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Complementary Connected Domination Number and Connectivity of a Graph

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Abstract

For any graph $G = (V, E)$, a subset S of V is a dominating set if every vertex in $V - S$ is adjacent to at least one vertex in S. A dominating set S is said to be a complementary connected dominating set if the induced subgraph $\langle V - S \rangle$ is connected. The minimum cardinality of a complementary connected dominating set is called the complementary connected domination number and is denoted by $\gamma_{cc}(G)$. The connectivity $\kappa(G)$ of a connected graph G is the minimum number of vertices whose removal results in a disconnected or trivial graph. In this paper we find an upper bound for the sum of the complementary connected domination number and connectivity of a graph and characterize the corresponding extremal graphs.

Keywords: *Domination number, Complementary connected domination number and Connectivity.*

1 Introduction

The graph $G = (V, E)$ we mean a finite, undirected and connected graph with neither loops nor multiple edges. The order and size of G are denoted by n and m respectively. The degree of any vertex u in G is the number of edges incident with u and is denoted by $d(u)$. The minimum and maximum degree of a graph G is denoted by $\delta(G)$ and $\Delta(G)$ respectively. For graph theoretic terminology we refer to Chartrand and Lesniak [1] and Haynes et.al [2, 3].

In a graph G, a subset $S \subseteq V$ is a dominating set if every vertex in $V - S$ is adjacent to at least one vertex in S. The minimum cardinality of a dominating set is called the domination number of G and is denoted by $\gamma(G)$. T. Tamizh Chelvam and B. Jayaprasad [6] introduced the concept of complementary connected domination in graphs. Also V.R. Kulli and B. Janakiram [4] studied the same concept in the name of the nonsplit domination number of a graph. A dominating set S is said to be a complementary connected dominating set if the induced subgraph $\langle V - S\rangle$ is connected. The minimum cardinality of a complementary connected dominating set is called the complementary connected domination number of G and is denoted by $\gamma_{cc}(G)$ and such a set S is called a $\gamma_{cc}-$ set. The connectivity $\kappa(G)$ of a connected graph G is the minimum number of vertices whose removal results in a disconnected or trivial graph.

Several authors have studied the problem of obtaining an upper bound for the sum of a domination parameter and a graph theoretic parameter and characterized the corresponding extremal graphs. J. Paulraj Joseph and S. Arumugam [5] proved that $\gamma(G) + \kappa(G) \leq n$ and characterized the corresponding extremal graphs. In this paper, we obtain an upper bound for the sum of the complementary connected domination number and connectivity of a graph and characterize the corresponding extremal graphs. We use the following theorems and notations.

Theorem 1.1 *[6] For any graph G, $\gamma_{cc}(G) \leq n - 1$ and equality holds if and only if G is a star.*

Theorem 1.2 *[1] For a graph G, $\kappa(G) \leq \delta(G)$.*

Notation 1.3 *$H(m_1, m_2, ..., m_n)$ denotes the graph obtained from the graph H by attaching m_i pendant edges to the vertex $v_i \in V(H), 1 \leq i \leq n$. The graph $K_2(m_1, m_2)$ is called a bistar and it is also denoted by $B(m_1, m_2)$.*

Notation 1.4 *$H(P_{m_1}, P_{m_2}, ..., P_{m_n})$ is the graph obtained from the graph H by attaching an end vertex of P_{m_i} to the vertex v_i in $H, 1 \leq i \leq n$.*

Notation 1.5 *Let G be a regular graph. The graph $G(r)$ is obtained from the graph $G \cup K_1$ by adding r number of edges between the vertex of K_1 and any r vertices of G.*

2 Main Results

Observation 2.1 *Suppose $n \geq 3$, Y is a matching of K_n and $G = K_n - Y$ then $\gamma_{cc}(G) \leq 2$.*

Theorem 2.2 *For any connected graph G, $\gamma_{cc}(G) + \kappa(G) \leq 2n - 2$ and equality holds if and only if G is isomorphic to K_2.*

Proof: $\gamma_{cc}(G) + \kappa(G) \leq n - 1 + \delta \leq n - 1 + n - 1 = 2n - 2$.

Let $\gamma_{cc}(G) + \kappa(G) = 2n - 2$. Then $\gamma_{cc}(G) = n - 1$ and $\kappa(G) = n - 1$ which gives G is a complete graph as well as a star graph. Hence G is isomorphic to K_2. The converse is obvious. □

Theorem 2.3 *For any connected graph G, $\gamma_{cc}(G) + \kappa(G) = 2n - 3$ if and only if G is isomorphic to $K_{1,2}$ or K_3.*

Proof: Let $\gamma_{cc}(G) + \kappa(G) = 2n - 3$. Then there are two cases to consider (*i*) $\gamma_{cc}(G) = n - 1$ and $\kappa(G) = n - 2$ (*ii*) $\gamma_{cc}(G) = n - 2$ and $\kappa(G) = n - 1$.

Case 1. $\gamma_{cc}(G) = n - 1$ and $\kappa(G) = n - 2$

Then G is a star graph and hence $\kappa(G) = 1$ which gives $n = 3$. Thus G is isomorphic to $K_{1,2}$.

Case 2. $\gamma_{cc}(G) = n - 2$ and $\kappa(G) = n - 1$

Since $\kappa(G) = n-1$, we have G is a complete graph. But $\gamma_{cc}(K_n) = 1$ which gives $n = 3$. Hence G is isomorphic to K_3. The converse is obvious. □

Theorem 2.4 *For any connected graph G, $\gamma_{cc}(G) + \kappa(G) = 2n - 4$ if and only if G is isomorphic to $K_{1,3}$ or K_4 or C_4.*

Proof: Let $\gamma_{cc}(G) + \kappa(G) = 2n - 4$. Then there are three cases to consider (*i*) $\gamma_{cc}(G) = n - 1$ and $\kappa(G) = n - 3$, (*ii*) $\gamma_{cc}(G) = n - 2$ and $\kappa(G) = n - 2$, (*iii*) $\gamma_{cc}(G) = n - 3$ and $\kappa(G) = n - 1$.

Case 1. $\gamma_{cc}(G) = n - 1$ and $\kappa(G) = n - 3$

Then G is a star graph and hence $\kappa(G) = 1$ which gives $n = 4$. Thus G is isomorphic to $K_{1,3}$.

Case 2. $\gamma_{cc}(G) = n - 2$ and $\kappa(G) = n - 2$

Then $n - 2 \leq \delta$. If $\delta = n - 1$ then G is a complete graph which is a contradiction. Hence $\delta = n - 2$. Then G is isomorphic to $K_n - Y$ where Y is any matching in K_n. Then $\gamma_{cc} \leq 2$. If $\gamma_{cc} = 1$ then $n = 3$ and hence G is isomorphic to $K_{1,2}$ which is a contradiction. If $\gamma_{cc} = 2$ then $n = 4$. Hence G is isomorphic to C_4 or $K_4 - e$. But $\gamma_{cc}(K_4 - e) = 1 \neq n - 2$ which gives G is isomorphic to C_4.

Case 3. $\gamma_{cc}(G) = n - 3$ and $\kappa(G) = n - 1$

Since $\kappa(G) = n-1$, we have G is a complete graph. But $\gamma_{cc}(K_n) = 1$ which gives $n = 4$. Hence G is isomorphic to K_4. The converse is obvious. □

Theorem 2.5 *For any connected graph G, $\gamma_{cc}(G) + \kappa(G) = 2n - 5$ if and only if G is isomorphic to any one of the following graphs (i) $K_{1,4}$ (ii) K_5 (iii) $K_4 - e$ (iv) C_5 (v) P_4 (vi) $K_3(1, 0, 0)$.*

Proof: Let $\gamma_{cc}(G) + \kappa(G) = 2n - 5$. Then there are four cases to consider (*i*) $\gamma_{cc}(G) = n - 1$ and $\kappa(G) = n - 4$, (*ii*) $\gamma_{cc}(G) = n - 2$ and $\kappa(G) = n - 3$, (*iii*) $\gamma_{cc}(G) = n - 3$ and $\kappa(G) = n - 2$, (*iv*) $\gamma_{cc}(G) = n - 4$ and $\kappa(G) = n - 1$.

Case 1. $\gamma_{cc}(G) = n - 1$ and $\kappa(G) = n - 4$

Then G is a star graph and hence $\kappa(G) = 1$ which gives $n = 5$. Thus G is isomorphic to $K_{1,4}$.

Case 2. $\gamma_{cc}(G) = n - 2$ and $\kappa(G) = n - 3$

Then $n - 3 \leq \delta$. If $\delta = n - 1$ then G is a complete graph which is a contradiction. If $\delta = n - 2$ then G is isomorphic to $K_n - Y$ where Y is a matching in K_n. Then $\gamma_{cc} \leq 2$ and hence $n = 4$ which gives G is isomorphic to either C_4 or $K_4 - e$ which is a contradiction to $\kappa(G) = n - 3$. Hence $\delta = n - 3$.

Let $X = \{v_1, v_2, \cdots, v_{n-3}\}$ be a minimum vertex cut of G and let $V - X = \{x_1, x_2, x_3\}$.

Sub Case 2.1. $\langle V - X \rangle = \overline{K_3}$

Then every vertex of $V - X$ is adjacent to all the vertices in X. Suppose $E(\langle X \rangle) = \emptyset$. Then G is isomorphic to $K_{1,3}$ or $K_{2,3}$ or $K_{3,3}$ which is a contradiction to $\gamma_{cc}(G) = n - 2$.

Suppose $E(\langle X \rangle) \neq \emptyset$. Let $v_1 v_2 \in E(G)$. Then $V - \{x_1, x_2, x_3, v_1\}$ is a complementary connected dominating set of G which is a contradiction.

Sub Case 2.2. $\langle V - X \rangle = K_1 \cup K_2$

Let $x_1 x_2 \in E(G)$. Then x_3 is adjacent to all the vertices in X and x_1, x_2 are not adjacent to at most one vertex in X. If $|X| \geq 3$ then there exists an vertex $v_1 \in X$ such that $v_1 x_1$, $v_1 x_2 \in E(G)$. Then $V - \{x_1, x_2, v_1\}$ is a complementary connected dominating set of G which is a contradiction. If $|X| = 1$ then G is either P_4 or $K_3(1, 0, 0)$. Suppose $|X| = 2$ and let $X = \{v_1, v_2\}$. If x_1 and x_2 are adjacent to all the vertices in X. Then G is a graph obtained from $(K_4 - e) \cup K_1$ by joining a vertex of K_1 to two vertices of $K_4 - e$ of degree 2 or $K_4(2)$. But for these graphs $\gamma_{cc} \neq n - 2$. If x_1 and x_2 are adjacent to v_1 and not adjacent to v_2 then also $\gamma_{cc} \neq n - 2$. If x_1 is not adjacent to v_1 and x_2 is not adjacent to v_2 then G is isomorphic to C_5 or $C_4(2)$. But $\gamma_{cc}(C_4(2)) = 2 \neq n - 2$.

Hence G is isomorphic to C_5.

Case 3. $\gamma_{cc}(G) = n-3$ and $\kappa(G) = n-2$

Then $n-2 \leq \delta$. If $\delta = n-1$ then G is a complete graph which is a contradiction. Hence $\delta = n-2$. Then G is isomorphic to $K_n - Y$ where Y is any matching in K_n. Then $\gamma_{cc} \leq 2$. If $\gamma_{cc} = 1$ then $n = 4$ and hence G is isomorphic to either C_4 or $K_4 - e$. But $\gamma_{cc}(C_4) = 2 \neq n-3$. Hence G is isomorphic to $K_4 - e$.

Case 4. $\gamma_{cc}(G) = n-4$ and $\kappa(G) = n-1$

Since $\kappa(G) = n-1$ we have G is a complete graph. But $\gamma_{cc}(K_n) = 1$ which gives $n = 5$. Hence G is isomorphic to K_5. The converse is obvious. □

Theorem 2.6 *For any connected graph G, $\gamma_{cc}(G) + \kappa(G) = 2n-6$ if and only if G is isomorphic to any one of the following graphs (i) $K_{1,5}$ (ii) K_6 (iii) C_6 (iv) P_5 (v) $B(2,1)$ (vi) $C_3(1,1,0)$ (vii) $K_3(2,0,0)$ (viii) $K_{2,3}$ (ix) $C_4(2)$ (x) $C_4(3)$ (xi) $K_5 - M$ where M is a matching in K_5 (xii) $K_6 - Y$ where Y is a perfect matching in K_6.*

Proof: Let $\gamma_{cc}(G) + \kappa(G) = 2n-6$. Then there are five cases to consider (*i*) $\gamma_{cc}(G) = n-1$ and $\kappa(G) = n-5$ (*ii*) $\gamma_{cc}(G) = n-2$ and $\kappa(G) = n-4$ (*iii*) $\gamma_{cc}(G) = n-3$ and $\kappa(G) = n-3$ (*iv*) $\gamma_{cc}(G) = n-4$ and $\kappa(G) = n-2$ (*v*) $\gamma_{cc}(G) = n-5$ and $\kappa(G) = n-1$

Case 1. $\gamma_{cc}(G) = n-1$ and $\kappa(G) = n-5$

Then G is a star graph and hence $\kappa(G) = 1$ which gives $n = 6$. Thus G is isomorphic to $K_{1,5}$.

Case 2. $\gamma_{cc}(G) = n-2$ and $\kappa(G) = n-4$

Then $n-4 \leq \delta(G)$. If $\delta(G) = n-1$ then G is a complete graph which is a contradiction to $\kappa(G) = n-4$. If $\delta(G) = n-2$ then G is isomorphic to $K_n - Y$ where Y is a matching in K_n. Hence $\gamma_{cc}(G) \leq 2$. Then $n \leq 4$ which is a contradiction to $\kappa(G) = n-4$. Suppose $\delta(G) = n-3$. Let $X = \{v_1, v_2, \cdots, v_{n-4}\}$ be a minimum vertex cut of G and let $V - X = \{x_1, x_2, x_3, x_4\}$. If $\langle V - X \rangle$ contains at least one isolated vertex then $\delta(G) \leq n-4$ which is a contradiction. Hence $\langle V - X \rangle$ is isomorphic to $K_2 \cup K_2$. Let us assume $x_1x_2, x_3x_4 \in E(G)$. Also every vertex of $V - X$ is adjacent to all the vertices of X. If $|X| \geq 2$ then $(X - \{v_1\}) \cup \{x_1, x_2\}$ is a complementary connected dominating set of

G which is a contradiction. If $|X| = 1$ then $\{x_2, x_3\}$ is a complementary connected dominating set of G which is a contradiction. Thus $\delta(G) = n - 4$.

Sub Case 2.1. $\langle V - X\rangle = \overline{K_4}$

Then every vertex of $V - X$ is adjacent to all the vertices in X. Suppose $E(\langle X\rangle) = \phi$. Then $|X| \leq 4$ and hence G is isomorphic to $K_{s,4}, 1 \leq s \leq 4$. But $\gamma_{cc}(G) + \kappa(G) \neq 2n - 6$.

Suppose $E(\langle X\rangle) \neq \phi$. If any one of the vertex in X say v_1 is adjacent to all the vertices in X and hence $\gamma_{cc}(G) = 1$. Then $n = 3$ which is impossible. Hence every vertex in X is not adjacent to at least one vertex in X. Hence $\gamma_{cc}(G) = 2$. Then $n = 4$ which is also impossible.

Sub Case 2.2. $\langle V - X\rangle = P_3 \cup K_1$

Let x_1 be the isolated vertex in $\langle V - X\rangle$ and let (x_2, x_3, x_4) be the path in $\langle V - X\rangle$. Then x_1 is adjacent to all the vertices in X and x_2, x_4 are not adjacent to at most one vertex in X and x_3 is not adjacent to at most two vertices in X. If $|X| \geq 3$ then $X \cup \{x_1\}$ is a complementary connected dominating set of cardinality $n - 3$ which is a contradiction. If $|X| = 2$ then $\{x_3, x_4, v_2\}$ is a complementary connected dominating set of G or G is isomorphic to C_6.Thus G is isomorphic to C_6. If $|X| = 1$ then G is isomorphic to P_5 or $B(2,1)$ or $C_3(1,1,0)$ or $C_4(1,0,0)$ or the graph G_1 which is obtained from $(K_4 - e) \cup K_1$ by adding an edge between a vertex of K_1 and a vertex of degree three in $K_4 - e$. But $\gamma_{cc}(C_4(1,0,0)) = \gamma_{cc}(G_1) = 2 \neq n - 2$. Hence G is isomorphic to P_5 or $B(2,1)$ or $C_3(1,1,0)$.

Sub Case 2.3. $\langle V - X\rangle = K_3 \cup K_1$

Let x_1 be the isolated vertex in $\langle V - X\rangle$ and let $\langle\{x_2, x_3, x_4\}\rangle$ be the complete graph. Then x_1 is adjacent to all the vertices in X and x_2, x_3, x_4 are not adjacent to at most two vertices in X. If $|X| \geq 3$ then $X \cup \{x_1\}$ is a complementary connected dominating set of cardinality $n - 3$ which is a contradiction. If $|X| = 2$ then $\{v_1, x_1, x_2\}$ or $\{v_1, x_1, x_3\}$ or $\{v_1, x_1, x_4\}$ is a complementary connected dominating set of G. Hence $\gamma_{cc}(G) \leq 3$. Then $n \leq 5$ which is a contradiction. If $|X| = 1$ then $\gamma_{cc}(G) \leq 2$ and hence $n \leq 4$ which is a contradiction.

Sub Case 2.4. $\langle V - X\rangle = K_2 \cup K_2$

Let $x_1 x_2$, $x_3 x_4 \in E(G)$. Since $\delta(G) = n - 4$ each $x_i, 1 \leq i \leq 4$ is non-adjacent to at most one vertex in X. If $|X| \geq 3$ then $N(x_1) \cap N(x_3) \cap X \neq \phi$. Let $v_1 \in N(x_1) \cap N(x_3) \cap X$. Then $V - \{x_1, x_3, v_1\}$ is a complementary connected dominating set of G which is a contradiction. Let $|X| = 2$. If $\{N(x_1) \cup N(x_2)\} \cap \{N(x_3) \cup N(x_4)\} = \phi$ then $\kappa(G) = 1 \neq n - 4$ which is a contradiction. Hence we assume with out loss of generality x_1 and x_3 are adjacent to v_1. Then $\{v_2, x_2, x_4\}$ is a complementary connected dominating set of G which is a contradiction. Hence $|X| = 1$. Then G is isomorphic to P_5 or $C_3(P_3, P_1, P_1)$ or the graph G_2 which is obtained from $C_3(2, 0, 0)$ by joining the pendant vertices by an edge. But $\gamma_{cc}(C_3(P_3, P_1, P_1)) = \gamma_{cc}(G_2) = 2 \neq n-2$ which is a contradiction. Hence G is isomorphic to P_5.

Sub Case 2.5. $\langle V - X \rangle = K_2 \cup \overline{K_2}$

Let $x_1 x_2 \in E(G)$ and $x_3 x_4 \in E(\overline{G})$. Then each x_i, $i = 1$ or 2 is non adjacent to at most one vertex in X and each x_j, $j = 3$ or 4 is adjacent to all the vertices in X. For this graph $\gamma_{cc}(G) \leq 3$ and hence $n \leq 5$. Thus $n = 5$. Then $|X| = 1$. Hence G is isomorphic to $B(2, 1)$ or $K_3(2, 0, 0)$.

Case 3. $\gamma_{cc}(G) = n - 3$ and $\kappa(G) = n - 3$

Then $n-3 \leq \delta(G)$. If $\delta = n-1$ then G is a complete graph which is a contradiction to $\kappa(G) = n-3$. If $\delta = n-2$ then G is isomorphic to $K_n - Y$ where Y is a matching in K_n. Then $\gamma_{cc}(G) \leq 2$. If $\gamma_{cc}(G) = 1$ then $n = 4$. Hence G is isomorphic to $K_4 - e$. But $\kappa(K_4 - e) = 2 \neq n - 3$ which is a contradiction. If $\gamma_{cc}(G) = 2$ then $n = 5$. But $\gamma_{cc}(K_5 - Y) = 1$. Hence there is no graph satisfy the given conditions. Hence $\delta(G) = n - 3$. Let $X = \{v_1, v_2, \cdots, v_{n-3}\}$ be a minimum vertex cut of G and let $V - X = \{x_1, x_2, x_3\}$.

Sub Case 3.1. $\langle V - X \rangle = \overline{K_3}$

Then every vertex of $V - X$ is adjacent to all the vertices in X. Suppose $E(\langle X \rangle) = \phi$. Then $|X| \leq 3$ and hence G is isomorphic to $K_{2,3}$ or $K_{3,3}$. But $\gamma_{cc}(K_{3,3}) = 2 \neq n - 3$. Hence G is isomorphic to $K_{2,3}$. Suppose $E(\langle X \rangle) \neq \phi$. If $v_i \in X$ for some i, is adjacent to all the vertices in X and hence $\gamma_{cc}(G) = 1$. Then $n = 4$ which is a contradiction. Hence every vertex in X is not adjacent to at least one vertex in X. Hence $\gamma_{cc}(G) = 2$. Then $n = 5$. Hence G is isomorphic to $K_{2,3}$.

Sub Case 3.2. $\langle V - X \rangle = K_1 \cup K_2$

Let $x_1 x_2 \in E(G)$. Since $\delta = n-3$ we have x_3 is adjacent to all the vertices of X and x_1, x_2 are non adjacent to at most one vertex in X. Suppose x_1 is adjacent to all the vertices of X. Then $\{x_2, x_3\}$ is a complementary connected dominating set of G and hence $\gamma_{cc}(G) \leq 2$. If $\gamma_{cc}(G) = 1$ then $n = 4$. Hence G is isomorphic to either P_4 or $K_3(1,0,0)$. But $\gamma_{cc}(P_4) = \gamma_{cc}(K_3(1,0,0)) = 2 \neq n-3$ which is a contradiction. If $\gamma_{cc}(G) = 2$ then $n = 5$. Hence G is isomorphic to $C_4(2)$ or $C_4(3)$. Suppose $d(x_i) = n-3, 1 \leq i \leq 2.$. Then $\gamma_{cc}(G) = 2$ or 3. If $\gamma_{cc}(G) = 3$ then $n = 6$. Then we get the graphs with $\gamma_{cc}(G)+\kappa(G) \neq 2n-6$. If $\gamma_{cc}(G) = 2$ then $n = 5$. Hence G is isomorphic to C_5 or $C_4(2)$ or $C_3(P_3, P_1, P_1)$ or the graph G_3 which is obtained from $C_3(2,0,0)$ by joining the pendant vertices by an edge. If G is isomorphic to C_5 or $C_3(P_3, P_1, P_1)$ or G_3 then $\gamma_{cc}(G) + \kappa(G) \neq 2n-6$. Hence G is isomorphic to $C_4(2)$.

Case 4. $\gamma_{cc}(G) = n-4$ and $\kappa(G) = n-2$

Then $\delta(G) \geq n-2$. If $\delta(G) = n-1$ then G is a complete graph which is a contradiction. Hence $\delta(G) = n-2$. Then G is isomorphic to $K_n - M$ where M is a matching in K_n. Thus $\gamma_{cc}(G) \leq 2$. If $\gamma_{cc}(G) = 1$ then $n = 5$. Hence G is isomorphic to $K_5 - M$ where M is a matching in K_5. If $\gamma_{cc}(G) = 2$ then $n = 6$ and hence G is isomorphic to $K_6 - Y$ where Y is a perfect matching in K_6.

Case 5. $\gamma_{cc}(G) = n-5$ and $\kappa(G) = n-1$

Since $\kappa(G) = n-1$ we have G is a complete graph. But $\gamma_{cc}(K_n) = 1$ which gives $n = 6$. Hence G is isomorphic to K_6. The converse is obvious. □

3 Conclusion

In this paper we found an upper bound for the sum of complementary connected domination number and connectivity of graphs and characterized the corresponding extremal graphs. Similarly complementary connected domination number with other graph theoretical parameters can be considered.

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