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Research Article

A Note on Hölder Continuity of Solution Set for Parametric Vector Quasiequilibrium Problems

Xian-Fu Hu

School of Information Engineering, Chongqing City Management Vocational College, Chongqing 401331, China

Correspondence should be addressed to Xian-Fu Hu, xianfuhu@126.com

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By using a scalarization technique, we extend and sharpen the results in S. Li and X. Li (2011) on the Hölder continuity of the solution sets of parametric vector equilibrium problems to the case of parametric vector quasiequilibrium problems in metric spaces. Furthermore, we also give an example to illustrate that our main results are applicable.

1. Introduction

The vector equilibrium problem has been attracting great interest because it provides a unified model for several important problems such as vector variational inequalities, vector complementarity problems, vector optimization problems, vector min-max inequality, and vector saddle point problems. Many different types of vector equilibrium problems have been intensively studied for the past years. For the details, we can refer to [1–7] and the reference therein.

It is well known that stability analysis of solution mapping for parametric vector equilibrium problems or variational inequalities is another important topic in optimization theory and applications. Stability may be understood as lower or upper semicontinuity, continuity, and Lipschitz or Hölder continuity. Recently, the semicontinuity, especially the lower semicontinuity, of solution mappings to parametric vector variational inequalities and parametric vector equilibrium problems has been intensively studied in the literature, such as [8–16]. We observe that rather few works in the literature on the Hölder continuity for parametric vector equilibrium problems, and for this direction one can only refer to [17–29]. Most of the research in the area of stability analysis for parametric variational inequalities and parametric equilibrium problems has been performed under assumptions which implied the local uniqueness of perturbed solutions so that the solution mapping was single valued, see [17–23, 29] and the references therein. Yen [17] obtained the Hölder continuity of the unique solution of a classic perturbed variational inequality by the metric projection method. Mansour and Riahi [18] proved the Hölder continuity of the unique solution for a parametric vector equilibrium problem under the concepts of strong monotonicity and Hölder continuity. Bianchi and Pini [19] introduced the concept of strong pseudomonotonicity and got the Hölder continuity of the unique solution of a parametric vector equilibrium problem. Bianchi and Pini [20] extend the results of [19] to vector equilibrium problems. Anh and Khanh [21] generalized the main results of [19] to the vector case and obtained Hölder continuity of the unique solutions for two classes of perturbed generalized vector equilibrium problem. Anh and Khanh [22] further discussed uniqueness and Hölder continuity of the solutions for perturbed generalized vector equilibrium problems, which improve remarkably the results in [19, 21], and which become properly stronger than the result of [17], when applied to variational inequalities. Anh and Khanh [23] extended the results of [22] to the case of perturbed generalized vector quasiequilibrium problems and obtained the Hölder continuity of the unique solutions.

For general perturbed vector quasiequilibrium problems, it is well known that a solu-tion mapping is, in general, a set valued one. There have also been a few papers to study more general situations where the solution sets of variational inequalities or equilibrium problems may be set-valued. Under the Hausdorff distance and the strong quasimonotonicity, Lee et al. [24] first showed that the set-valued solution mapping for a parametric vector variational inequality is Hölder continuous. Recently, by virtue of the strong quasimonotonicity, Ait Mansour and Aussel [25] have discussed the Hölder continuity of set-valued solution mappings for a parametric generalized variational inequalities. Li et al. [26] introduced an assumption, which is weaker than the corresponding ones in the literature, and established the Hölder continuity of the set-valued solution mappings for two classes of parametric generalized vector quasiequilibrium problems in general metric spaces. Li et al. [27] extended the results of [26] to perturbed generalized vector quasiequilibrium problems, and improved the main results in [27]. Later, S. Li and X. Li [28] use a scalarization technique to obtain the Hölder continuity of the set-valued solution mappings for a parametric weak vector equilibrium problems with set-valued mappings in general metric spaces.

Motivated by the work reported in [24, 26–28], this paper aims at establishing the Hölder continuity of a solution mapping, which is set valued in general, to a parametric weak vector quasiequilibrium problems, by using a scalarization technique. Of course, the main consequences of our results are different from corresponding results in [26, 27] and overcome the drawback, which requires the knowledge of detailed values of the solution mapping in a neighborhood of the point under consideration. Our main results also extend and improve the corresponding ones in [28]. However, in this paper, the main conditions are quite explicit under which we can verify directly. Moreover, we compare our results with corresponding ones in [22, 23] and give an example to illustrate the application of our results.

The rest of the paper is organized as follows. In Section 2, we introduce the parametric vector quasiequilibrium problem and define the solution and ξ-solution to parametric vector quasiequilibrium problem. Then, we recall some definitions and their properties which are needed in the sequel. In Section 3, we discuss the Hölder continuity of the solution mapping for the parametric vector quasiequilibrium problem and compare our main results with the corresponding ones in the recent literature. We also give an example to illustrate that our main results are applicable.

2. Preliminaries

Throughout this paper, if not other specified, $\|\cdot\|$ and $d(\cdot,\cdot)$ denote the norm and metric in any metric space, respectively. Let $B(0,\delta)$ denote the closed ball with radius $\delta \geq 0$ and center 0 in any metric linear spaces. Let X, Y, Λ, M be three metric linear spaces. Let Y^* be the topological dual space of Y. For any $\xi \in Y^*$, we introduce the norm $\|\xi\| = \sup\{\|\langle \xi, x\rangle\| : \|x\| = 1\}$, where $\langle \xi, x\rangle$ denotes the value of ξ at y. Let $C \subset Y$ be a pointed, closed and convex cone with $\operatorname{int} C \neq \emptyset$, where $\operatorname{int} C$ stands for the interior of a subset C. Let $C^* = \{\xi \in Y^* : \langle \xi, y\rangle \geq 0$, for all $y \in C\}$ be the dual cone of C. Since $\operatorname{int} C \neq \emptyset$, the dual cone C^* of C has a weak* compact base. Let $B^* := \{\xi \in C^* : \|\xi\| = 1\}$, which is a weak* compact base of C^*.

Let $N(\lambda_0) \subset \Lambda$ and $N(\mu_0) \subset M$ be neighborhoods of considered points λ_0 and μ_0, respectively. Let $K : X \times \Lambda \rightrightarrows X$ be a set-valued mapping and $f : X \times X \times M \to Y$ be a vector-valued mapping. For each $\lambda \in N(\lambda_0)$ and each $\mu \in N(\mu_0)$, consider the following parameterized vector quasiequilibrium problem of finding $\overline{x} \in K(\overline{x}, \lambda)$ such that

$$f(\overline{x}, y, \mu) \notin -\operatorname{int} C, \quad \forall y \in K(\overline{x}, \lambda). \tag{PVQEP}$$

For each $\lambda \in N(\lambda_0)$ and each $\mu \in N(\mu_0)$, let

$$E(\lambda) := \{x \in X \mid x \in K(x, \lambda)\}. \tag{2.1}$$

Let $S(\lambda, \mu)$ be the solution set of (PVQEP), that is,

$$S(\lambda, \mu) := \{x \in E(\lambda) \mid f(x, y, \mu) \notin -\operatorname{int} C,\ \forall y \in K(x, \lambda)\}. \tag{2.2}$$

For each $\xi \in C^* \setminus \{0\}$, each $\lambda \in N(\lambda_0)$, and each $\mu \in N(\mu_0)$, let $S_\xi(\lambda, \mu)$ denote the set of ξ-solution set to (PVQEP), that is,

$$S_\xi(\lambda, \mu) := \{x \in E(\lambda) : \xi(f(x, y, \mu)) \geq 0,\ \forall y \in K(x, \lambda)\}. \tag{2.3}$$

Now we recall some basic definitions and their properties which are needed in this paper.

Definition 2.1. A vector-valued function $g : X \to Y$ is said to be C convex on X if and only if for any $x, y \in X$ and $t \in [0,1]$, $tf(x) + (1-t)f(y) - f(tx + (1-t)y) \in C$.

Definition 2.2 (see [11]). A set-valued map $G : X \times \Lambda \rightrightarrows Y$ is said to be $(l_1 \cdot \alpha_1, l_2 \cdot \alpha_2)$-Hölder continuous at (x_0, λ_0) if and only if there exist neighborhoods $N(x_0)$ of x_0 and $N(\lambda_0)$ of λ_0 such that, for all $x_1, x_2 \in N(x_0)$, for all $\lambda_1, \lambda_2 \in N(\lambda_0)$,

$$G(x_1, \lambda_1) \subseteq G(x_2, \lambda_2) + (l_1 d^{\alpha_1}(x_1, x_2) + l_2 d^{\alpha_2}(\lambda_1, \lambda_2)) B(0,1), \tag{2.4}$$

where $l_1, l_2 \geq 0$ and $\alpha_1, \alpha_2 > 0$.

Definition 2.3. A set-valued mapping $G : X \times X \times M \rightrightarrows Y$ is said to be $l \cdot \alpha$-Hölder continuous at $\mu_0 \in M$, θ-relative to X if and only if there exists a neighborhood $N(\mu_0)$ of μ_0 such that, for all $\mu_1, \mu_2 \in N(\mu_0)$, for all $x, y \in X$,

$$G(x, y, \mu_1) \subseteq G(x, y, \mu_2) + l d^{\alpha}(\mu_1, \mu_2) d^{\theta}(x, y) B(0, 1), \tag{2.5}$$

where $l \geq 0$, $\theta > 0$ and $\alpha > 0$.

From of [15, Lemma 3.1], we have

Lemma 2.4. *If for each $\mu \in N(\mu_0)$ and each $x \in E(N(\lambda_0))$, $f(x, E(N(\lambda_0)), \mu)$ is a C-convex set, that is, $f(x, E(N(\lambda_0)), \mu) + C$ is a convex set, then*

$$S(\lambda, \mu) = \cup_{\xi \in C^* \setminus \{0\}} S_{\xi}(\lambda, \mu) = \cup_{\xi \in B^*} S_{\xi}(\lambda, \mu). \tag{2.6}$$

Remark 2.5. If for each $\mu \in N(\mu_0)$ and each $x \in E(N(\lambda_0))$, $f(x, \cdot, \mu)$ is C convex on $E(N(\lambda_0))$, then $f(x, E(N(\lambda_0)), \mu)$ is a C-convex set.

In this paper, we use the following notation, for any $A, B \subset X$,

$$\rho(A, B) := \sup\{d(a, b) : a \in A, b \in B\}. \tag{2.7}$$

If A or B is unbounded, then $\rho(A, B) = +\infty$. It is known [30] that solution sets of quasicomplementarity problems are in general unbounded. Hence, so are solution sets of quasiequilibrium problems, which are more general problems.

3. Main Results

In this section, we mainly discuss the Hölder continuity of the solution sets to (PVQEP).

Lemma 3.1. *Suppose that $N(\lambda_0)$ and $N(\mu_0)$ are the given neighborhoods of λ_0 and μ_0, respectively.*

(a) *If $f(\cdot, \cdot, \cdot)$ is $m_1 \cdot \gamma_1$-Hölder continuous at $\mu_0 \in M$, θ-relative to $E(N(\lambda_0))$, then for each $\xi \in B^*$, $\xi(f(\cdot, \cdot, \cdot))$ is also $m_1 \cdot \gamma_1$-Hölder continuous at $\mu_0 \in M$, θ-relative to $E(N(\lambda_0))$.*

(b) *If for each $x \in E(N(\lambda_0))$ and $\mu \in N(\mu_0)$, $f(x, \cdot, \mu)$ is $m_2 \cdot \gamma_2$-Hölder continuous in $E(N(\lambda_0))$, then for each $\xi \in B^*$, $\xi(f(x, \cdot, \mu))$ is also $m_2 \cdot \gamma_2$-Hölder continuous in $E(N(\lambda_0))$.*

Proof. (a) By assumption, there exits a neighborhood of μ_0, denoted without loss of generality by $N(\mu_0)$, such that, for all $\mu_1, \mu_2 \in N(\mu_0)$, for all $x, y \in E(N(\lambda_0)) : x \neq y$,

$$f(x, y, \mu_1) \in f(x, y, \mu_2) + m_1 d^{\gamma_1}(\mu_1, \mu_2) d^{\theta}(x, y) B(0, 1). \tag{3.1}$$

Then, for each $\xi \in B^*$, we obtain that

$$\begin{aligned} \left|\xi(f(x, y, \mu_1)) - \xi(f(x, y, \mu_2))\right| &\leq m_1 d^{\gamma_1}(\mu_1, \mu_2) d^{\theta}(x, y) \sup\{\xi(b) : b \in B(0, 1)\} \\ &= m_1 d^{\gamma_1}(\mu_1, \mu_2) d^{\theta}(x, y). \end{aligned} \tag{3.2}$$

(b) As the proof of (b) is similar to (a), we omit it. Then the proof is complete. □

Theorem 3.2. *Assume that for each $\xi \in B^*$, the ξ-solution set for* (PVQEP) *exists in a neighborhood $N(\lambda_0)\times N(\mu_0)$ of the considered point $(\lambda_0, \mu_0) \in \Lambda\times M$. Assume further that the following conditions hold*

(i) *$K(\cdot,\cdot)$ is $(l_1 \cdot \alpha_1, l_2 \cdot \alpha_2)$-Hölder continuous in $E(N(\lambda_0)) \times N(\lambda_0)$;*

(ii) *for all $\xi \in B^*$, for all $\mu \in N(\mu_0)$, for all $x, y \in E(N(\lambda_0)) : x \neq y$, there exist two constants $h > 0$ and $\beta > 0$ such that*

$$hd^{\beta}(x,y) \leq d(\xi(f(x,y,\mu)),\mathbb{R}_+) + d(\xi(f(y,x,\mu)),\mathbb{R}_+); \tag{3.3}$$

(iii) *f is $m_1 \cdot \gamma_1$-Hölder continuous at $\mu_0 \in M$, θ-relative to $E(N(\lambda_0))$, and for all $x \in E(N(\lambda_0))$, for all $\mu \in N(\mu_0)$, $f(x,\cdot,\mu)$ is $m_2 \cdot \gamma_2$-Hölder continuous in $E(N(\lambda_0))$;*

(iv) *$\alpha_1\gamma_2 = \beta > \theta$ and $h > 2m_2 l_1^{\gamma_2}$.*

Then, for any $\bar{\xi} \in B^*$, there exist open neighborhoods $N'(\bar{\xi})$ of $\bar{\xi}$,$N'_{\bar{\xi}}(\lambda_0)$ of λ_0 and $N'_{\bar{\xi}}(\mu_0)$ of μ_0, such that, the ξ-solution set $S(\cdot,\cdot)$ on $N'(\bar{\xi}) \times N'_{\bar{\xi}}(\lambda_0) \times N'_{\bar{\xi}}(\mu_0)$ is a singleton and satisfies the following Hölder condition: for all $\xi \in N'(\bar{\xi})$, for all $(\lambda_1,\mu_1),(\lambda_2,\mu_2) \in N'_{\bar{\xi}}(\lambda_0) \times N'_{\bar{\xi}}(\mu_0)$,

$$\begin{aligned} d(x_\xi(\lambda_1,\mu_1), x_\xi(\lambda_2,\mu_2)) \leq{} & \left(\frac{m_1}{h - 2m_2 l_1^{\gamma_2}}\right)^{1/(\beta-\theta)} d^{\gamma_1/(\beta-\theta)}(\mu_1,\mu_2) \\ & + \left(\frac{2m_2 l_2^{\gamma_2}}{h - 2m_2 l_1^{\gamma_2}}\right)^{1/\beta} d^{\alpha_2\gamma_2/\beta}(\lambda_1,\lambda_2), \end{aligned} \tag{3.4}$$

where $x_\xi(\lambda_i,\mu_i) \in S_\xi(\lambda_i,\mu_i)$, $i = 1,2$.

Proof. For any $\bar{\xi} \in B^*$, let $N'(\bar{\xi}) \times N'_{\bar{\xi}}(\lambda_0) \times N'_{\bar{\xi}}(\mu_0) \subset B^* \times N(\lambda_0) \times N(\mu_0)$ be open (where $N'_{\bar{\xi}}(\lambda_0), N'_{\bar{\xi}}(\mu_0)$ depend on $\bar{\xi}$). Obviously, $S_\xi(\lambda,\mu)$ is nonempty for each $(\xi,\lambda,\mu) \in N'(\bar{\xi}) \times N'_{\bar{\xi}}(\lambda_0) \times N'_{\bar{\xi}}(\mu_0)$. Let $(\lambda_1,\mu_1),(\lambda_2,\mu_2) \in N'_{\bar{\xi}}(\lambda_0) \times N'_{\bar{\xi}}(\mu_0)$ and $\xi \in N'(\bar{\xi})$ be fixed. For any $\xi \in B^*$, $x, y \in X$, and $\mu \in M$, we set $g^{\xi}(x,y,\mu) := \xi(f(x,y,\mu))$ for the sake of convenient statement in the sequel. We shall divide the proof of (3.4) into three steps.

Step 1. We first show that, for all $x_\xi(\lambda_1,\mu_1) \in S_\xi(\lambda_1,\mu_1)$, for all $x_\xi(\lambda_1,\mu_2) \in S_\xi(\lambda_1,\mu_2)$,

$$d(x_\xi(\lambda_1,\mu_1), x_\xi(\lambda_1,\mu_2)) \leq \left(\frac{m_1}{h - 2m_2 l_1^{\gamma_2}}\right)^{1/(\beta-\theta)} d^{\gamma_1/(\beta-\theta)}(\mu_1,\mu_2). \tag{3.5}$$

Obviously, the conclusion (3.5) is trivial when $x_\xi(\lambda_1,\mu_1) = x_\xi(\lambda_1,\mu_2)$. So we suppose that $x_\xi(\lambda_1,\mu_1) \neq x_\xi(\lambda_1,\mu_2)$. Since $x_\xi(\lambda_1,\mu_1) \in K(x_\xi(\lambda_1,\mu_1),\lambda_1)$, $x_\xi(\lambda_1,\mu_2) \in K(x_\xi(\lambda_1,\mu_2),\lambda_1)$ and by the Hölder continuity of $K(\cdot,\lambda_1)$, there exist $x_1 \in K(x_\xi(\lambda_1,\mu_1),\lambda_1)$ and $x_2 \in K(x_\xi(\lambda_1,\mu_2),\lambda_1)$ such that

$$\begin{aligned} d(x_\xi(\lambda_1,\mu_1),x_2) &\le l_1 d^{\alpha_1}(x_\xi(\lambda_1,\mu_1),x_\xi(\lambda_1,\mu_2)),\\ d(x_\xi(\lambda_1,\mu_2),x_1) &\le l_1 d^{\alpha_1}(x_\xi(\lambda_1,\mu_1),x_\xi(\lambda_1,\mu_2)). \end{aligned} \tag{3.6}$$

Noting that $x_\xi(\lambda_1,\mu_1), x_\xi(\lambda_1,\mu_2)$ are the ξ-solution to (PVQEP) at parameters $(\lambda_1,\mu_1), (\lambda_1,\mu_2)$, respectively, we obtain

$$\begin{aligned} g^\xi(x_\xi(\lambda_1,\mu_1),x_1,\mu_1) &\ge 0,\\ g^\xi(x_\xi(\lambda_1,\mu_2),x_2,\mu_2) &\ge 0. \end{aligned} \tag{3.7}$$

By (ii), we have

$$\begin{aligned} &hd^\beta(x_\xi(\lambda_1,\mu_1),x_\xi(\lambda_1,\mu_2))\\ &\quad\le d\Big(g^\xi(x_\xi(\lambda_1,\mu_2),x_\xi(\lambda_1,\mu_1),\mu_1)\Big),\mathbb{R}_+ + d\Big(g^\xi(x_\xi(\lambda_1,\mu_1),x_\xi(\lambda_1,\mu_2),\mu_1)\Big),\mathbb{R}_+, \end{aligned} \tag{3.8}$$

which together with (3.7) yield that

$$\begin{aligned} &hd^\beta(x_\xi(\lambda_1,\mu_1),x_\xi(\lambda_1,\mu_2))\\ &\quad\le \Big|g^\xi(x_\xi(\lambda_1,\mu_2),x_\xi(\lambda_1,\mu_1),\mu_1) - g^\xi(x_\xi(\lambda_1,\mu_2),x_2,\mu_2)\Big|\\ &\qquad + \Big|g^\xi(x_\xi(\lambda_1,\mu_1),x_\xi(\lambda_1,\mu_2),\mu_1) - g^\xi(x_\xi(\lambda_1,\mu_1),x_1,\mu_1)\Big|\\ &\quad\le \Big|g^\xi(x_\xi(\lambda_1,\mu_2),x_\xi(\lambda_1,\mu_1),\mu_1) - g^\xi(x_\xi(\lambda_1,\mu_2),x_\xi(\lambda_1,\mu_1),\mu_2)\Big|\\ &\qquad + \Big|g^\xi(x(\lambda_1,\mu_2),x_\xi(\lambda_1,\mu_1),\mu_2) - g^\xi(x_\xi(\lambda_1,\mu_2),x_2,\mu_2)\Big|\\ &\qquad + \Big|g^\xi(x_\xi(\lambda_1,\mu_1),x_\xi(\lambda_1,\mu_2),\mu_1) - g^\xi(x_\xi(\lambda_1,\mu_1),x_1,\mu_1)\Big|. \end{aligned} \tag{3.9}$$

Therefore, it follows from Lemma 3.1, (3.6) that

$$\begin{aligned} &hd^\beta(x_\xi(\lambda_1,\mu_1),x_\xi(\lambda_1,\mu_2))\\ &\quad\le m_1 d^\theta(x_\xi(\lambda_1,\mu_1),x_\xi(\lambda_1,\mu_2))d^{\gamma_1}(\mu_1,\mu_2) + m_2 d^{\gamma_2}(x_\xi(\lambda_1,\mu_1),x_2) + m_2 d^{\gamma_2}(x_\xi(\lambda_1,\mu_2),x_1)\\ &\quad\le m_1 d^\theta(x_\xi(\lambda_1,\mu_1),x_\xi(\lambda_1,\mu_2))d^{\gamma_1}(\mu_1,\mu_2) + 2m_2 l_1{}^{\gamma_2} d^{\alpha_1\gamma_2}(x_\xi(\lambda_1,\mu_1),x_\xi(\lambda_1,\mu_2)). \end{aligned} \tag{3.10}$$

This, by (iv), implies that

$$d^{\beta-\theta}(x_\xi(\lambda_1,\mu_1),x_\xi(\lambda_1,\mu_2)) \le \left(\frac{m_1}{h-2m_2l_1^{\gamma_2}}\right)d^{\gamma_1}(\mu_1,\mu_2), \tag{3.11}$$

and hence (3.5) holds.

Step 2. Now we show that, for all $x_\xi(\lambda_1,\mu_2) \in S_\xi(\lambda_1,\mu_2)$, for all $x_\xi(\lambda_2,\mu_2) \in S_\xi(\lambda_2,\mu_2)$,

$$d(x_\xi(\lambda_1,\mu_2),x_\xi(\lambda_2,\mu_2)) \le \left(\frac{2m_2l_2^{\gamma_2}}{h-2m_2l_1^{\gamma_2}}\right)^{1/\beta} d^{\alpha_2\gamma_2/\beta}(\lambda_1,\lambda_2). \tag{3.12}$$

Without loss of generality, we assume that $x_\xi(\lambda_1,\mu_2) \ne x_\xi(\lambda_2,\mu_2)$. Thanks to (i), there exist $x_1' \in K(x_\xi(\lambda_2,\mu_2),\lambda_1)$ and $x_2' \in K(x_\xi(\lambda_1,\mu_2),\lambda_2)$ such that

$$\begin{aligned} &d(x_\xi(\lambda_1,\mu_2),x_2') \le l_2 d^{\alpha_2}(\lambda_1,\lambda_2),\\ &d(x_\xi(\lambda_2,\mu_2),x_1') \le l_2 d^{\alpha_2}(\lambda_1,\lambda_2). \end{aligned} \tag{3.13}$$

By the Hölder continuity of $K(\cdot,\cdot)$, there exist $x_1'' \in K(x_\xi(\lambda_1,\mu_2),\lambda_1)$ and $x_2'' \in K(x_\xi(\lambda_2,\mu_2),\lambda_2)$ such that

$$\begin{aligned} &d(x_1',x_1'') \le l_1 d^{\alpha_1}(x_\xi(\lambda_1,\mu_2),x_\xi(\lambda_2,\mu_2)),\\ &d(x_1',x_2'') \le l_1 d^{\alpha_1}(x_\xi(\lambda_1,\mu_2),x_\xi(\lambda_2,\mu_2)). \end{aligned} \tag{3.14}$$

From the definition of ξ-solution for (PVQEP), we have

$$\begin{aligned} &g^\xi(x_\xi(\lambda_1,\mu_2),x_1'',\mu_2) \ge 0,\\ &g^\xi(x_\xi(\lambda_2,\mu_2),x_2'',\mu_2) \ge 0. \end{aligned} \tag{3.15}$$

It follows from (ii), (3.15) that

$$
\begin{aligned}
& h d^{\beta}\left(x_{\xi}(\lambda_1, \mu_2), x_{\xi}(\lambda_2, \mu_2)\right) \\
& \quad \leq d\left(g^{\xi}\left(x_{\xi}(\lambda_1, \mu_2), x_{\xi}(\lambda_2, \mu_2), \mu_2\right), \mathbb{R}_+\right) + d\left(g^{\xi}\left(x_{\xi}(\lambda_2, \mu_2), x_{\xi}(\lambda_1, \mu_2), \mu_2\right), \mathbb{R}_+\right) \\
& \quad \leq \left| g^{\xi}\left(x_{\xi}(\lambda_1, \mu_2), x_{\xi}(\lambda_2, \mu_2), \mu_2\right) - g^{\xi}\left(x_{\xi}(\lambda_1, \mu_2), x''_1, \mu_2\right) \right| \\
& \qquad + \left| g^{\xi}\left(x_{\xi}(\lambda_2, \mu_2), x_{\xi}(\lambda_1, \mu_2), \mu_2\right) - g^{\xi}\left(x_{\xi}(\lambda_2, \mu_2), x''_2, \mu_2\right) \right| \\
& \quad \leq \left| g^{\xi}\left(x_{\xi}(\lambda_1, \mu_2), x_{\xi}(\lambda_2, \mu_2), \mu_2\right) - g^{\xi}\left(x_{\xi}(\lambda_1, \mu_2), x'_1, \mu_2\right) \right| \\
& \qquad + \left| g^{\xi}\left(x_{\xi}(\lambda_1, \mu_2), x'_1, \mu_2\right) - g^{\xi}\left(x_{\xi}(\lambda_1, \mu_2), x''_1, \mu_2\right) \right| \\
& \qquad + \left| g^{\xi}\left(x_{\xi}(\lambda_2, \mu_2), x_{\xi}(\lambda_1, \mu_2), \mu_2\right) - g^{\xi}\left(x_{\xi}(\lambda_2, \mu_2), x'_2, \mu_2\right) \right| \\
& \qquad + \left| g^{\xi}\left(x_{\xi}(\lambda_2, \mu_2), x'_2, \mu_2\right) - g^{\xi}\left(x_{\xi}(\lambda_2, \mu_2), x''_2, \mu_2\right) \right|,
\end{aligned} \tag{3.16}
$$

which together with (ii) and Lemma 3.1 yields that

$$
\begin{aligned}
h d^{\beta}\left(x_{\xi}(\lambda_1, \mu_2), x(\lambda_2, \mu_2)\right) \leq{}& m_2 d^{\gamma_2}\left(x_{\xi}(\lambda_2, \mu_2), x'{}_1\right) + m_2 d^{\gamma_2}\left(x'_1, x''_1\right) \\
& + m_2 d^{\gamma_2}\left(x_{\xi}(\lambda_1, \mu_2), x'{}_2\right) + m_2 d^{\gamma_2}\left(x'_2, x''_2\right).
\end{aligned} \tag{3.17}
$$

From (3.13), (3.14), and (3.17), we have

$$
\begin{aligned}
h d^{\beta}\left(x_{\xi}(\lambda_1, \mu_2), x_{\xi}(\lambda_2, \mu_2)\right) \leq{}& m_2 l_2^{\gamma_2} d^{\alpha_2 \gamma_2}(\lambda_1, \lambda_2) + m_2 l_1^{\gamma_2} d^{\alpha_1 \gamma_2}\left(x_{\xi}(\lambda_1, \mu_2), x_{\xi}(\lambda_2, \mu_2)\right) \\
& + m_2 l_2^{\gamma_2} d^{\alpha_2 \gamma_2}(\lambda_1, \lambda_2) + m_2 l_1^{\gamma_2} d^{\alpha_1 \gamma_2}\left(x_{\xi}(\lambda_1, \mu_2), x_{\xi}(\lambda_2, \mu_2)\right).
\end{aligned} \tag{3.18}
$$

Therefore, it follows from (iv) and (3.18) that

$$
d^{\beta}\left(x_{\xi}(\lambda_1, \mu_2), x_{\xi}(\lambda_2, \mu_2)\right) \leq \left(\frac{2 m_2 l_2^{\gamma_2}}{h - 2 m_2 l_1^{\gamma_2}}\right) d^{\alpha_2 \gamma_2}(\lambda_1, \lambda_2), \tag{3.19}
$$

and the conclusion (3.12) holds.

Step 3. Finally, we easily see from (3.5) and (3.12) that for all $x_\xi(\lambda_1,\mu_1) \in S_\xi(\lambda_1,\mu_1)$, for all $x_\xi(\lambda_2,\mu_2) \in S_\xi(\lambda_2,\mu_2)$,

$$\begin{aligned} d(x_\xi(\lambda_1,\mu_1), x_\xi(\lambda_2,\mu_2)) &\le d(x_\xi(\lambda_1,\mu_1), x_\xi(\lambda_1,\mu_2)) + d(x_\xi(\lambda_1,\mu_2), x_\xi(\lambda_2,\mu_2)) \\ &\le \left(\frac{m_1}{h - 2m_2 l_1^{\gamma_2}}\right)^{1/(\beta-\theta)} d^{\gamma_1/(\beta-\theta)}(\mu_1,\mu_2) \\ &\quad + \left(\frac{2m_2 l_2^{\gamma_2}}{h - 2m_2 l_1^{\gamma_2}}\right)^{1/\beta} d^{\alpha_2\gamma_2/\beta}(\lambda_1,\lambda_2), \end{aligned} \tag{3.20}$$

and the conclusion (3.4) holds. This implies that

$$\begin{aligned} \rho(S_\xi(\lambda_1,\mu_1), S_\xi(\lambda_2,\mu_2)) &\le \left(\frac{m_1}{h - 2m_2 l_1^{\gamma_2}}\right)^{1/(\beta-\theta)} d^{\gamma_1/(\beta-\theta)}(\mu_1,\mu_2) \\ &\quad + \left(\frac{2m_2 l_2^{\gamma_2}}{h - 2m_2 l_1^{\gamma_2}}\right)^{1/\beta} d^{\alpha_2\gamma_2/\beta}(\lambda_1,\lambda_2). \end{aligned} \tag{3.21}$$

Taking $\lambda_2 = \lambda_1$ and $\mu_2 = \mu_1$ in (3.21), we can get the diameter of $S_\xi(\lambda_1,\mu_1)$ is 0, that is, this set is singleton $\{x_\xi(\lambda_1,\mu_1)\}$. $S_\xi(\lambda_2,\mu_2)$ is similar. Therefore, the uniqueness of ξ-solution for (PVQEP) has been demonstrated, and the proof is complete. □

Remark 3.3. If $E(N(\lambda_0))$ in (iii) of Theorem 3.2 is bounded, then we can take $\theta = 0$ in (iii), since $d(x,y) \le M$ for some $M > 0$, for all $x, y \in E(N(\lambda_0))$. Thus, the condition $\beta > \theta$ in Theorem 3.2 can be omitted.

Assumption (ii) of Theorem 3.2 look seemingly complicated. Now, we give a sufficient condition for (ii) of Theorem 3.2. Namely, we have the following result.

Corollary 3.4. *Theorem 3.2 is still valid if assumption (ii) is replaced with*

(ii') *for all $\mu \in N(\mu_0)$, $f(\cdot,\cdot,\mu)$ is $h \cdot \beta$-Hölder strongly monotone in $E(N(\lambda_0))$, that is, there exist two constants $h > 0$ and $\beta > 0$ such that, for all $x, y \in E(N(\lambda_0)) : x \ne y$,*

$$f(x,y,\mu) + f(y,x,\mu) + hd^\beta(x,y)B(0,1) \subset -C. \tag{3.22}$$

Proof. We only need to prove that assumption (ii) of Theorem 3.2 holds. Indeed, it follows from (ii') that for all $\xi \in B^*$,

$$\xi\Big(f(x,y,\mu) + f(y,x,\mu) + hd^\beta(x,y)b\Big) \le 0, \quad \forall b \in B(0,1). \tag{3.23}$$

Noting that linearity of ξ, we have

$$\xi(f(x,y,\mu)) + \xi(f(y,x,\mu)) + hd^{\beta}(x,y) \sup_{b\in B(0,1)} \xi(b) \le 0, \tag{3.24}$$

which together with $\|\xi\| = 1$ implies that

$$\xi(f(x,y,\mu)) + \xi(f(y,x,\mu)) + hd^{\beta}(x,y) \le 0. \tag{3.25}$$

Therefore,

$$\begin{aligned} hd^{\beta}(x,y) &\le -\xi(f(x,y,\mu)) - \xi(f(y,x,\mu)) \\ &\le d(\xi(f(x,y,\mu)),\mathbb{R}_+) + d(\xi(f(y,x,\mu)),\mathbb{R}_+), \end{aligned} \tag{3.26}$$

and the proof is complete. □

Remark 3.5. When $f : X \times X \times M \to \mathbb{R}$, (PVQEP) collapses to the quasiequilibrium problem (QEP) considered by Anh and Khanh [29]. In this case, Corollary 3.4 is same as the Theorem 2.1 of [29]. From Proposition 1.1 of [22] or Corollary 3.4, we can easily see that Theorem 3.2 improves the Theorem 2.1 of [29].

Theorem 3.6. *Assume that for each $\xi \in B^*$, the ξ-solution set for the problem* (PVQEP) *exists in a neighborhood $N(\lambda_0) \times N(\mu_0)$ of the considered point $(\lambda_0, \mu_0) \in \Lambda \times M$. Assume further that the following conditions hold:*

(i) *$K(\cdot,\cdot)$ is $(l_1 \cdot \alpha_1, l_2 \cdot \alpha_2)$-Hölder continuous in $E(N(\lambda_0)) \times N(\lambda_0)$;*

(ii) *for all $\xi \in B^*$, for all $\mu \in N(\mu_0)$, for all $x, y \in E(N(\lambda_0)) : x \ne y$, there exist two constants $h > 0$ and $\beta > 0$ such that*

$$hd^{\beta}(x,y) \le d(\xi(f(x,y,\mu)),\mathbb{R}_+) + d(\xi(f(y,x,\mu)),\mathbb{R}_+); \tag{3.27}$$

(iii) *f is $m_1 \cdot \gamma_1$-Hölder continuous at $\mu_0 \in M$, θ-relative to $E(N(\lambda_0))$, and for all $x \in E(N(\lambda_0))$, for all $\mu \in N(\mu_0)$, $f(x,\cdot,\mu)$ is $m_2 \cdot \gamma_2$-Hölder continuous in $E(N(\lambda_0))$;*

(iv) *$\alpha_1\gamma_2 = \beta > \theta$ and $h > 2m_2 l_1^{\gamma_2}$;*

(v) *for all $x \in E(N(\lambda_0))$, for all $\mu \in N(\mu_0)$, $f(x,\cdot,\mu)$ is C convex on $E(N(\lambda_0))$.*

Then there exist neighborhoods $\widetilde{N}(\lambda_0)$ of λ_0 and $\widetilde{N}(\mu_0)$ of μ_0, such that, the solution set $S(\cdot,\cdot)$ on $\widetilde{N}(\lambda_0) \times \widetilde{N}(\mu_0)$ is nonempty and satisfies the following Hölder continuous condition, for all (λ_1,μ_1), $(\lambda_2,\mu_2) \in \widetilde{N}(\lambda_0) \times \widetilde{N}(\mu_0)$:

$$\begin{aligned} &S(\lambda_1,\mu_1) \subset S(\lambda_2,\mu_2) \\ &\quad + \left(\left(\frac{m_1}{h - 2m_2 l_1^{\gamma_2}} \right)^{1/\beta-\theta} d^{\gamma_1/\beta-\theta}(\mu_1,\mu_2) + \left(\frac{2m_2 l_2^{\gamma_2}}{h - 2m_2 l_1^{\gamma_2}} \right)^{1/\beta} d^{\alpha_2\gamma_2/\beta}(\lambda_1,\lambda_2) \right) B(0,1). \end{aligned} \tag{3.28}$$

Proof. Since the system of $\{N'(\bar{\xi})\}_{\bar{\xi}\in B^*}$, which are given by Theorem 3.2, is an open covering of the weak∗ compact set B^*, there exist a finite number of points (ξ_i) $(i = 1, 2, \ldots, n)$ from B^* such that

$$B^* \subset \cup_{i=1}^{n} N'(\xi_i). \tag{3.29}$$

Hence, let $\widetilde{N}(\lambda_0) = \cap_{i=1}^{n} N'_{\xi_i}(\lambda_0)$ and $\widetilde{N}(\mu_0) = \cap_{i=1}^{n} N'_{\xi_i}(\mu_0)$. Then $\widetilde{N}(\lambda_0)$ and $\widetilde{N}(\mu_0)$ are desired neighborhoods of λ_0 and μ_0, respectively. Indeed, let $(\lambda, \mu) \in \widetilde{N}(\lambda_0) \times \widetilde{N}(\mu_0)$ be given arbitrarily. For any $\xi \in B^*$, by virtue of (3.29), there exists $i_0 \in \{1, 2, \ldots, n\}$ such that $\xi \in N'(\xi_{i_0})$. From the construction of the neighborhoods $\widetilde{N}(\lambda_0)$ and $\widetilde{N}(\mu_0)$, one has

$$(\lambda, \mu) \in N'_{\xi_{i_0}}(\lambda_0) \times N'_{\xi_{i_0}}(\mu_0). \tag{3.30}$$

Then, according to Theorem 3.2, the ξ-solution $S_\xi(\lambda, \mu)$ is a nonempty singleton. Hence, in view of Lemma 2.4, $S(\lambda, \mu) = \cup_{\xi\in B^*} S(\xi, \lambda, \mu)$ is nonempty.

Now, we show that (3.28) holds. Indeed, taking any $(\lambda_1, \mu_1), (\lambda_2, \mu_2) \in \widetilde{N}(\lambda_0) \times \widetilde{N}(\mu_0)$, we need to show that for any $x_1 \in S(\lambda_1, \mu_1)$, there exists $x_2 \in S(\lambda_2, \mu_2)$ satisfying

$$\begin{aligned} d(x_1, x_2) \leq {} & \left(\frac{m_1}{h - 2m_2 l_1^{\gamma_2}}\right)^{1/(\beta-\theta)} d^{\gamma_1/(\beta-\theta)}(\mu_1, \mu_2) \\ & + \left(\frac{2m_2 l_2^{\gamma_2}}{h - 2m_2 l_1^{\gamma_2}}\right)^{1/\beta} d^{\alpha_2\gamma_2/\beta}(\lambda_1, \lambda_2). \end{aligned} \tag{3.31}$$

Since $x_1 \in S(\lambda_1, \mu_1) = \cup_{\xi\in B^*} S_\xi(\lambda_1, \mu_1)$, there exists $\widehat{\xi} \in B^*$ such that

$$x_1 = x_{\widehat{\xi}}(\lambda_1, \mu_1) \in S_{\widehat{\xi}}(\lambda_1, \mu_1). \tag{3.32}$$

Thanks to (3.29), there exists $i_0 \in \{1, 2, \ldots, n\}$ such that $\widehat{\xi} \in N'(\xi_{i_0})$. Thus, by the construction of the neighborhoods $\widetilde{N}(\lambda_0)$ and $\widetilde{N}(\mu_0)$, we have

$$(\lambda_1, \mu_1),\ (\lambda_2, \mu_2) \in N_{\xi_{i_0}}(\lambda_0) \times N_{\xi_{i_0}}(\mu_0). \tag{3.33}$$

Then, it follows from Theorem 3.2 that

$$\begin{aligned} d\Big(x_{\widehat{\xi}}(\lambda_1, \mu_1), x_{\widehat{\xi}}(\lambda_2, \mu_2)\Big) \leq {} & \left(\frac{m_1}{h - 2m_2 l_1^{\gamma_2}}\right)^{1/(\beta-\theta)} d^{\gamma_1/(\beta-\theta)}(\mu_1, \mu_2) \\ & + \left(\frac{2m_2 l_2^{\gamma_2}}{h - 2m_2 l_1^{\gamma_2}}\right)^{1/\beta} d^{\alpha_2\gamma_2/\beta}(\lambda_1, \lambda_2). \end{aligned} \tag{3.34}$$

Let $x_2 = x_{\widehat{\xi}}(\lambda_2, \mu_2)$. Then, (3.31) holds and the proof is complete. □

From Corollary 3.4, Theorems 3.2 and 3.6, we can easily obtain the following result.

Corollary 3.7. *Assume that for each $\xi \in B^*$, the ξ-solution set for the problem* (PVQEP) *exists in a neighborhood $N(\lambda_0) \times N(\mu_0)$ of the considered point $(\lambda_0, \mu_0) \in \Lambda \times M$. Assume further that the conditions (i), (iii), (iv), and (v) in Theorem 3.6 hold and the condition (ii) in Theorem 3.6 is replaced with*

(ii′) *for all $\mu \in N(\mu_0)$, $f(\cdot,\cdot,\mu)$ is $h \cdot \beta$-Hölder strongly monotone in $E(N(\lambda_0))$, that is, there exist two constants $h > 0$ and $\beta > 0$ such that, for all $x, y \in E(N(\lambda_0)) : x \neq y$,*

$$f(x,y,\mu) + f(y,x,\mu) + hd^{\beta}(x,y)B(0,1) \subset -C. \tag{3.35}$$

Then, there exist neighborhoods $\widetilde{N}(\lambda_0)$ of λ_0 and $\widetilde{N}(\mu_0)$ of μ_0, such that, the weak solution set $S(\cdot,\cdot)$ on $\widetilde{N}(\lambda_0) \times \widetilde{N}(\mu_0)$ is nonempty and satisfies the following Hölder continuous condition: for all $(\lambda_1, \mu_1), (\lambda_2, \mu_2) \in \widetilde{N}(\lambda_0) \times \widetilde{N}(\mu_0)$,

$$\begin{aligned} S(\lambda_1,\mu_1) &\subset S(\lambda_2,\mu_2) \\ &\quad + \left(\left(\frac{m_1}{h - 2m_2 l_1^{\gamma_2}} \right)^{1/(\beta-\theta)} d^{\gamma_1/(\beta-\theta)}(\mu_1,\mu_2) + \left(\frac{2m_2 l_2^{\gamma_2}}{h - 2m_2 l_1^{\gamma_2}} \right)^{1/\beta} d^{\alpha_2\gamma_2/\beta}(\lambda_1,\lambda_2) \right) B(0,1). \end{aligned} \tag{3.36}$$

Remark 3.8. Theorem 3.6 and Corollary 3.7 generalize and improve the corresponding results of S. Li and X. Li [28] in the following three aspects.

(i) The assumption (H_4) of Theorem 3.1 in S. Li and X. Li [28] is removed.

(ii) The Hölder degree of the solution set is remarkably sharpened since the assumption (H_3) of Theorem 3.1 in S. Li and X. Li [28] is replaced with (iii) of Theorem 3.6 and Corollary 3.7.

(iii) We extend the result of S. Li and X. Li [28] on the Hölder continuity of the solution set of parametric vector equilibrium problems to parametric vector quasiequilibrium problems in metric spaces.

Moreover, it is easy to see that the assumption (H_1) of Theorem 3.1 in S. Li and X. Li [28] implies the assumption (ii) of Theorem 3.6. However, the converse may not hold. Therefore, Theorem 3.2 generalizes and improves Theorem 3.1 of S. Li and X. Li [28] by weakening the corresponding Hölder-related assumptions.

Now, we give an example to illustrate Theorem 3.6, or Corollary 3.7 is applicable when the solution mapping is set valued.

Example 3.9. Let $X = \mathbb{R}$, $Y = \mathbb{R}^2$, $\Lambda = M = [0,1]$, and $C = \mathbb{R}^2_+$. Let $K : X \times M \rightrightarrows Y$ be defined by

$$K(x,\lambda) = K(\lambda) = \left[\lambda^2, 1\right] \tag{3.37}$$

and $f : X \times X \times M \to Y$ defined by

$$f(x, y, \lambda) = \left((1+\lambda)y(x-y), (1+\lambda)x(y-x)\right). \tag{3.38}$$

Consider that $\lambda_0 = 0.5$ and $N(\lambda_0) = \Lambda$. Obviously, $E(\Lambda) = E(N(\lambda_0)) = [0,1]$; $K(\cdot,\cdot)$ is $(0 \cdot \alpha_1, 1 \cdot 1)$-Hölder continuous in $E(\Lambda)$; $f(x, y, \cdot)$ is $\sqrt{2}.1$-Hölder continuous 1-relative to $E(\Lambda)$; for all $\lambda \in \Lambda$ and $x \in E(\lambda)$, $f(x, \cdot, \lambda)$ is $2\sqrt{2}.1$-Hölder continuous; for all $\lambda \in \Lambda$, $f(\cdot,\cdot,\lambda)$ is 2.2-Hölder strongly monotone in $E(\Lambda)$. Here, $l_1 = 0, \alpha_1$ is arbitrary, $l_2 = \alpha_2 = \theta = 1, m_1 = \sqrt{2}, m_2 = 2\sqrt{2}, \gamma_1 = \gamma_2 = 1$ and $h = \beta = 2$. Hence, we take $\alpha_1 = 2$ to see (iv) is satisfied. Therefore, Theorem 3.6 (or Corollary 3.7) derives the Holder continuity of the solution around λ_0 (in fact, $S(\lambda) = [\lambda^2, 1]$, for all $\lambda \in \Lambda$).

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