

Research Article

Nonexistence Results for the Cauchy Problem for Nonlinear Ultraparabolic Equations

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Nonexistence of global solutions to ultraparabolic equations and systems is presented. Our results fill a gap in the literature on ultraparabolic equations. The method of proof we use relies on a choice of a suitable test function in the weak formulation of the solutions of the problems under-study.

1. Introduction

In this paper, we will present first nonexistence results for the two-time nonlinear equation

$$\mathcal{L}u := u_{t_1} + u_{t_2} - \Delta(|u|^m) = |u|^p, \quad (1.1)$$

posed for $(t_1, t_2, x) \in Q = (0, +\infty) \times (0, +\infty) \times \mathbb{R}^d$, $d \in \mathbb{N}$, and subject to the initial conditions

$$u(t_1, 0; x) = \varphi_1(t_1; x), \quad u(0, t_2; x) = \varphi_2(t_2; x). \quad (1.2)$$

Here $p > 1$, $m > 0$ are real numbers. Then we extend our results to systems of the form

$$\begin{aligned} u_{t_1} + u_{t_2} - \Delta(|u|^m) &= |v|^p, \\ v_{t_1} + v_{t_2} - \Delta(|v|^n) &= |u|^q, \end{aligned} \quad (1.3)$$

for $(t_1, t_2, x) \in Q$, subject to the initial conditions

$$\begin{aligned} u(t_1, 0, x) &= \varphi_1(t_1, x), & u(0, t_2, x) &= \varphi_2(t_2, x), \\ v(t_1, 0, x) &= \psi_1(t_1, x), & v(0, t_2, x) &= \psi_2(t_2, x), \end{aligned} \quad (1.4)$$

and where $p > 1$, $q > 1$, $m > 0$, and $n > 0$ are real numbers. We take the nonlinearities $|u|^p$ in (1.1) and $(|v|^p, |u|^q)$ in (1.3) as prototypes; we could consider much more general nonlinearities.

Before we present our results, let us dwell a while on the existing literature on nonlinear ultraparabolic parabolic equations known also as pluri-parabolic equations or multitime parabolic equations which we are aware of. These types of equations started in the case of linear equations with Kolmogoroff [1] in 1934; he introduced them in order to describe the probability density of a system with 2d degrees of freedom. A lot of generalizations have been made by a large number of authors since then. Nonlinear ultraparabolic equations arise in the kinetic theory of gases [2, 3]. Some stochastic processes models lead also to ultraparabolic equations [4–7]. The analysis of nonlinear ultraparabolic equations have been studied first by Ugowski [8] who studied differential inequalities of parabolic type with multidimensional time; he established, for example, a maximum principle which is very useful for applications. His results were reformulated, in a less general setting, by Walter in [9]. Many nice works on different aspects on ultraparabolic nonlinear equations have been conducted by Lavrenyuk and his collaborators [10–12], Lanconelli and his collaborators [13, 14], and Citti et al. [15]; see also [16, 17]. In the absence of diffusion, an interesting application is mentioned in [18]. Our equation and system have their applications in diffusion theory in porous media.

For better positioning of our results, let us recall the pioneering results of Fujita [19] and their complementary results by Hayakawa [20], Kobayashi et al. [21], and Samarskii et al. [22] concerning nonexistence results for the equation

$$u_t - \Delta(u^m) = |u|^p, \quad t > 0, \quad x \in \mathbb{R}^d, \quad (1.5)$$

which corresponds to (1.1) in the absence of t_2 and with $t_1 = t$.

In his article [19] corresponding to $m = 1$, Fujita proved that

- (i) if $1 < p < 1 + 2/d$, then no global positive solutions for any nonnegative initial data u_0 exist;
- (ii) if $p > 1 + 2/d$, global small data solutions exist while global solutions for large data do not exist.

The borderline case $p = 1 + 2/d$ has been decided by Hayakawa [20] for $d = 1, 2$ and then by Kobayashi et al. [21] for any $d \geq 1$; In case $m = 1$, the exponent $p_{\text{crit}} = 1 + 2/d$ is called the critical exponent.

For (1.5), Samarskii et al. [22] showed that the critical exponent is $p_{\text{crit}} = m + 2/d$.

The aim of this paper is to obtain the critical exponent in the sense of Fujita for (1.1) and for system (1.3). Moreover, we present critical exponents for systems of two equations.

2. Results

Solutions to (1.1) subject to conditions (1.2) are meant in the following weak sense.

Definition 2.1. A function $u \in L^m_{\text{loc}}(Q) \cap L^p_{\text{loc}}(Q)$ is called a weak solution to (1.1) if

$$\begin{aligned} & \int_Q |u|^p \varphi \, dP + \int_S u(0, t_2; x) \varphi(0, t_2; x) \, dP_2 + \int_S u(t_1, 0; x) \varphi(t_1, 0; x) \, dP_1 \\ & = - \int_Q u \varphi_{t_1} \, dP - \int_Q u \varphi_{t_2} \, dP - \int_Q |u|^m \Delta \varphi \, dP \end{aligned} \quad (2.1)$$

for any test function $\varphi \in C_0^\infty(Q)$; $S = \mathbb{R}_+ \times \mathbb{R}^d$, $P = (t_1, t_2, x)$ and $P_1 = (t_1, x)$, $P_2 = (t_2, x)$.

Note that every weak solution is classical near the points (t_1, t_2, x) where $u(t_1, t_2, x)$ is positive.

Two words about the local existence of solutions are in order: as it is a rule, one regularizes (1.1) by adding first a vanishing diffusion term as follows:

$$\mathcal{L}_\varepsilon u = \mathcal{L}u - \varepsilon D_{t_1 t_1}, \quad \varepsilon > 0, \quad (2.2)$$

and then by regularizing the degenerate term $\Delta(|u|^m)$; so, the regular equation

$$u_{t_1} + u_{t_2} - \Delta(\min\{ku, |u|^m\}) - \varepsilon D_{t_1 t_1} = |u|^p, \quad \varepsilon > 0, \quad k = 1, 2, \dots \quad (2.3)$$

is obtained. Consequently, one obtains, for small time t_1 , (ε, k) -uniform estimates of solutions, namely, estimates which are independent on the ‘‘parabolicity’’ constants of the equation as it is clearly explained in [23], see also [10, 24].

Our main first result is dealing with (1.1) subject to (1.2); it is given by the following theorem.

Theorem 2.2. *Assume that $\int_S u(0, t_2; x) \, dP_2 + \int_S u(t_1, 0; x) \, dP_1 > 0$. If $1 \leq m < p \leq m + 2m/(2 + d)$, then Problem (1.1)-(1.2) does not admit global weak solutions.*

Proof. Our strategy of proof is to use the weak formulation of the solution with a suitable choice of the test function which we learnt from [25]. Assume u is a global solution.

If we write

$$u \varphi_{t_i} = u \varphi^{1/p} \varphi^{-1/p} \varphi_{t_i}, \quad i = 1, 2, \quad (2.4)$$

and estimate $\int_Q u \varphi_{t_i} \, dP$ using the ε -Young inequality, we obtain

$$\int_Q u \varphi_{t_i} \, dP \leq \varepsilon \int_Q |u|^p \varphi \, dP + C_\varepsilon \int_Q \varphi^{-1/(p-1)} |\varphi_{t_i}|^{p/(p-1)} \, dP. \quad (2.5)$$

Similarly, we have

$$\int_Q |u|^m \Delta \varphi dP \leq \varepsilon \int_Q |u|^p \varphi dP + C_\varepsilon \int_Q \varphi^{-m/(p-m)} |\Delta \varphi|^{p/(p-m)} dP, \quad (2.6)$$

where $p > m$.

Now, using (2.5) and (2.6), we obtain

$$\begin{aligned} & \int_Q |u|^p \varphi dP + \int_S u(0, t_2; x) \varphi(0, t_2; x) dP_2 + \int_S u(t_1, 0; x) \varphi(t_1, 0; x) dP_1 \\ & \leq 2\varepsilon \int_Q |u|^p \varphi dP + C_\varepsilon \int_Q \left(\varphi^{-1/(p-1)} \left(|\varphi_{t_1}|^{p/(p-1)} + |\varphi_{t_2}|^{p/(p-1)} \right) + \varphi^{-m/(p-m)} |\Delta \varphi|^{p/(p-m)} \right) dP. \end{aligned} \quad (2.7)$$

If we choose $\varepsilon = 1/4$, then we get the estimate

$$\begin{aligned} & \int_Q |u|^p \varphi dP + 2 \int_S u(0, t_2; x) \varphi(0, t_2; x) dP_2 + 2 \int_S u(t_1, 0; x) \varphi(t_1, 0; x) dP_1 \\ & \leq C \int_Q \left(\varphi^{-1/(p-1)} \left(|\varphi_{t_1}|^{p/(p-1)} + |\varphi_{t_2}|^{p/(p-1)} \right) + \varphi^{-m/(p-m)} |\Delta \varphi|^{p/(p-m)} \right) dP =: \mathcal{A}(\varphi), \end{aligned} \quad (2.8)$$

for some positive constant C . Observe that the right-hand side of (2.8) is free of the unknown function u .

At this stage, we introduce the smooth nonincreasing function $\theta : \mathbb{R}_+ \rightarrow [0, 1]$ such that

$$\theta(z) = \begin{cases} 1, & 0 \leq z \leq 1, \\ 0, & 2 \leq z. \end{cases} \quad (2.9)$$

Let us take in (2.8)

$$\varphi(t_1, t_2; x) = \theta^\lambda \left(\frac{t_1}{R^2} + \frac{t_2}{R^2} + \frac{|x|^2}{R^2} \right), \quad (2.10)$$

with $\lambda > \max\{p/(p-1), 2p/(p-m)\}$ and R being positive real number.

Let us now pass to the new variables

$$\tau_1 = R^{-2}t_1, \quad \tau_2 = R^{-2}t_2, \quad y = R^{-1}x. \quad (2.11)$$

We have

$$\varphi_{t_i} = R^{-2}\varphi_{\tau_i}, \quad i = 1, 2, \quad \Delta_x \varphi = R^{-2}\Delta_y \varphi. \quad (2.12)$$

Whereupon

$$\begin{aligned} & \int_Q |u|^p \varphi dP + 2 \int_S u(0, t_2; x) \varphi(0, t_2, x) dP_2 + 2 \int_S u(t_1, 0; x) \varphi(t_1, 0, x) dP_1 \\ & \leq L \left(R^{4+d-2p/(p-1)} + R^{4+d-2p/(p-m)} \right), \end{aligned} \tag{2.13}$$

with

$$\begin{aligned} L := C \int_{\Omega_1} & \left(\theta^{((\lambda-1)p-\lambda)/(p-1)} |\theta'|^{p/(p-1)} + |\theta'|^{2p/(p-m)} \theta^{((\lambda-2)p-\lambda m)/(p-m)} \right. \\ & \left. + |\theta''|^{p/(p-m)} \theta^{((\lambda-1)p-\lambda m)/(p-m)} \right) < +\infty, \end{aligned} \tag{2.14}$$

where $\Omega_1 = \{(\tau_1, \tau_2, y) : 1 \leq |\tau_1| + |\tau_2| + y \leq 2\}$.

Now, we want to pass to the limit as $R \rightarrow +\infty$ in (2.13) under the constraint $2p/(p-m) - 4 - d \geq 0$. We have to consider two cases.

(i) Either $2p/(p-m) - 4 - d > 0 \Leftrightarrow 1 < p < m + 2m/(2+d) = p_{\text{crit}}$ and in this case, the right-hand side of (2.13) will go to zero while the left-hand side is positive. Contradiction.

(ii) Or $p = p_{\text{crit}}$, and in this case, we get in particular

$$\int_{\mathbb{R}_+^2 \times \mathbb{R}^d} |u|^p \varphi dP \leq C \implies \lim_{R \rightarrow +\infty} \int_{C_R} |u|^p \varphi dP = 0, \tag{2.15}$$

where $C_R = \{(t_1, t_2; x) \mid R^2 \leq t_1 + t_2 + |x|^2 \leq 2R^2\}$.

Now, to conclude, we rely on the estimate

$$\begin{aligned} & \int_Q |u|^p \varphi dP + \int_S u(0, t_2; x) \varphi(0, t_2, x) dP_2 + \int_S u(t_1, 0; x) \varphi(t_1, 0, x) dP_1 \\ & \leq \left(\int_{C_R} |u|^p \varphi dP \right)^{1/p} \mathcal{L}(\varphi), \end{aligned} \tag{2.16}$$

which is obtained by using the Hölder inequality.

Passing to the limit as $R \rightarrow +\infty$ in (2.16), we obtain

$$\int_{\mathbb{R}_+^2 \times \mathbb{R}^d} |u|^p dP + \int_S u(0, t_2; x) dP_2 + \int_S u(t_1, 0; x) dP_1 = 0. \tag{2.17}$$

Contradiction. □

Remark 2.3. Notice that the critical exponent for the ultraparabolic equation is *smaller* than the one of the corresponding parabolic equation.

2.1. The Case of a 2×2 -System with a 2-Dimensional Time

In this section, we extend the analysis of the previous section to the case of a 2×2 -system of 2-time equations. More precisely, we consider the system

$$\begin{aligned} u_{t_1} + u_{t_2} - \Delta(|u|^m) &= |v|^p, \\ v_{t_1} + v_{t_2} - \Delta(|v|^n) &= |u|^q, \end{aligned} \quad (2.18)$$

for $(t_1, t_2; x) \in Q$, subject to the initial conditions

$$\begin{aligned} u(0, t_2, x) &= \varphi_1(t_2, x), & u(t_1, 0, x) &= \varphi_2(t_1, x), \\ v(0, t_2, x) &= \psi_1(t_2, x), & v(t_1, 0, x) &= \psi_2(t_1, x), \end{aligned} \quad (2.19)$$

and where $0 < m < p$, $0 < n < q$, and $p, q > 1$ are real numbers.

To lighten the presentation, let us set

$$\begin{aligned} I_0 &:= \int_S u(t_1, 0; x) \varphi(t_1, 0; x) dP_1 + \int_S u(0, t_2; x) \varphi(0, t_2; x) dP_2, \\ J_0 &:= \int_S v(t_1, 0; x) \varphi(t_1, 0; x) dP_1 + \int_S v(0, t_2; x) \varphi(0, t_2; x) dP_2. \end{aligned} \quad (2.20)$$

Let us start with the following definition.

Definition 2.4. We say that $(u, v) \in (L^q_{\text{loc}}(Q) \cap L^m_{\text{loc}}(Q)) \times (L^p_{\text{loc}}(Q) \cap L^n_{\text{loc}}(Q))$ is a weak solution to system (2.18) if

$$\begin{aligned} \int_Q |v|^p \varphi dP + I_0 &= - \int_Q u \varphi_{t_1} dP - \int_Q u \varphi_{t_2} dP - \int_Q |u|^m \Delta \varphi dP, \\ \int_Q |u|^q \varphi dP + J_0 &= - \int_Q v \varphi_{t_1} dP - \int_Q v \varphi_{t_2} dP - \int_Q |v|^n \Delta \varphi dP \end{aligned} \quad (2.21)$$

for any test function $\varphi \in C_0^\infty(Q)$.

Note that every weak solution is classical near the points (t_1, t_2, x) where $u(t_1, t_2, x)$ and $v(t_1, t_2, x)$ are positive.

Let us set

$$\begin{aligned} \sigma_1(p, q) &= \frac{q(2 - (d + 2)p) + 4 + d}{pq - 1}, & \sigma_2(p, q) &= \frac{q(2 - (d + 2)p) + (4 + d)m}{pq - m}, \\ \sigma_3(p, q) &= \frac{q(2n - (d + 2)p) + (4 + d)n}{pq - n}, & \sigma_4(p, q) &= \frac{q(2 - (d + 2)p) + (4 + d)mn}{pq - mn}. \end{aligned} \tag{2.22}$$

Theorem 2.5. *Let $p > 1$, $q > 1$, $p > n$, $q > m$, and assume that*

$$\begin{aligned} \int_S u(t_1, 0; x) dP_1 + \int_S u(0, t_2; x) dP_2 &> 0, \\ \int_S v(t_1, 0; x) dP_1 + \int_S v(0, t_2; x) dP_2 &> 0. \end{aligned} \tag{2.23}$$

Then system (2.18)-(2.19) admits no global weak solution whenever

$$\max\{\sigma_1(p, q), \dots, \sigma_4(p, q), \sigma_1(q, p), \dots, \sigma_4(q, p)\} \leq 0. \tag{2.24}$$

Proof. Assume that the solution is global. Using Hölder's inequality, we obtain

$$\begin{aligned} \int_Q |u|^m |\Delta\varphi| dP &= \int_Q |u|^m \varphi^{m/q} \varphi^{-m/q} |\Delta\varphi| dP \\ &\leq \left(\int_Q |u|^q \varphi dP \right)^{m/q} \left(\int_Q \varphi^{-m/(q-m)} |\Delta\varphi|^{q/(q-m)} dP \right)^{(q-m)/q}, \end{aligned} \tag{2.25}$$

$$\int_Q u \varphi_i dP \leq \left(\int_Q |u|^q \varphi dP \right)^{1/q} \left(\int_Q \varphi^{-1/(q-1)} |\varphi_i|^{(q-1)/q} dP \right)^{(q-1)/q}, \tag{2.26}$$

for $i = 1, 2$. Similarly, we have

$$\int_Q |v|^n |\Delta\varphi| dP \leq \left(\int_Q |v|^p \varphi dP \right)^{n/p} \left(\int_Q \varphi^{-n/(p-n)} |\Delta\varphi|^{p/(p-n)} dP \right)^{(p-n)/p}. \tag{2.27}$$

If we set

$$\begin{aligned}\mathcal{J} &:= \int_Q |u|^q \varphi \, dP, & \mathcal{J} &:= \int_Q |v|^p \varphi \, dP, \\ \mathcal{A}(p, n) &= \left(\int_Q \varphi^{-n/(p-n)} |\Delta \varphi|^{p/(p-n)} \, dP \right)^{(p-n)/p}, \\ \mathcal{B}_i(q) &= \left(\int_Q \varphi^{-1/(q-1)} |\varphi_{t_i}|^{q/(q-1)} \, dP \right)^{(q-1)/q}, \\ \mathcal{B}(q) &= \mathcal{B}_1(q) + \mathcal{B}_2(q),\end{aligned}\tag{2.28}$$

then, using (2.23), inequalities (2.26) and (2.27) in (2.21), we may write

$$\begin{aligned}\mathcal{J} &\leq \mathcal{J}^{1/p} \mathcal{B}(p) + \mathcal{J}^{n/p} \mathcal{A}(p, n), \\ \mathcal{J} &\leq \mathcal{J}^{1/q} \mathcal{B}(q) + \mathcal{J}^{m/q} \mathcal{A}(q, m),\end{aligned}\tag{2.29}$$

so

$$\mathcal{J}^{n/p} \leq C \left\{ \mathcal{J}^{n/pq} \mathcal{B}^{n/q}(q) + \mathcal{J}^{mn/pq} \mathcal{A}^{n/p}(q, m) \right\}\tag{2.30}$$

for some positive constant C .

Whereupon

$$\begin{aligned}\mathcal{J} &\leq C \left\{ \mathcal{J}^{1/pq} \mathcal{B}^{1/p}(q) \mathcal{B}(p) + \mathcal{J}^{m/pq} \mathcal{A}^{1/p}(q, m) \mathcal{B}(p) \right. \\ &\quad \left. + \mathcal{J}^{n/pq} \mathcal{B}^{n/p}(q) \mathcal{A}(p, n) + \mathcal{J}^{mn/pq} \mathcal{A}^{n/p}(q, m) \mathcal{A}(p, n) \mathcal{B}(p) \right\}.\end{aligned}\tag{2.31}$$

Using Hölder's inequality, we may write

$$\begin{aligned}\mathcal{J} &\leq C \left\{ \left(\mathcal{B}^{1/p}(q) \mathcal{B}(p) \right)^{pq/(pq-1)} + \left(\mathcal{A}^{1/p}(q, m) \mathcal{B}(p) \right)^{pq/(pq-m)} \right. \\ &\quad \left. + \left(\mathcal{B}^{n/p}(q) \mathcal{A}(p, n) \right)^{pq/(pq-n)} + \left(\mathcal{A}^{n/p}(q, m) \mathcal{A}(p, n) \right)^{pq/(pq-mn)} \right\}.\end{aligned}\tag{2.32}$$

At this stage, using the scaled variables (2.11), we obtain

$$\begin{aligned}\mathcal{A}(p, n) &= CR^{-2+(4+d)(1-n/p)}, \\ \mathcal{B}_i(q) &= CR^{-2+(4+d)(1-1/q)}, \quad i = 1, 2.\end{aligned}\tag{2.33}$$

Hence, for \mathcal{J} , we get the estimate

$$\mathcal{J} \leq C \left\{ R^{-\sigma_1(p,q)} + R^{-\sigma_2(p,q)} + R^{-\sigma_3(p,q)} + R^{-\sigma_4(p,q)} \right\}. \quad (2.34)$$

Observe that, following the same lines, we can also obtain the following estimate for \mathcal{J} :

$$\mathcal{J} \leq C \left\{ R^{-\sigma_1(q,p)} + R^{-\sigma_2(q,p)} + R^{-\sigma_3(q,p)} + R^{-\sigma_4(q,p)} \right\}. \quad (2.35)$$

To conclude, we have to consider two cases.

Case 1. If $\max\{\sigma_1(p,q), \dots, \sigma_4(p,q), \sigma_1(q,p), \dots, \sigma_4(q,p)\} < 0$ then

$$\begin{aligned} \lim_{R \rightarrow +\infty} \mathcal{J} = \int_Q |u|^q dP = 0 &\implies u = 0, \text{ p.p.} \\ \lim_{R \rightarrow +\infty} \mathcal{J} = \int_Q |v|^p dP = 0 &\implies v = 0, \text{ p.p.} \end{aligned} \quad (2.36)$$

A contradiction.

Case 2. If $\max\{\sigma_1(p,q), \dots, \sigma_4(p,q), \sigma_1(q,p), \dots, \sigma_4(q,p)\} = 0$, we conclude following the same argument used for one equation. \square

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