Then $np+p-1=(n+m)q+q+1$ *i.e.*,$mq=(n+1)(p-q)-2\ge n-1$ *i.e.*,$q\ge\frac{n-1}{m}$. So $p\ge\frac{n+m-1}{m}$.

Let $z_1(\ne 0,\infty)$ be a zero of $P(f)$ of order p_1 and be a zero of $f-\beta_i$ of order q_i for $i=1,2,\ldots,s$. Then $p_1=\gamma_iq_i$ for $i=1,2,\ldots,s$. Suppose that z_1 is a pole of g of order q. Again by (9) we can obtain $q_i\gamma_i+q_i-1=nq+mq+q+1$, *i.e.*, $q_i\ge\frac{n+m+3}{\gamma_i+1}$ for $i=1,2,\ldots,s$.

Let $z_2(\ne 0,\infty)$ be a zero of f' of order p_2 that is not a zero of $fP(f)$. Similarly, we get $p_2\ge n+m+2$.

Therefore we can get that a pole of f is either a zero of $gP(g)$ or a zero of g', we get

$$\begin{aligned}\overline{N}(r,\infty;f) &\le \overline{N}(r,0;g)+\overline{N}(r,\beta_1;g)+\cdots+\overline{N}(r,\beta_i;g)+\cdots\\ &\quad+\overline{N}(r,\beta_s;g)+\overline{N}_0(r,0;g')\\ &\le \tfrac{m}{n+m+1}T(r,g)+\tfrac{m+s}{n+m+3}T(r,g)+\overline{N}_0(r,1/g'),\end{aligned}$$

where $\overline{N}_0(r,0;g')$ is the reduced counting function of those zeros of g' which are not the zeros of $gP(g)$.

By the second fundamental theorem we obtain

$$(10)\quad \begin{aligned} sT(r,f) &\le \overline{N}(r,\infty;f)+\overline{N}(r,0;f)+\overline{N}(r,\beta_1;f)+\cdots+\overline{N}(r,\beta_i;f)\\ &\quad +\cdots+\overline{N}(r,\beta_i;g)-\overline{N}_0(r,0;f')+S(r,f)\\ &\le (\tfrac{m}{n+m+1}+\tfrac{m+s}{n+m+3})T(r,g)+(\tfrac{m}{n+m+1}+\tfrac{m+s}{n+m+3})T(r,f)\\ &\quad +\overline{N}_0(r,0;g')-\overline{N}_0(r,0;f')+2\log r+S(r,f). \end{aligned}$$

Similarly we get

$$(11)\quad \begin{aligned} sT(r,g) &\le (\tfrac{m}{n+m+1}+\tfrac{m+s}{n+m+3})T(r,f)+(\tfrac{m}{n+m+1}+\tfrac{m+s}{n+m+3})T(r,g)\\ &\quad +\overline{N}_0(r,0;f')-\overline{N}_0(r,0;g')+2\log r+S(r,g). \end{aligned}$$

Adding (10) and (11) we get

$$(s-\frac{2m}{n+m-1}-\frac{2m+2s}{n+m+3})\{T(r,f)+T(r,g)\}\le 4\log r+S(r,f)+S(r,g).$$

From $1\le s\le m$ and $n\ge 3m+2$, we can get a contradiction.

Thus, we can get the conclusion of this lemma.

Lemma 2.13 *Let f and g be two transcendental meromorphic functions, and let n and m be three positive integers with $n\ge m+3$, $F=\frac{f^nP(f)f'}{z}$ and $G=\frac{g^nP(g)g'}{z}$, where $n(\ge 4)$ is a positive integer. If $F\equiv G$, then either $f\equiv tg$ for a constant t such that $t^d=1$, where $d=(n+m+1,\dots,n+m+1-i,\dots,n+1)$, $a_{m-i}\ne 0$ for some $i=0,1,\dots,m$, or f and g satisfy the algebraic equation $R(f,g)\equiv 0$, where $R(\omega_1,\omega_2)$ is the definition of Theorem 1.4.*

Proof: Let

$$F^*=\frac{a_mf^{n+m+1}}{n+m+1}+\frac{a_{m-1}f^{n+m}}{n+m}+\cdots+\frac{a_0f^{n+1}}{n+1},$$

and

$$G^*=\frac{a_mg^{n+m+1}}{n+m+1}+\frac{a_{m-1}g^{n+m}}{n+m}+\cdots+\frac{a_0g^{n+1}}{n+1}.$$

From $F\equiv G$, we can get

$$(12)\qquad F^*\equiv G^*+C,$$

where C is a constant. Then we have $T(r,f)=T(r,g)+S(r,f)$.

Suppose that $C \neq 0$, by the second fundamental theorem, we have

$$\begin{aligned}(n+m+1)T(r,f) = T(r,F^*) \leq \overline{N}(r,0;F^*) + \overline{N}(r,\infty;F^*)\\ +\overline{N}(r,C;F^*) + S(r,f) \leq (2m+3)T(r,g) + S(r,g).\end{aligned} \tag{13}$$

By $n \geq m+3$, we can get a contradiction. Thus, we can get $F^* \equiv G^*$, *i.e.*,

$$\begin{aligned}&f^{n+1}(\tfrac{a_m f^m}{n+m+1} + \tfrac{a_{m-1} f^{m-1}}{n+m} + \cdots + \tfrac{a_0}{n+1})\\ &\equiv g^{n+1}(\tfrac{a_m g^m}{n+m+1} + \tfrac{a_{m-1} g^{m-1}}{n+m} + \cdots + \tfrac{a_0}{n+1}.\end{aligned} \tag{14}$$

Let $h = \frac{f}{g}$. If h is a constant, substituting $f = gh$ into (14) we get

$$\frac{a_m g^{n+m+1}(h^{n+m+1}-1)}{n+m+1} + \frac{a_{m-1} f^{n+m}(h^{n+m}-1)}{n+m} + \cdots + \frac{a_0 f^{n+1}(h^{n+1}-1)}{n+1} = 0,$$

which implies $h^\mu = 1$, where $\mu = (n+m+1, \ldots, n+m+1-i, \ldots, n+1)$, $a_{m-i} \neq 0$ for some $i = 0, 1, \ldots, m$. Thus $f \equiv tg$ for a constant t such that $t^\mu = 1$, where $\mu = (n+m+1, \ldots, n+m+1-i, \ldots, n+1)$,$a_{m-i} \neq 0$ for some $i = 0, 1, \ldots, m$.

If h is not a constant, then we can get that f and g satisfy the algebraic equation $R(f,g) = 0$,where $R(\omega_1, \omega_2) = \omega_1^n(\frac{a_m \omega_1^m}{n+m+1} + \frac{a_{m-1}\omega_1^{m-1}}{n+m} + \cdots + \frac{a_0}{n+1}) - \omega_2^n(\frac{a_m \omega_2^m}{n+m+1} + \frac{a_{m-1}\omega_2^{m-1}}{n+m} + \cdots + \frac{a_0}{n+1})$.

Thus, we complete the proof of Lemma 2.13.

Lemma 2.14 [15] Let f and g be two nonconstant meromorphic functions. If $h \equiv 0$ where $h \equiv (\frac{f''}{f'} - \frac{2f'}{f-1}) - (\frac{g''}{g'} - \frac{2g'}{g-1})$, then f, g share 1 CM.

3 The Proofs of Theorems

Let F, G, F^* and G^* be the definition of Lemma 2.13, and F_1, G_1 be the definition of Lemma 2.11.

The Proof of Theorem 1.1: From the condition of Theorem 1.1, we have F, G share 1 CM.

By Lemma 2.1, we have

$$\begin{aligned}T(r,F^*) = (n+m+1)T(r,f) + S(r,f),\\ T(r,G^*) = (n+m+1)T(r,g) + S(r,g).\end{aligned} \tag{15}$$

Since $(F^*)' = Fz$, we deduce

$$m(r,\frac{1}{F^*}) \leq m(r,\frac{1}{zF}) + S(r,f) \leq m(r,\frac{1}{F}) + \log r + S(r,f),$$

and by the first fundamental theorem

$$
\begin{aligned}
T(r,F^*) &\leq T(r,F)+N(r,0;F^*)-N(r,0;F)+\log r+S(r,f)\\
&\leq T(r,F)+N(r,0;f)+N(r,b_1;f)+\cdots+N(r,b_m;f)\\
&\quad -N(r,c_1;f)-\cdots-N(r,c_m;f)-N(r,0;f')\\
&\quad +\log r+S(r,f),
\end{aligned}
\tag{16}
$$

where $b_1,b_2,\dots,b_m$ are roots of the algebraic equation $\frac{a_m z^m}{n+m+1}+\frac{a_{m-1}z^{m-1}}{n+m}+\cdots+\frac{a_0}{n+1}=0$, and $c_1,c_2,\dots,c_m$ are roots of the algebraic equation $a_m z^m+a_{m-1}z^{m-1}+\cdots+a_1 z+a_0=0$.

By the definition of F,G, we have

$$
\begin{aligned}
&N_2(r,0;F)+N_2(r,\infty;F)\\
&\leq 2\overline{N}(r,\infty;f)+2\overline{N}(r,0;f)+N(r,c_1;f)\\
&+\cdots+N(r,c_m;f)+N(r,0;f')+2\log r.
\end{aligned}
\tag{17}
$$

Similarly, we obtain

$$
\begin{aligned}
&N_2(r,0;G)+N_2(r,\infty;G)\\
&\leq 2\overline{N}(r,\infty;g)+2\overline{N}(r,0;g)+N(r,c_1;g)\\
&+\cdots+N(r,c_m;g)+N(r,0;g')+2\log r.
\end{aligned}
\tag{18}
$$

If Lemma 2.3(i) holds, from (17),(18), we have for $\varepsilon(>0)$

(19)

$$
\begin{aligned}
T(r,F^*) &\leq (m+3)T(r,f)+(m+4)T(r,g)+2\overline{N}(r,\infty;f)\\
&\quad +2\overline{N}(r,\infty;g)+5\log r+S(r,f)+S(r,g)\\
&\leq (m+3)T(r,f)+(m+4)T(r,g)+(2-2\Theta(\infty;f)+\varepsilon)T(r,f)\\
&\quad +(2-2\Theta(\infty;g)+\varepsilon)T(r,g)+5\log r+S(r,f)+S(r,g)\\
&\leq [2m+11-(2\Theta(\infty;f)+2\Theta(\infty;g)+2\varepsilon)]T(r)+5\log r+S(r),
\end{aligned}
$$

where $T(r)=\max\{T(r,f),T(r,g)\}$ and $S(r)=\max\{S(r,f),S(r,g)\}$.

Similarly, we obtain

$$
T(r,G^*)\leq [2m+11-(2\Theta(\infty;f)+2\Theta(\infty;g)+2\varepsilon)]T(r)+5\log r+S(r). \tag{20}
$$

By (15),(19) and (20), we have

$$
[n-m-10+(2\Theta(\infty;f)+2\Theta(\infty;g)+2\varepsilon)]T(r)\leq 5\log r+S(r). \tag{21}
$$

Since f,g are two transcendental meromorphic functions and $n>m+10-(2\Theta(\infty;f)+2\Theta(\infty;g))$, we can obtain a contradiction.

If Lemma 2.3(ii) holds, then $F\equiv G$. By Lemma 2.13, we can get the conclusion of Theorem 1.1.

If Lemma 2.2(iii) holds, then $F\cdot G\equiv 1$. By Lemma 2.12 and $n>\max\{m+10-(2\Theta(\infty;f)+2\Theta(\infty;g)),3m+1\}$, we can get a contradiction.

Therefore, we complete the proof of Theorem 1.1.

The Proof of Theorem 1.2: From the condition of Theorem 1.2 and the definition of F,G, we have $\overline{E}_{l)}(1,F)=\overline{E}_{l)}(1,G)$, $E_{1)}(1,F)=E_{1)}(1,G)$ where $l\geq 3$ and

$$(22)\qquad \begin{array}{lcl} \overline{N}(r,\infty;F) & \leq & \overline{N}(r,\infty;f)+\log r,\\ \overline{N}(r,0;F) & \leq & \overline{N}(r,0;f)+N(r,c_1;f)+\cdots+N(r,c_m;f)\\ & & +N(r,0;f')+\log r, \end{array}$$

where $c_1,c_2,\ldots,c_m$ are the definition of **Subsection** 3.1.

Suppose that $H\not\equiv 0$. From (16)-(18),(22) and Lemma 2.9, we have

$$(23)\qquad [n-\frac{5}{3}m-\frac{38}{3}+(\frac{8}{3}\Theta(\infty;f)+2\Theta(\infty;g)]T(r)\leq\frac{19}{3}\log r+S(r).$$

or

$$(24)\qquad [n-\frac{5}{3}m-\frac{38}{3}+(\frac{8}{3}\Theta(\infty;g)+2\Theta(\infty;f)]T(r)\leq\frac{19}{3}\log r+S(r).$$

From $n>\frac{5}{3}m+\frac{38}{3}-2(\Theta(\infty;g)+\Theta(\infty;f)-\frac{2}{3}\min\{\Theta(\infty;g),\Theta(\infty;f)$ and f,g are two transcendental meromorphic functions, we can get a contradiction.

Therefore, we can get $H\equiv 0$. From Lemma 2.14, we have that F,G share 1 CM. By $n>\max\{\frac{5}{3}m+\frac{38}{3}-2(\Theta(\infty;g)+\Theta(\infty;f)-\frac{2}{3}\min\{\Theta(\infty;g),\Theta(\infty;f)\},3m+1\}$ and Theorem 1.1, we can obtain the conclusion of Theorem 1.2.

Therefore, we complete the proof of Theorem 1.2

The Proof of Theorem 1.3: From the condition of Theorem 1.2 and the definition of F,G, we have $\overline{E}_{l)}(1,F)=\overline{E}_{l)}(1,G)$, and $E_{2)}(1,F)=E_{2)}(1,G)$ where $l\geq 4$.

Suppose that $H\not\equiv 0$. From (16)-(18) and Lemma 2.10, we have

$$(25)\qquad [n-m-10+(2\Theta(\infty;f)+2\Theta(\infty;g)+2\varepsilon)]T(r)\leq 5\log r+S(r).$$

Since f,g are two transcendental meromorphic functions and $n>m+10-(2\Theta(\infty;f)+2\Theta(\infty;g))$, we can get a contradiction.

Therefore, we can get $H\equiv 0$. From Lemma 2.14, we have that F,G share 1 CM. From Theorem 1.1 and $n>\max\{m+10-(2\Theta(\infty;f)+2\Theta(\infty;g)),3m+1\}$, we can obtain the conclusion of Theorem 1.3.

Therefore, we complete the proof of Theorem 1.3

4 Remarks

It follows from the proof of Theorem 1.1 that if the condition $f^nP(f)f'$ and $g^nP(g)g'$ share z CM is replaced by the condition $f^nP(f)f'$ and $g^nP(g)g'$ share $\alpha(z)$ CM , where $\alpha(z)$ is a meromorphic function such that $\alpha(z) \not\equiv 0, \infty$ and $\alpha(z) \in S(f) \bigcap S(g)$, the conclusion of Theorem 1.1 still holds.

Similarly, we can get the following results.

Theorem 4.1 *Let f and g be two transcendental meromorphic functions, and let n, l and m be three positive integers with $n > \max\{\frac{5}{3}m + \frac{38}{3} - 2(\Theta(\infty; g) + \Theta(\infty; f) - \frac{2}{3}\min\{\Theta(\infty; g), \Theta(\infty; f)\}, 3m+1\}$. If $\overline{E}_{l)}(\alpha(z), f^nP(f)f') = \overline{E}_{l)}(\alpha(z), g^nP(g)g')$ and $E_{1)}(\alpha(z), f^nP(f)f') = E_{1)}(\alpha(z), g^nP(g)g')$, where $l \geq 3$, then the conclusion of Theorem 1.1 holds.*

Theorem 4.2 *Let f and g be two transcendental meromorphic functions, and let n, l and m be three positive integers with $n > \max\{m + 10 - (2\Theta(\infty; f) + 2\Theta(\infty; g)), 3m+1\}$. If $\overline{E}_{l)}(\alpha(z), f^nP(f)f') = \overline{E}_{l)}(\alpha(z), g^nP(g)g')$ and $E_{2)}(\alpha(z), f^nP(f)f') = E_{2)}(\alpha(z), g^nP(g)g')$, where $l \geq 4$, then the conclusion of Theorem 1.1 holds.*

5 Acknowledgements

The authors want to thanks the referee for his/her thorough review. The research was supported by the NSF of Jiangxi of China (2008GQS0075) and the Soft Scientific and Technological Research Projects of Jiangxi Province ([2008]147).

References

[1] A.Banerjee, *Weighted sharing of a small function by a meromorphic function and its derivative*, Comput. Math. Appl. 53(2007), 1750-1761.

[2] A.Banerjee, *On uniqueness of meromorphic functions when two differentail monomials share one value*, Bull. Korean Math. Soc. 44(4) (2007): 607-622

[3] C.-Y.Fang, M.-L.Fang, *Uniqueness of meromorphic functions and differential polynomials*, Comput. Math. Appl. 44(2002) 607-617.

[4] M.-L.Fang, *Uniqueness and value-sharing of entire functions*, Comput. Math. Appl. 44(2002), 823-831.

[5] M.-L.Fang, X.-H.Hua, *Entire functions that share one value*, J.Nanjing Unvi. Math. Biquarterly 13(1996):44-48.

[6] W.K.Hayman, *Meromorphic Functions*, The Clarendon Press, Oxford,1964.

[7] I.Lahiri, *Weighted sharing and uniqueness of meromorphic functions*, Nagoya Math.J. 161 (2001), 193-206.

[8] I.Lahiri, *Weighted value sharing and uniqueness of meromorphic functions*, Complex Variables Theory Appl. 46 (2001), No. 3, 241-253.

[9] I.Lahiri, S.Dewan, *Value distribution of the product of a meromorphic function and its derivative*, Kodai Math.J., 26(2003)(1): 95-100.

[10] W.-C.Lin, H.-X.Yi, *Uniqueness theorems for meromorphic function*, Indian J. Pure Appl. Math. 35(2004)(2),121-132.

[11] W.-C.Lin, H.-X.Yi, *Uniqueness theorems for meormorphic functions concerning fixed-points*, Complex Variables Theory Appl. 49(2004),No.11, 793-806.

[12] H.-Y.Xu, C.-F.Yi, T.-B.Cao, *Uniqueness of meromorphic functions and differential polynomials sharing one value with finite weight*, Ann.Polon.Math., 95 (2009), 51-66.

[13] C.-C.Yang, X.-H.Hua, *Uniqueness and value-sharing of meromorphic functions*, Ann. Acad. Sci. Fenn. Math. 22(1997), No.2, 395-406.

[14] H.-X.Yi, *Meromorphic functions that share one or two values II*, Kodai Math. J. 22(1999), No.2, 264-272.

[15] H.-X.Yi, *Some further results on uniqueness of meromorphic functions*, Complex Variables Theory Appl. 38 (4),(1999): 375-385.

[16] H.-X.Yi, C.-C.Yang,*Uniqueness theory of meromorphic functions*, Science Press, Beijing,(1995).

[17] L.Yang, *Value distribution theory.*Springer-Verlag.Berlin(1993).

[18] X.-Y.Zhang, J.-F.Chen, W.-C.Lin, *Entire or meromorphic functions sharing one value*, Comput. Math. Appl. 56(2008), 1876-1883.

Hong-Yan Xu
Jingdezhen Ceramic Institute
Department of Informatics and Engineering
Jingdezhen,Jiangxi 333403, China
e-mail: xhyhhh@126.com

Ting-Bin Cao
Nanchang University
Department of Mathematics
Nanchang, Jiangxi 330031, China
e-mail: tbcao@ncu.edu.cn or ctb97@163.com

Tang-Sen Zhan
Jingdezhen Ceramic Institute
Department of Informatics and Engineering
Jingdezhen,Jiangxi 333403, China
e-mail: ztangsen@yahoo.com.cn