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Complementary Nil Domination in Interval-valued Intuitionistic Fuzzy Graph

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Abstract

The aim of this paper is to introduce the concept of complementary nil domination in interval-valued intuitionistic fuzzy graph and to obtain some results related to this concept.

Keywords: *Interval-valued Intuitionistic Fuzzy graph, degree, neighborhood of vertex, dominating set, Complementary nil domination set.*

1 Introduction

Zadeh [8] introduced the notion of interval-valued fuzzy set as an extension of fuzzy set [7] which gives a more precise tool to modal uncertainty in real life situations. Some recent work of Zadeh in connection with the importance of fuzzy logic may be found in [9, 10]. Interval-valued fuzzy sets have widely used in many areas of science and engineering, e.g. in approximate reasoning, medical diagnosis, multi valued logic, intelligent control, topological spaces, etc.

Atanassov and Gargov [6] introduced Interval-valued intuitionistic fuzzy set which is helpful to modal the problem precisely. In 1975, Rosenfeld [1] introduced the concept of fuzzy graph. Yeh and Bang [16] also introduced fuzzy graphs independently. Domination in fuzzy graphs was introduced and studied by Somasundaram and Somasundaram [2] in 1998. Interval-valued fuzzy graphs (IVFG) are defined by Akram and Dudek [11] in 2011. Atanassov [5] introduced the concept of intuitionistic fuzzy relations and intuitionistic fuzzy graph (IFG).

In fact, interval-valued fuzzy graphs and interval-valued intuitionistic fuzzy graphs are two different models that extend the theory of fuzzy graph. Recently, complementary nil domination in fuzzy graph was introduced by Ismayil and Mohideen [18] in 2014. Further, In 2014, Hussain and Mohamed [15] introduced the complementary nil domination in intuitionistic fuzzy graph. Motivated by the concept of complementary nil domination and interval-valued intuitionistic fuzzy graph, we introduce complementary nil domination in interval-valued intuitionistic fuzzy graph and obtain some results related to this concept.

2 Preliminaries

Definition 2.1 *A fuzzy set σ is a mapping $\sigma: V \rightarrow [0, 1]$. A fuzzy graph G is a pair of functions $G = (\sigma, \mu)$ where σ is a fuzzy subset of non-empty set V and μ is a symmetric fuzzy relation on σ, i.e.* $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$. *The underlying crisp graph of $G = (\sigma, \mu)$ is denoted by $G^* = (V, E)$ where $E \in V \times V$.*

Let D[0,1] be the set of all closed subintervals of the interval [0,1] and element of this set are denoted by uppercase letters. If $M \in D[0,1]$ then it can be represented as $M = [M_L, M_U]$, where M_L and M_U are the lower and upper limits of M.

Definition 2.2 *An interval valued intuitionistic fuzzy graph with underlying set V is defined to be a pair G = (A, B) where*

i) The functions $M_A: V \rightarrow D[0,1]$ and $N_A : V \rightarrow D[0,1]$ denote the degree of membership and non membership of the element $x \in V$, respectively, such that

$$0 \leq M_A(x) + N_A(x) \leq 1 \text{ for all } x \in V.$$

ii) The functions $M_B: E \subseteq V \times V \rightarrow D[0,1]$ and $N_B: E \subseteq V \times V \rightarrow D[0,1]$ are defined by

$$M_{BL}((x, y)) \leq \min\{M_{AL}(x), M_{AL}(y)\} \text{ and}$$
$$N_{BL}((x, y)) \geq \max\{N_{AL}(x), N_{AL}(y)\}$$
$$M_{BU}((x, y)) \leq \min\{M_{AU}(x), M_{AU}(y)\} \text{ and}$$
$$N_{BU}((x, y)) \geq \max\{N_{AU}(x), N_{AU}(y)\}$$

such that $0 \leq M_{BU}(x, y) + N_{BU}(x, y) \leq 1$ for all $(x, y) \in E$.

Example 2.3 *Figure 2.1 is example for IVIFG, G = (A,B) defined on a graph G^* = (V,E) such that V = {a, b, c, d}, E = {ab, bc, cd, da}, A is an interval valued intuitionistic fuzzy set of V and let B is an interval-valued intuitionistic fuzzy set of E ⊆ V×V. Here*

A = {< a,[0.5,0.7],[0.1,0.2] >, < b,[0.3,0.5],[0.2,0.4] >, < c,[0.4,0.6],[0.2,0.4]>, <d,[0.2,0.5],[0.2,0.5]> },

B = {<ab,[0.3,0.5],[0.2,0.4]>,<bc,[0.3,0.4],[0.2,0.4 >, <cd,[0.2,0.4],[0.3,0.5]>, < da,[0.2,0.5],[0.2,0.5] > }

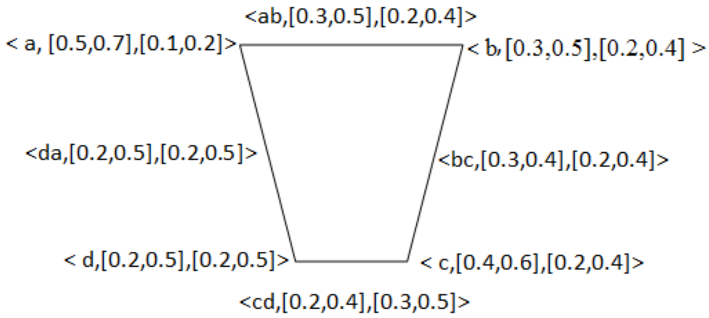

Fig. 2.1

Definition 2.4 *The vertex cardinality of interval valued intuitionistic fuzzy graph G = (A, B) of the graph G^* = (V, E) is defined*

$$|V| = \sum_{x \in V} \left(\frac{1 + M_{AU}(x) - M_{AL}(x)}{2} + \frac{1 + N_{AU}(x) - N_{AL}(x)}{2} \right) = p$$

And Edge cardinality of interval valued intuitionistic fuzzy graph is defined

$$|E| = \sum_{(x,y) \in V} \left(\frac{1 + M_{BU}(x) - M_{BL}(x)}{2} + \frac{1 + N_{BU}(x) - N_{BL}(x)}{2} \right) = q$$

The vertex cardinality of interval valued intuitionistic fuzzy graph (IVIFG) is called the order of G and denoted by O(G). The edge cardinality of IVIFG is called the size of G and denoted by S(G).

Definition 2.5 *An edge e =(x, y) of an interval valued intuitionistic fuzzy graph is called an effective edge if*

$$M_{BL}((x,y)) = \min\{M_{AL}(x), M_{AL}(y)\} \quad \textit{and} \quad N_{BL}((x,y)) = \max\{N_{AL}(x), N_{AL}(y)\}$$
$$M_{BU}((x,y)) = \min\{M_{AU}(x), M_{AU}(y)\} \quad \textit{and} \quad N_{BU}((x,y)) = \max\{N_{AU}(x), N_{AU}(y)\}$$

In this case, the vertex x is called a neighbor of y.

N(x) = {y∈ V: y is a neighbor of x}. N[x] = N(x) ∪ N(y) is called the closed neighbourhood of x.

Definition 2.6 *The complement of an IVIFG G = (A,B) of the graph G = (V,E) is the IVIFG* $\overline{G} = (\overline{A}, \overline{B})$ *where*

i) $\overline{M}_A(x) = M_A(x)$ and $\overline{N}_A(x) = N_A(x)$

ii) $\overline{M}_{BL}(x, y) = \min\{M_A(x), M_A(y)\} - M_{BL}(x, y)\}$ and

$\overline{N}_{BL}(x, y) = \max\{N_A(x), N_A(y)\} - N_{BL}(x, y)\}$

$\overline{M}_{BU}(x, y) = \min\{M_A(x), M_A(y)\} - M_{BU}(x, y)$ and

$\overline{N}_{BU}(x, y) = \max\{N_A(x), N_A(y)\} - N_{BU}(x, y)\}$ for x, y in V.

Definition 2.7 *An interval valued intuitionistic fuzzy graph G = (A, B) of the graph* G^* *= (V, E) is said to be complete if*

$M_{BL}((x, y)) = \min\{M_{AL}(x), M_{AL}(y)\}$ *and* $N_{BL}((x, y)) = \max\{N_{AL}(x), N_{AL}(y)\}$

$M_{BU}((x, y)) = \min\{M_{AU}(x), M_{AU}(y)\}$ *and* $N_{BU}((x, y)) = \max\{N_{AU}(x), N_{AU}(y)\}$

for all (x,y) ∈ E.

Example 2.8 *Figure 2.2 is a complete IVIFG G = (A, B), where*

A = {< a,[0.3,0.5],[0.2,0.4]>, < b,[0.4,0.6],[0.1,0.3]>, < c,[0.2,0.4],[0.3,0.5]> }

B = {<ab,[0.3,0.5],[0.2,0.4]>,<bc,[0.2,0.4],[0.3,0.5]>,< ac,[0.2,0.4],[0.3,0.5]>}

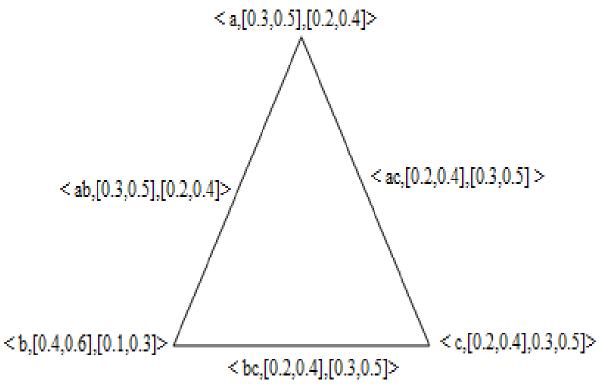

Fig. 2.2

Definition 2.9 *An interval valued intuitionistic fuzzy graph G = (A, B) is said to be strong interval valued intuitionistic fuzzy graph if*

$$M_{BL}((x,y)) = \min\{M_{AL}(x), M_{AL}(y)\} \quad and \quad N_{BL}((x,y)) = \max\{N_{AL}(x), N_{AL}(y)\}$$
$$M_{BU}((x,y)) = \min\{M_{AU}(x), M_{AU}(y)\} \quad and \quad N_{BU}((x,y)) = \max\{N_{AU}(x), N_{AU}(y)\}$$
for all (x,y) $\in$ *E. i.e Every edge is effective edge.*

Example 2.10 *Figure 2.3 is an SIVIFG G = (A,B), where*

A = {< a,[0.4,0.7],[0.1,0.2]>, < b,[0.3,0.6],[0.1,0.4] >, < c,[0.1,0.3],[0.4,0.6] >, < d,[0.2,0.5],[0.3,0.4] > }

B = {<ab,[0.3,0.6],[0.1,0.4]>,<bc,[0.1,0.3],[0.4,0.6]>, < cd,[0.1,0.3],[0.4,0.6]>, < da,[0.2,0.5],[0.3,0.4] > }

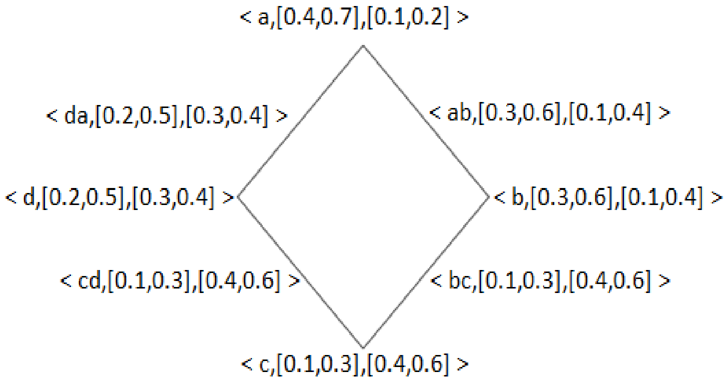

Fig. 2.3

Definition 2.11 *Let G = (A,B) be an IVIFG on V and let u,v* $\in$ *V, we say that v in G if there exist a strong arc between them. A subset D* $\subseteq$ *V is said to be dominating set in G if for every v* $\notin$ *D there exist u* $\in$ *D such that u dominates v. The minimum cardinality of a dominating set in G is called domination number of G and denoted by* $\gamma(G)$*. The maximum cardinality of a minimal domination set is called upper domination number and is denoted by* $\Gamma(G)$*.*

3 Complementary Nil Domination Set in IVIFG

Definition 3.1 *Let G = (A,B) be an IVIFG of graph G = (V,E). A set X* $\subset$ *V is said to be a complementary nil domination set (or simply cnd-set) of an IVIFG G, if X is a dominating set and its complement V-X is not a dominating set. The minimum scalar cardinality over all cnd-set is called a complementary nil domination number and is denoted by* γ_{cnd}*, the corresponding minimum cnd-set is denote by* γ_{cnd}*-set.*

Example 3.2 *Let G = (A,B) be an IVIFG of graph* G^* *= (V,E) be defined as follows:*

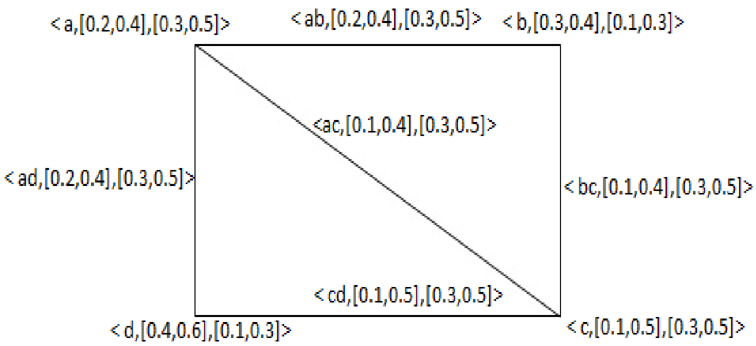

Fig. 3.1

Here $X_1 = \{a, b, c\}$ and $X_2 = \{a, c, d\}$ are minimal cnd-sets

Definition 3.3 *Let $X \subseteq V$ in connected IVIFG $G = (A,B)$ be an IVIFG of graph $G = (V,E)$. A vertex $x \in X$ is said to be an enclave of X if*

$$M_{BL}((x,y)) < \min\{M_{AL}(x), M_{AL}(y)\} \text{ and } N_{BL}((x,y)) < \max\{N_{AL}(x), N_{AL}(y)\}$$
$$M_{BU}((x,y)) < \min\{M_{AU}(x), M_{AU}(y)\} \text{and } N_{BU}((x,y)) < \max\{N_{AU}(x), N_{AU}(y)\}$$

for all $y \in V\text{-}X$. That is $N[x] \subseteq X$

In above figure, *b* is enclave of the set X_1 and d is enclave of the set X_2.

Theorem 3.4 *A dominating set X of interval-valued IFG is a cnd-set if and only if it contains at least one enclave*

Proof: Let *X* be a cnd-set of interval valued intuitionistic fuzzy graph G = *(A, B)* of the graph $G^* = (V, E)$. The *V-X* is not a dominating set which implies that there exist a vertex $x \in X$ such that

$$M_{BL}((x,y)) < \min\{M_{AL}(x), M_{AL}(y)\} \text{ and } N_{BL}((x,y)) < \max\{N_{AL}(x), N_{AL}(y)\}$$
$$M_{BU}((x,y)) < \min\{M_{AU}(x), M_{AU}(y)\} \text{and } N_{BU}((x,y)) < \max\{N_{AU}(x), N_{AU}(y)\}$$

for all $y \in V\text{-}X$. Therefore x is enclave of *X*.

Hence *X* contains at least one enclave.

Conversely: Suppose the dominating set *X* contains enclaves. Without loss of generality let us take x be the enclave of *X*.

$$M_{BL}((x,y)) < \min\{M_{AL}(x), M_{AL}(y)\} \text{ and } N_{BL}((x,y)) < \max\{N_{AL}(x), N_{AL}(y)\}$$
$$M_{BU}((x,y)) < \min\{M_{AU}(x), M_{AU}(y)\} \text{and } N_{BU}((x,y)) < \max\{N_{AU}(x), N_{AU}(y)\}$$

for all $y \in V\text{-}X$.

Hence *V- X* is not a dominating set.

Hence dominating set *X* is cnd-set.

Remark 3.5 *For any IVIFG G = (A,B) of graph G = (V,E).*

1. *Every super set of a cnd-set is also a cnd-set.*
2. *Complement of a cnd-set is not a cnd-set.*
3. *Complement of a domination set is not a cnd-set.*
4. *A cnd-set need not be unique.*

Theorem 3.6 *In any interval valued intuitionistic fuzzy graph G = (A, B) of the graph G^* = (V, E), every complementary nil dominating set of G intersects with every dominating set of G.*

Proof: Let *X* be complementary nil dominating set and *D* be a dominate set of interval valued intuitionistic fuzzy graph *G = (A, B)* of the graph $G^* = (V, E)$. Suppose $D \cap X = \phi$, then $D \subseteq V\text{-}X$ and *V-X* contains a dominating set *D*. Therefore *V-X*, a super of *D*, is dominating set.

Which is contradiction.

Hence $X \cap D \neq \phi$.

Theorem 3.7 *If X is a complementary nil dominating set of an interval valued intuitionistic fuzzy graph G = (A, B) of the graph G^* = (V, E), then there is a vertex $x \in X$ such that X – {x} is dominating set.*

Proof: Let *X* be a cnd-set. By theorem 3.4, every cnd-set contains at least one enclave of *X*. Then

$$M_{BL}((x,y)) < \min\{M_{AL}(x), M_{AL}(y)\} \text{ and } N_{BL}((x,y)) < \max\{N_{AL}(x), N_{AL}(y)\}$$
$$M_{BU}((x,y)) < \min\{M_{AU}(x), M_{AU}(y)\} \text{and } N_{BU}((x,y)) < \max\{N_{AU}(x), N_{AU}(y)\}$$

for all $y \in V\text{-}X$.

Since *G* is connected interval valued intuitionistic fuzzy graph, there exist at least a vertex $z \in X$ such that

$$M_{BL}((x,z)) = \min\{M_{AL}(x), M_{AL}(z)\} \quad \text{and} \quad N_{BL}((x,z)) = \max\{N_{AL}(x), N_{AL}(z)\}$$
$$M_{BU}((x,z)) = \min\{M_{AU}(x), M_{AU}(z)\} \text{ and } \quad N_{BU}((x,z)) = \max\{N_{AU}(x), N_{AU}(z)\} .$$

Hence *X-{x}* is a dominating set.

Theorem 3.8 *A complementary nil dominating set of an interval valued intuitionistic fuzzy graph G = (A, B) of the graph G^* = (V, E) is not singleton.*

Proof: If *X* is a complementary nil dominating set. By theorem 3.4, every cnd-set contains at least one enclave of *X*.

Let $x \in X$ be an enclave of *X*. Then

$$M_{BL}((x, y)) < \min\{M_{AL}(x), M_{AL}(y)\} \text{ and } N_{BL}((x, y)) < \max\{N_{AL}(x), N_{AL}(y)\}$$
$$M_{BU}((x, y)) < \min\{M_{AU}(x), M_{AU}(y)\} \text{and } N_{BU}((x, y)) < \max\{N_{AU}(x), N_{AU}(y)\}$$
for all $y \in V\text{-}X$.

Suppose X contains only one vertex x, then it must be isolated in G.

This is contradiction to connectedness.

Hence complementary nil dominating set contains more than one vertex.

Theorem 3.9 *An interval valued intuitionistic fuzzy graph G = (A, B) of the graph G^* = (V, E) and X be a cnd-set of G. If x and y are two enclaves in X then*

(i) $N[x] \cap N[y] \neq \phi$ *and*
(ii) x and y are adjacent.

$$M_{BL}((x, y)) = \min\{M_{AL}(x), M_{AL}(y)\} \text{ and } N_{BL}((x, y)) = \max\{N_{AL}(x), N_{AL}(y)\}$$

$$M_{BU}((x, y)) = \min\{M_{AU}(x), M_{AU}(y)\} \text{ and } N_{BU}((x, y)) = \max\{N_{AU}(x), N_{AU}(y)\} .$$

Proof: Let *X* be a minimum cnd-set of IVIFG, *G = (A,B)* of graph *G = (V,E)*.
Let x and y are two enclaves of *X*.

Suppose $N[x] \cap N[y] = \phi$, then x is an enclave of *X-N[y]* which implies that *V-(X-N[y])* is not a dominating set.

Therefore *X-N[y]* is not a cnd-set of *G* and $|X - N[y]| < |x| = \gamma_{cnd}(G)$ which is contradiction to the minimality of *X*.

Then $N[x] \cap N[y] \neq \phi$

Suppose

$$M_{BL}((x, y)) < \min\{M_{AL}(x), M_{AL}(y)\} \text{ and } N_{BL}((x, y)) < \max\{N_{AL}(x), N_{AL}(y)\}$$
$$M_{BU}((x, y)) < \min\{M_{AU}(x), M_{AU}(y)\} \text{and } N_{BU}((x, y)) < \max\{N_{AU}(x), N_{AU}(y)\}$$
for all $y \in V\text{-}X$.

that is x and y are non-adjacent. Then $x \notin N[y]$ and x is an enclave of *X-{y}*.

V-(X-{y}) is not a dominating set.

Hence *X-{y}* is a cnd-set, which is contradiction to minimality of *X*.

Hence x and y are adjacent.

Theorem 3.10 *For any Interval-valued fuzzy graph G = (A, B) of graph G = (V,E),* $\Gamma(G) + \gamma_{cnd}(G) \le O(G) + Max\{|v_i|\}$ *for every* $v_i \in V$.

Proof: If *G* is a IVIFG then cnd-set $\subseteq V$ and super set of γ - set

$$\text{i.e. } \gamma_{cnd}(G) \le O(G)$$

and *Γ(G)* = Max {minimal dominating set}

$$\text{i.e. } \Gamma(G) \le Max\{|v_i|\}$$

Therefore, we have

$$\Gamma(G) + \gamma_{cnd}(G) \le O(G) + Max\{|v_i|\}.$$

Example 3.11 *Let G = (A, B) of graph G* = (V, E), where V = {a, b, c, d, e, f} and E = {ab, ad, af, be, cd, cf, ef }. Let A is an interval valued intuitionistic fuzzy set of V and let B is an interval-valued intuitionistic fuzzy set of E* $\subseteq$ *V×V defined by*

A = { < a,[0.2,0.4],[0.1,0.5]>, < b,[0.1,0.5],[0.2,0.3]>, < c,[0.4,0.5],[0.1,0.3]>, < d,[0.3,0.6],[0.2,0.4]>, < e,[0.4,0.6],[0.2,0.4]>, < f,[0.3,0.5],[0.1,0.4]> } and B = {<ab,[0.1,0.4],[0.2,0.5]>,<ad,[0.2,0.4],[0.2,0.5]>, < af,[0.2,0.4],[0.1,0.5]>, < be,[0.1,0.5],[0.2,0.4]>, < cd,[0.3,0.4],[0.2,0.5]>, < cf,[0.3,0.5],[0.1,0.4]>, < ef,[0.3,0.5],[0.2,0.4]> }.

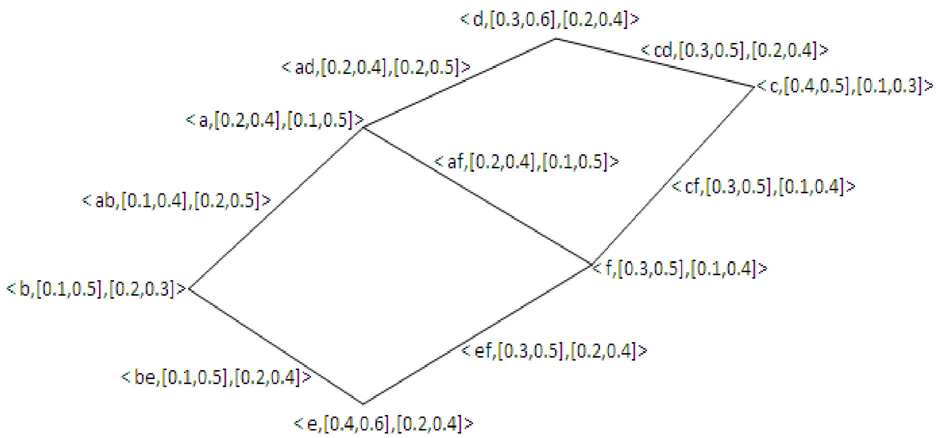

Fig. 3.2

Here X_1 = {a, b, c, d} and X_2 = {b, c, e, f} are minimal complementary nil dominating set. Also minimal dominating set (γ-set) = {b, c}.

(i) $|a| = 1.3$, $|b| = 1.25$, $|c| = 1.15$, $|d| = 1.25$, $|e| = 1.2$, $|f| = 1.25$,
$O(G) = p = 7.4$

(ii) $\gamma_{cnd}(G) = 4.85$ and $\Gamma_{cnd}(G)) = 4.95$

(iii) $\gamma(G) = \Gamma(G) = 2.4$

(iv) The vertices a and d are two enclaves with respect to X_1. The vertices e and f are two enclaves with respect to X_2.

(v) $N[a]$ = {a, b, d }, $N[d]$ = {a, c, d }

i.e. $N[a] \cap N[d] \neq \phi$

Also a and d are adjacent.

(vi) The vertex $d \in X_1$ and X_1-{d} is a dominating set.

(vii) $Min\{|v_i|\} = 1.15$, $Max\{|v_i|\} = 1.3$ for every $v_i \in V$

Also $\Gamma(G) + \gamma_{cnd}(G) = 4.85+2.4 = 7.25$

$O(G) + Max\{|v_i|\} = 7.4+1.3 = 8.7$

$$i.e \quad \Gamma(G) + \gamma_{cnd}(G) \leq O(G) + Max\{|v_i|\}$$

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