Research Article

Berinde-Type Generalized Contractions on Partial Metric Spaces

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We consider generalized Berinde-type contractions in the context of partial metric spaces. Such contractions are also known as generalized almost contractions in the literature. In this paper, we extend, generalize, and enrich the results in this direction. Some examples are presented to illustrate our results.

1. Introduction and Preliminaries

Matthews [1] introduced the notion of a partial metric space as a part of the study of denotational semantics of data for networks, showing that the contraction mapping principle [2] can be generalized to the partial metric context for applications in program verifications. Later, there have been several recent extensive researches on (common) fixed points for different contractions on partial metric spaces, see [3–28].

First, we recall some basic concepts and notations.

Definition 1. A partial metric on a nonempty set X is a function $p : X \times X \rightarrow [0, +\infty)$ such that for all $x, y, z \in X$:

($p1$) $x = y \Leftrightarrow p(x, x) = p(x, y) = p(y, y)$,

($p2$) $p(x, x) \leq p(x, y)$,

($p3$) $p(x, y) = p(y, x)$,

($p4$) $p(x, y) \leq p(x, z) + p(z, y) - p(z, z)$.

A partial metric space is a pair (X, p) such that X is a nonempty set and p is a partial metric on X.

Example 2 (see [1]). Let $X = \mathbb{R}^+$ and p defined on X by $p(x, y) = \max\{x, y\}$ for all $x, y \in X$. Then (X, p) is a partial metric space.

Example 3 (see [20, 26]). Let (X, d) and (X, p) be a metric space and a partial metric space, respectively. Functions $\rho_i : X \times X \rightarrow \mathbb{R}^+$ ($i \in \{1, 2, 3\}$) given by

$$\rho_1(x, y) = d(x, y) + p(x, y),$$
$$\rho_2(x, y) = d(x, y) + \max\{u(x), u(y)\}, \qquad (1)$$
$$\rho_3(x, y) = d(x, y) + a,$$

define partial metrics on X, where $u : X \rightarrow \mathbb{R}^+$ is an arbitrary function and $a \geq 0$.

Example 4 (see [1]). Let $X = \{[a, b] : a, b \in \mathbb{R}, a \leq b\}$ and define $p([a, b], [c, d]) = \max\{b, d\} - \min\{a, c\}$. Then (X, p) is a partial metric space.

Example 5 (see [1]). Let $X = [0, 1] \cup [2, 3]$ and define $p : X \times X \rightarrow \mathbb{R}^+$ by

$$p(x, y) = \begin{cases} \max\{x, y\}, & \text{if } \{x, y\} \cap [2, 3] \neq \emptyset, \\ |x - y|, & \text{if } \{x, y\} \subset [0, 1]. \end{cases} \qquad (2)$$

Then (X, p) is a partial metric space.

Remark 6. It is clear that, if $p(x, y) = 0$, then from ($p1$) and ($p2$), we get $x = y$. On the other hand, $p(x, y)$ may not be 0 even if $x = y$.

Each partial metric p on X generates a T_0 topology τ_p on X which has as a base the family of open p-balls $\{B_p(x, \varepsilon), x \in X, \varepsilon > 0\}$, where $B_p(x, \varepsilon) = \{y \in X : p(x, y) < p(x, x) + \varepsilon\}$ for all $x \in X$ and $\varepsilon > 0$.

If p is a partial metric on X, then the functions d_p, $d_m^p :$ $X \times X \to \mathbb{R}_+$ given by

$$d_p(x, y) = 2p(x, y) - p(x, x) - p(y, y),$$

$$d_m^p(x, y) = \max\{p(x, y) - p(x, x), p(x, y) - p(y, y)\}$$

$$= p(x, y) - \min\{p(x, x), p(y, y)\}$$

$$(3)$$

are equivalent metrics on X.

Definition 7 (see [1]). Let (X, p) be a partial metric space.

(1) A sequence $\{x_n\}_{n\in\mathbb{N}}$ in X is called a Cauchy sequence in (X, p) if $\lim_{n,m\to+\infty} p(x_n, x_m)$ exists and is finite.

(2) (X, p) is called complete if every Cauchy sequence $\{x_n\}_{n\in\mathbb{N}}$ converges with respect to τ_p to a point $x \in X$ such that $p(x, x) = \lim_{n,m\to+\infty} p(x_n, x_m)$.

Lemma 8 (see [1]). *Let (X, p) be a partial metric space.*

(1) *$\{x_n\}_{n\in\mathbb{N}}$ is a Cauchy sequence in (X, p) if and only if it is a Cauchy sequence in the metric space (X, d_p).*

(2) *A partial metric space (X, p) is complete if and only if the metric space (X, d_p) is complete. Furthermore, $\lim_{n\to+\infty} d_p(x_n, x) = 0$ if and only if*

$$p(x, x) = \lim_{n\to+\infty} p(x_n, x) = \lim_{n,m\to+\infty} p(x_n, x_m). \quad (4)$$

Lemma 9 (see [20]). *Let $\{x_n\}_{n\in\mathbb{N}}$ be a convergent sequence in a partial metric space X such that $x_n \to x$ and $x_n \to y$ with respect to τ_p. If*

$$\lim_{n\to+\infty} p(x_n, x_n) = p(x, x) = p(y, y), \quad (5)$$

then $x = y$.

Lemma 10 (see [20]). *Let $\{x_n\}_{n\in\mathbb{N}}$ and $\{y_n\}_{n\in\mathbb{N}}$ be two sequences in a partial metric space X such that*

$$\lim_{n\to+\infty} p(x_n, x) = \lim_{n\to+\infty} p(x_n, x_n) = p(x, x),$$

$$\lim_{n\to+\infty} p(y_n, y) = \lim_{n\to+\infty} p(y_n, y_n) = p(y, y), \quad (6)$$

then $\lim_{n\to+\infty} p(x_n, y_n) = p(x, y)$. In particular, $\lim_{n\to+\infty} p(x_n, z) = p(x, z)$ for every $z \in X$.

Lemma 11 (see [3]). *Let (X, p) be a partial metric space and $x_n \to z$, with respect to τ_p, with $p(z, z) = 0$. Then $\lim_{n\to+\infty} p(x_n, y) = p(z, y)$ for all $y \in X$.*

The concept of almost contractions was introduced by Berinde [29, 30] on metric spaces. Other results on almost contractions could be found in [31–34]. Recently, Altun

and Acar [35] characterized this concept in the setting of partial metric space and proved some fixed point theorems using these concepts. Very recently, Turkoglu and Ozturk [27] established a fixed point theorem for four mappings satisfying an almost generalized contractive condition on partial metric spaces. In this paper, we generalize the results given in [27, 35] by presenting some fixed point results for self mappings involving some almost generalized contractions in the setting of partial metric spaces. Also, we give some illustrative examples making our results proper.

2. Main Results

We start to this section by defining some sets of auxiliary functions. Let $\mathcal{F}$ denote all functions $f : [0, +\infty) \to [0, +\infty)$ such that $f(t) = 0$ if and only if $t = 0$. We denote by Ψ and Φ be subsets of $\mathcal{F}$ such that

$$\Psi = \{\psi \in \mathcal{F} : \psi \text{ is continuous and nondecreasing}\},$$

$$\Phi = \{\phi \in \mathcal{F} : \phi \text{ is lower semicontinuous}\}. \quad (7)$$

Let (X, p) a partial metric space. We consider the following expressions:

$$M(x, y) = \max\left\{p(x, y), p(x, Tx), p(y, Ty),\right.$$

$$\left.\frac{1}{2}[p(x, Ty) + p(y, Tx)]\right\},$$

$$N(x, y) = \min\{d_m^p(x, Tx), d_m^p(y, Ty),$$

$$d_m^p(x, Ty), d_m^p(y, Tx)\}, \quad (8)$$

for all $x, y \in X$.

Our first result is the following.

Theorem 12. *Let (X, p) be a complete partial metric space. Let $T : X \to X$ be a self mapping. Suppose there exist $\psi \in \Psi$, $\phi \in \Phi$ and $L \geq 0$ such that for all $x, y \in X$*

$$\psi(p(Tx, Ty)) \leq \psi(M(x, y)) - \phi(M(x, y)) + LN(x, y). \quad (9)$$

Then T has a unique fixed point, say $u \in X$. Also, one has $p(u, u) = 0$.

Proof. Let $x_0 \in X$. We construct a sequence $\{x_n\}_{n\in\mathbb{N}}$ in X in a way that $x_n = Tx_{n-1}$ for all $n \geq 1$. Suppose that $p(x_{n_0}, x_{n_0+1}) = 0$ for some $n_0 \geq 0$. So we have $x_{n_0} = x_{n_0+1} = Tx_{n_0}$, that is, x_{n_0} is the fixed point of T.

From now on, assume that $p(x_n, x_{n+1}) > 0$ for all $n \geq 0$. By (9), we have

$$\psi(p(x_n, x_{n+1})) \leq \psi(p(Tx_{n-1}, Tx_n))$$

$$\leq \psi(M(x_{n-1}, x_n)) - \phi(M(x_{n-1}, x_n)) \quad (10)$$

$$+ LN(x_{n-1}, x_n),$$

where

$$N(x_{n-1}, x_n) = \min\{d_m^p(x_{n-1}, x_n), d_m^p(x_n, x_{n+1}),$$
$$d_m^p(x_{n-1}, x_{n+1}), d_m^p(x_n, x_n)\}. \tag{11}$$

Since $d_m^p(x_n, x_n) = 0$, we get $N(x_{n-1}, x_n) = 0$. Hence, it follows that

$$\psi(p(x_n, x_{n+1})) \leq \psi(M(x_{n-1}, x_n)) - \phi(M(x_{n-1}, x_n)), \tag{12}$$

which yields that

$$\psi(p(x_n, x_{n+1})) \leq \psi(M(x_{n-1}, x_n)). \tag{13}$$

Since ψ is nondecreasing, then

$$p(x_n, x_{n+1}) \leq M(x_{n-1}, x_n), \tag{14}$$

where

$$M(x_{n-1}, x_n)$$
$$= \max\left\{p(x_{n-1}, x_n), p(x_{n-1}, Tx_{n-1}), p(x_n, Tx_n),\right.$$
$$\left.\frac{1}{2}[p(x_{n-1}, Tx_n) + p(x_n, Tx_{n-1})]\right\}$$
$$= \max\left\{p(x_{n-1}, x_n), p(x_{n-1}, x_n), p(x_n, x_{n+1}),\right.$$
$$\left.\frac{1}{2}[p(x_{n-1}, x_{n+1}) + p(x_n, x_n)]\right\}. \tag{15}$$

Due to $(p4)$, we have

$$p(x_{n-1}, x_{n+1}) + p(x_n, x_n) \leq p(x_{n-1}, x_n) + p(x_n, x_{n+1}). \tag{16}$$

Hence, the expression (15) turns into

$$M(x_{n-1}, x_n) = \max\{p(x_{n-1}, x_n), p(x_n, x_{n+1})\}. \tag{17}$$

If for some n,

$$\max\{p(x_{n-1}, x_n), p(x_n, x_{n+1})\} = p(x_n, x_{n+1}) \tag{18}$$

then by (12)

$$\psi(p(x_n, x_{n+1})) \leq \psi(p(x_n, x_{n+1})) - \phi(p(x_n, x_{n+1})), \tag{19}$$

so $\phi(p(x_n, x_{n+1})) = 0$. By (ϕ_2), we get $p(x_n, x_{n+1}) = 0$, which is a contradiction with respect to $p(x_n, x_{n+1}) > 0$ for all $n \geq 0$. Thus

$$M(x_{n-1}, x_n) = p(x_{n-1}, x_n), \quad \text{for each } n \geq 1, \tag{20}$$

so from (14)

$$p(x_n, x_{n+1}) \leq p(x_{n-1}, x_n), \quad \text{for each } n \geq 1. \tag{21}$$

Thus, the sequence $\{p(x_n, x_{n+1})\}$ is non-increasing and so there exists $\delta \geq 0$ such that

$$\lim_{n \to +\infty} p(x_n, x_{n+1}) = \delta. \tag{22}$$

Suppose that $\delta > 0$. Taking $\limsup_{n \to +\infty}$ in inequality (12) and using (20), we get

$$\limsup_{n \to +\infty} \psi(p(x_n, x_{n+1})) \leq \limsup_{n \to +\infty} \psi(p(x_{n-1}, x_n))$$
$$- \liminf_{n \to +\infty} \phi(p(x_{n-1}, x_n)). \tag{23}$$

By continuity of ψ and lower semicontinuity of ϕ, we get $\psi(\delta) \leq \psi(\delta) - \phi(\delta)$, so $\phi(\delta) = 0$, that is, $\delta = 0$, a contradiction. We conclude that

$$\lim_{n \to +\infty} p(x_n, x_{n+1}) = 0. \tag{24}$$

We will show that $\{x_n\}_{n \in \mathbb{N}}$ is a Cauchy sequence in the partial metric space (X, p). From Lemma 8, we need to prove that $\{x_n\}_{n \in \mathbb{N}}$ is a Cauchy sequence in the metric space (X, d_p). Suppose to the contrary that $\{x_n\}_{n \in \mathbb{N}}$ is not a Cauchy sequence in the metric space (X, d_p). Then, there is a $\varepsilon > 0$ such that for an integer k there exist integers $m(k) > n(k) > k$ such that

$$d_p(x_{n(k)}, x_{m(k)}) > \varepsilon. \tag{25}$$

By definition of d_p, we have $d_p(x, y) \leq 2p(x, y)$ for each $x, y \in X$, so (25) gives us

$$p(x_{n(k)}, x_{m(k)}) > \frac{\varepsilon}{2}. \tag{26}$$

For every integer k, let $m(k)$ be the least positive integer exceeding $n(k)$ satisfying (26) then

$$p(x_{n(k)}, x_{m(k)-1}) \leq \frac{\varepsilon}{2}. \tag{27}$$

Now, using (26), (27), and the triangular inequality (which still holds for the partial metric p), we obtain

$$\frac{\varepsilon}{2} < p(x_{n(k)}, x_{m(k)})$$
$$\leq p(x_{n(k)}, x_{m(k)-1}) + p(x_{m(k)-1}, x_{m(k)})$$
$$- p(x_{m(k)-1}, x_{m(k)-1})$$
$$\leq p(x_{n(k)}, x_{m(k)-1}) + p(x_{m(k)-1}, x_{m(k)})$$
$$\leq \frac{\varepsilon}{2} + p(x_{m(k)-1}, x_{m(k)}). \tag{28}$$

Then by (24) it follows that

$$\lim_{k \to +\infty} p(x_{n(k)}, x_{m(k)}) = \frac{\varepsilon}{2}. \tag{29}$$

Also, by the triangle inequality, we have

$$|p(x_{n(k)}, x_{m(k)-1}) - p(x_{n(k)}, x_{m(k)})| \leq p(x_{m(k)-1}, x_{m(k)}). \tag{30}$$

From (24) and (29) we get

$$\lim_{k \to +\infty} p\left(x_{n(k)}, x_{m(k)-1}\right) = \frac{\varepsilon}{2}. \tag{31}$$

Similarly, by triangle inequality

$$
\begin{aligned}
p\left(x_{n(k)}, x_{m(k)}\right) &\leq p\left(x_{n(k)}, x_{n(k)+1}\right) + p\left(x_{n(k)+1}, x_{m(k)}\right) \\
&\leq p\left(x_{n(k)}, x_{n(k)+1}\right) + p\left(x_{n(k)+1}, x_{m(k)-1}\right) \\
&\quad + p\left(x_{m(k)-1}, x_{m(k)}\right) \\
&\leq 2p\left(x_{n(k)}, x_{n(k)+1}\right) + p\left(x_{n(k)}, x_{m(k)-1}\right) \\
&\quad + p\left(x_{m(k)-1}, x_{m(k)}\right)
\end{aligned}
\tag{32}
$$

and from (24), (29), and (31) we get

$$\lim_{k \to +\infty} p\left(x_{n(k)+1}, x_{m(k)}\right) = \frac{\varepsilon}{2}, \tag{33}$$

$$\lim_{k \to +\infty} p\left(x_{n(k)+1}, x_{m(k)-1}\right) = \frac{\varepsilon}{2}. \tag{34}$$

Having

$$
\begin{aligned}
d_m^p\left(x_{n(k)}, x_{n(k)+1}\right) &= p\left(x_{n(k)}, x_{n(k)+1}\right) \\
&\quad - \min\left\{p\left(x_{n(k)}, x_{n(k)}\right), p\left(x_{n(k)+1}, x_{n(k)+1}\right)\right\} \\
&\leq p\left(x_{n(k)}, x_{n(k)+1}\right),
\end{aligned}
\tag{35}
$$

so referring to (24), we get

$$\lim_{k \to +\infty} d_m^p\left(x_{n(k)}, x_{n(k)+1}\right) = 0. \tag{36}$$

Moreover

$$
\begin{aligned}
&M\left(x_{n(k)}, x_{m(k)-1}\right) \\
&= \max\left\{ p\left(x_{n(k)}, x_{m(k)-1}\right), p\left(x_{n(k)}, Tx_{n(k)}\right), \right. \\
&\qquad\quad p\left(x_{m(k)-1}, Tx_{m(k)-1}\right), \\
&\qquad\quad \left. \frac{1}{2}\left[p\left(x_{n(k)}, Tx_{m(k)-1}\right) + p\left(x_{m(k)-1}, Tx_{n(k)}\right)\right]\right\} \\
&= \max\left\{ p\left(x_{n(k)}, x_{m(k)-1}\right), p\left(x_{n(k)}, x_{n(k)+1}\right), \right. \\
&\qquad\quad p\left(x_{m(k)-1}, x_{m(k)}\right), \\
&\qquad\quad \left. \frac{1}{2}\left[p\left(x_{n(k)}, x_{m(k)}\right) + p\left(x_{m(k)-1}, x_{n(k)+1}\right)\right]\right\}.
\end{aligned}
\tag{37}
$$

Thus, from (24), (29), (31), and (34), we get

$$\lim_{k \to +\infty} M\left(x_{n(k)}, x_{m(k)-1}\right) = \max\left\{\frac{\varepsilon}{2}, 0, 0, \frac{\varepsilon}{2}\right\} = \frac{\varepsilon}{2}. \tag{38}$$

From (9), we have

$$
\begin{aligned}
&\psi\left(p\left(x_{n(k)+1}, x_{m(k)}\right)\right) \\
&\quad \leq \psi\left(p\left(Tx_{n(k)}, Tx_{m(k)-1}\right)\right) \\
&\quad \leq \psi\left(M\left(x_{n(k)}, x_{m(k)-1}\right)\right) \\
&\qquad - \phi\left(M\left(x_{n(k)}, x_{m(k)-1}\right)\right) + LN\left(x_{n(k)}, x_{m(k)-1}\right),
\end{aligned}
\tag{39}
$$

where

$$
\begin{aligned}
&N\left(x_{n(k)}, x_{m(k)-1}\right) \\
&= \min\left\{d_m^p\left(x_{n(k)}, Tx_{n(k)}\right), d_m^p\left(x_{m(k)-1}, Tx_{m(k)-1}\right), \right. \\
&\qquad\quad \left. d_m^p\left(x_{n(k)}, Tx_{m(k)-1}\right), d_m^p\left(x_{m(k)-1}, Tx_{n(k)}\right)\right\} \\
&= \min\left\{d_m^p\left(x_{n(k)}, x_{n(k)+1}\right), d_m^p\left(x_{m(k)-1}, x_{m(k)}\right), \right. \\
&\qquad\quad \left. d_m^p\left(x_{n(k)}, x_{m(k)}\right), d_m^p\left(x_{m(k)-1}, x_{n(k)+1}\right)\right\}.
\end{aligned}
\tag{40}
$$

By (36), we get

$$\lim_{k \to +\infty} N\left(x_{n(k)}, x_{m(k)-1}\right) = 0 \tag{41}$$

and referring to (33), (38) and letting $k \to +\infty$, we get

$$\psi\left(\frac{\varepsilon}{2}\right) \leq \psi\left(\frac{\varepsilon}{2}\right) - \phi\left(\frac{\varepsilon}{2}\right), \tag{42}$$

so $\phi(\varepsilon/2) = 0$, which is a contradiction with respect to $\varepsilon > 0$. Thus we proved that $\{x_n\}_{n \in \mathbb{N}}$ is a Cauchy sequence in the metric space (X, d_p).

Since (X, p) is complete, then from Lemma 8, (X, d_p) is a complete metric space. Therefore, the sequence $\{x_n\}_{n \in \mathbb{N}}$ converges to some $u \in X$ in (X, d_p), that is,

$$\lim_{n \to +\infty} d_p\left(x_n, u\right) = 0. \tag{43}$$

Again, from Lemma 8,

$$p\left(u, u\right) = \lim_{n \to +\infty} p\left(x_n, u\right) = \lim_{n \to +\infty} p\left(x_n, x_n\right). \tag{44}$$

On the other hand, thanks to (24) and the condition $(p2)$ from Definition 1,

$$\lim_{n \to +\infty} p\left(x_n, x_n\right) = 0, \tag{45}$$

so it follows that

$$p\left(u, u\right) = \lim_{n \to +\infty} p\left(x_n, u\right) = \lim_{n \to +\infty} p\left(x_n, x_n\right) = 0. \tag{46}$$

Now, we show that $p(u, Tu) = 0$. Assume this is not true, then from (9) we obtain

$$
\begin{aligned}
\psi\left(p\left(x_{n+1}, Tu\right)\right) &= \psi\left(p\left(Tx_n, Tu\right)\right) \\
&\leq \psi\left(M\left(x_n, u\right)\right) - \phi\left(M\left(x_n, u\right)\right) \\
&\quad + L\min\left\{d_m^p\left(x_n, Tx_n\right), d_m^p\left(u, Tu\right), \right. \\
&\qquad\quad \left. d_m^p\left(u, Tx_n\right), d_m^p\left(x_n, Tu\right)\right\},
\end{aligned}
\tag{47}
$$

where

$$
\begin{aligned}
M(x_n, u) = \max \Big\{ & p(x_n, u), p(x_n, Tx_n), p(u, Tu), \\
& \frac{1}{2} [p(x_n, Tu) + p(u, Tx_n)] \Big\} \\
= \max \Big\{ & p(x_n, u), p(x_n, x_{n+1}), p(u, Tu), \\
& \frac{1}{2} [p(x_n, Tu) + p(u, x_{n+1})] \Big\}.
\end{aligned}
\tag{48}
$$

Thanks to (46), it is obvious that $\lim_{n \to +\infty} p(x_n, Tu) = p(u, Tu)$. Therefore, using (24) and again (46), we deduce that

$$
\begin{aligned}
\lim_{n \to +\infty} M(x_n, u) &= \max \Big\{ 0, 0, p(u, Tu), \frac{1}{2} p(u, Tu) \Big\} \\
&= p(u, Tu).
\end{aligned}
\tag{49}
$$

Also

$$
\lim_{n \to +\infty} N(x_n, u) = 0
\tag{50}
$$

because (24) and (45) give $\lim_{n \to +\infty} d_m^p(x_n, Tx_n) = 0$. Now, taking the upper limit as $n \to +\infty$, we obtain using the properties of ψ and ϕ

$$
\psi(p(u, Tu)) \le \psi(p(u, Tu)) - \phi(p(u, Tu)),
\tag{51}
$$

so $\phi(p(u, Tu)) = 0$, that is, $p(u, Tu) = 0$, so $Tu = u$. We conclude that T has a fixed point $u \in X$ and $p(u, u) = 0$.

Now if $v \ne u$ (so $p(u, v) \ne 0$) is another fixed point of T (with $p(v, v) = 0$), then by (46),

$$
\begin{aligned}
N(u, v) &= \min \{ d_m^p(u, Tu), d_m^p(v, Tv), \\
&\qquad\quad d_m^p(u, Tv), d_m^p(v, Tu) \} \\
&= \min \{ d_m^p(u, u), d_m^p(v, v), d_m^p(u, v), d_m^p(v, u) \} \\
&= 0,
\end{aligned}
$$

$$
\begin{aligned}
M(u, v) &= \max \Big\{ p(u, v), p(u, Tu), p(v, Tv), \\
&\qquad\quad \frac{1}{2} [p(u, Tv) + p(v, Tu)] \Big\} \\
&= \max \Big\{ p(u, v), p(u, u), p(v, v), \\
&\qquad\quad \frac{1}{2} [p(u, v) + p(v, u)] \Big\} \\
&= \max \Big\{ p(u, v), 0, 0, \frac{1}{2} [p(u, v) + p(v, u)] \Big\} \\
&= p(u, v).
\end{aligned}
\tag{52}
$$

Hence, using (9) we obtain

$$
\begin{aligned}
\psi(p(u, v)) &= \psi(p(Tu, Tv)) \\
&\le \psi(M(u, v)) - \phi(M(u, v)) + LN(v, u) \\
&= \psi(p(u, v)) - \phi(p(u, v)),
\end{aligned}
\tag{53}
$$

that is, $p(u, v) = 0$, which is a contradiction. The proof of Theorem 12 is completed. $\qquad\square$

As a consequence of Theorem 12, we may state the following corollaries.

First, taking $L = 0$ in Theorem 12, we have the following.

Corollary 13. *Let (X, p) be a complete partial metric space. Let $T : X \to X$ be a self mapping. Suppose there exist $\psi \in \Psi$ and $\phi \in \Phi$ such that for all $x, y \in X$*

$$
\psi(p(Tx, Ty)) \le \psi(M(x, y)) - \psi(M(x, y)).
\tag{54}
$$

Then T has a unique fixed point, say $u \in X$. Also, one has $p(u, u) = 0$.

Corollary 14. *Let (X, p) be a complete partial metric space. Let $T : X \to X$ be a self mapping. Suppose there exist $k \in [0, 1)$ and $L \ge 0$ such that for all $x, y \in X$*

$$
\begin{aligned}
p(Tx, Ty) \le{} & kM(x, y) \\
& + L \min \{ d_m^p(x, Tx), d_m^p(y, Ty), \\
& \qquad\qquad d_m^p(x, Ty), d_m^p(y, Tx) \}.
\end{aligned}
\tag{55}
$$

Then T has a unique fixed point, say $u \in X$. Also, one has $p(u, u) = 0$.

Proof. It follows by taking $\psi(t) = t$ and $\phi(t) = (1 - k)(t)$ in Theorem 12. $\qquad\square$

Denote by Λ the set of functions $\lambda : [0, +\infty) \to [0, +\infty)$ satisfying the following hypotheses:

(1) λ is a Lebesgue-integrable mapping on each compact subset of $[0, +\infty)$,

(2) for every $\epsilon > 0$, we have $\int_0^\epsilon \lambda(s) ds > 0$.

We have the following result.

Corollary 15. *Let (X, p) be a complete partial metric space. Let $T : X \to X$ be a self mapping. Suppose there exist $\alpha, \beta \in \Lambda$ and $L \ge 0$ such that for all $x, y \in X$*

$$
\begin{aligned}
\int_0^{p(Tx, Ty)} \alpha(s) ds \le{} & \int_0^{p(Tx, Ty)} \alpha(s) ds - \int_0^{M(x, y)} \beta(s) ds \\
& + L \min \{ d_m^p(x, Tx), d_m^p(y, Ty), \\
& \qquad\qquad d_m^p(x, Ty), d_m^p(y, Tx) \}.
\end{aligned}
\tag{56}
$$

Then T has a unique fixed point, say $u \in X$. Also, one has $p(u, u) = 0$.

Proof. It follows from Theorem 12 by taking

$$
\begin{aligned}
\psi(t) &= \int_0^t \alpha(s) ds, \\
\phi(t) &= \int_0^t \beta(s) ds.
\end{aligned}
\tag{57}
$$

$\qquad\square$

Taking $L = 0$ in Corollary 15, we obtain the following result.

Corollary 16. *Let (X, p) be a complete partial metric space. Let $T : X \to X$ be a self mapping. Suppose there exist $\alpha, \beta \in \Lambda$ such that for all $x, y \in X$*

$$\int_0^{p(Tx,Ty)} \alpha(s)\,ds \leq \int_0^{p(Tx,Ty)} \alpha(s)\,ds - \int_0^{M(x,y)} \beta(s)\,ds. \tag{58}$$

Then T has a unique fixed point, say $u \in X$. Also, one has $p(u, u) = 0$.

Now, let $\mathcal{F}$ be the set of functions $\varphi : [0, +\infty) \to [0, +\infty)$ satisfying the following hypotheses:

(φ_1) φ is nondecreasing

(φ_2) $\sum_{n=0}^{+\infty} \varphi^n(t)$ converges for all $t > 0$.

Note that if $\varphi \in \mathcal{F}$, φ is said a (C)-comparison function. It is easily proved that if φ is a (C)-comparison function, then $\varphi(t) < t$ for any $t > 0$. Our second main result is as follows.

Theorem 17. *Let (X, p) be a complete partial metric space. Let $T : X \to X$ be a mapping such that there exist $\varphi \in \mathcal{F}$ and $L \geq 0$ such that for all $x, y \in X$*

$$p(Tx, Ty) \leq \varphi(M(x, y))$$
$$+ L \min\{d_m^p(x, Tx), d_m^p(y, Ty), \tag{59}$$
$$d_m^p(x, Ty), d_m^p(y, Tx)\}.$$

Then T has a unique fixed point, say $u \in X$. Also, one has $p(u, u) = 0$.

Proof. Let $x_0 \in X$. Let $\{x_n\}_{n \in \mathbb{N}}$ in X such that $x_n = Tx_{n-1}$ for all $n \geq 1$.

If for some $n \in \mathbb{N}$, $p(x_n, x_{n+1}) = 0$, the proof is completed. Assume that $p(x_n, x_{n+1}) \neq 0$ for all $n \geq 0$.

From (59)

$$p(x_n, x_{n+1}) = p(Tx_{n-1}, Tx_n)$$
$$\leq \varphi(M(x_{n-1}, x_n))$$
$$= L \min\{d_m^p(x_{n-1}, Tx_{n-1}), d_m^p(x_n, Tx_n),$$
$$d_m^p(x_{n-1}, Tx_n), d_m^p(x_n, Tx_{n-1})\}. \tag{60}$$

As explained in the proof of Theorem 12, we may get

$$\min\{d_m^p(x_{n-1}, Tx_{n-1}), d_m^p(x_n, Tx_n),$$
$$d_m^p(x_{n-1}, Tx_n), d_m^p(x_n, Tx_{n-1})\} = 0, \tag{61}$$
$$M(x_{n-1}, x_n) = \max\{p(x_{n-1}, x_n), p(x_n, x_{n+1})\}.$$

Therefore

$$p(x_n, x_{n+1}) \leq \varphi(\max\{p(x_{n-1}, x_n), p(x_n, x_{n+1})\}). \tag{62}$$

If for some $n \geq 1$, we have $p(x_{n-1}, x_n) \leq p(x_n, x_{n+1})$. So from (62), we obtain that

$$p(x_n, x_{n+1}) \leq \varphi(p(x_n, x_{n+1})) < p(x_n, x_{n+1}), \tag{63}$$

a contradiction. Thus, for all $n \geq 1$, we have

$$M(x_{n-1}, x_n) = \max\{p(x_{n-1}, x_n), p(x_n, x_{n+1})\}$$
$$= p(x_{n-1}, x_n). \tag{64}$$

Using (62) and (64), we get that

$$p(x_n, x_{n+1}) \leq \varphi(p(x_{n-1}, x_n)) \quad \forall n \geq 1. \tag{65}$$

By induction, we get

$$p(x_n, x_{n+1}) \leq \varphi^n(p(x_0, x_1)) \tag{66}$$

for all $n \geq 0$. By triangle inequality, we have for $m > n$

$$p(x_n, x_m) \leq \sum_{k=n}^{k=m-1} p(x_k, x_{k+1}) - \sum_{k=n+1}^{k=m-1} p(x_k, x_k)$$
$$\leq \sum_{k=n}^{k=m-1} p(x_k, x_{k+1})$$
$$\leq \sum_{k=n}^{k=+\infty} p(x_k, x_{k+1}) \tag{67}$$
$$\leq \sum_{k=n}^{k=+\infty} \varphi^k(p(x_0, x_1)).$$

Keeping in mind that φ is a (C)-comparison function, then $\lim_{n \to +\infty} \sum_{k=n}^{k=+\infty} \varphi^k(p(x_0, x_1)) = 0$ and so $\{x_n\}_{n \in \mathbb{N}}$ is a Cauchy sequence in (X, p) with $\lim_{n,m \to +\infty} p(x_n, x_m) = 0$. Since (X, p) is complete then $\{x_n\}_{n \in \mathbb{N}}$ converges, with respect to τ_p, to a point $u \in X$ such that

$$p(u, u) = \lim_{n \to +\infty} p(x_n, u) = \lim_{n,m \to +\infty} p(x_n, x_m) = 0. \tag{68}$$

Now we claim that $p(u, Tu) = 0$. Suppose the contrary, then $p(u, Tu) > 0$. By (59), we have

$$p(u, Tu) \leq p(u, x_{n+1}) + p(Tx_n, Tu)$$
$$\leq p(u, x_{n+1}) + \varphi(M(x_n, u))$$
$$+ L \min\{d_m^p(x_n, Tx_n), d_m^p(u, Tu), \tag{69}$$
$$d_m^p(u, Tx_n), d_m^p(x_n, Tu)\},$$

where

$$M(x_n, u) = \max\Big\{p(x_n, u), p(x_n, Tx_n), p(u, Tu),$$
$$\frac{1}{2}[p(x_n, Tu) + p(u, Tx_n)]\Big\}$$
$$= \max\Big\{p(x_n, u), p(x_n, x_{n+1}), p(u, Tu), \tag{70}$$
$$\frac{1}{2}[p(x_n, Tu) + p(u, x_{n+1})]\Big\}.$$

By (68), we have

$$\lim_{n \to +\infty} \min \left\{ d_m^p \left(x_n, T x_n \right), d_m^p \left(u, T u \right), \right.$$

$$\left. d_m^p \left(u, T x_n \right), d_m^p \left(x_n, T u \right) \right\} = 0, \qquad (71)$$

$$\lim_{n \to +\infty} M \left(x_n, u \right) = p \left(u, T u \right).$$

Therefore

$$p \left(u, T u \right) \le \varphi \left(p \left(u, T u \right) \right) < p \left(u, T u \right), \qquad (72)$$

which is a contradiction. That is $p(u, Tu) = 0$. Thus, we obtained that u is a fixed point for T and $p(u, u) = 0$.

Now if $v \ne u$ (so $p(u, v) \ne 0$) is another fixed point of T, then by (68),

$$\min \left\{ d_m^p \left(u, T u \right), d_m^p \left(v, T v \right), d_m^p \left(u, T v \right), d_m^p \left(v, T u \right) \right\}$$

$$= \min \left\{ d_m^p \left(u, u \right), d_m^p \left(v, v \right), d_m^p \left(u, v \right), d_m^p \left(v, u \right) \right\}$$

$$= 0,$$

$$M \left(u, v \right) = \max \left\{ p \left(u, v \right), p \left(u, T u \right), p \left(v, T v \right), \right.$$

$$\left. \frac{1}{2} \left[p \left(u, T v \right) + p \left(v, T u \right) \right] \right\}$$

$$= \max \left\{ p \left(u, v \right), p \left(u, u \right), p \left(v, v \right), \right.$$

$$\left. \frac{1}{2} \left[p \left(u, v \right) + p \left(v, u \right) \right] \right\}$$

$$= p \left(u, v \right).$$

$$(73)$$

Hence, using (59) we obtain

$$p \left(u, v \right) = p \left(T u, T v \right)$$

$$\le \varphi \left(M \left(u, v \right) \right)$$

$$+ L \min \left\{ d_m^p \left(u, T u \right), d_m^p \left(v, T v \right), \right.$$

$$\left. d_m^p \left(u, T v \right), d_m^p \left(v, T u \right) \right\}$$

$$\le \varphi \left(p \left(u, v \right) \right)$$

$$< p \left(u, v \right)$$

$$(74)$$

which is a contradiction. Thus $u = v$ and the proof of Theorem 17 is completed. □

Taking $L = 0$ in Theorem 17, we have the following.

Corollary 18. *Let (X, p) be a complete partial metric space. Let $T : X \to X$ be a mapping such that there exists $\varphi \in \mathcal{F}$ such that for all $x, y \in X$*

$$p \left(T x, T y \right) \le \varphi \left(M \left(x, y \right) \right). \qquad (75)$$

Then T has a unique fixed point, say $u \in X$. Also, one has $p(u, u) = 0$.

Taking $\varphi(t) = ht$ where $0 \le h < 1$ in Corollary 18, we obtain the Ćirić fixed point theorem [36] in the setting of metric spaces (by considering $p = d$ is a metric).

Corollary 19. *Let (X, d) be a complete metric space. Let $T : X \to X$ be a mapping such that there exists $h \in [0, 1)$ such that for all $x, y \in X$*

$$d \left(T x, T y \right) \le h \max \left\{ d \left(x, y \right), d \left(x, T x \right), d \left(y, T y \right), \right.$$

$$\left. \frac{1}{2} \left[d \left(x, T y \right) + d \left(y, T x \right) \right] \right\}. \qquad (76)$$

Then T has a unique fixed point.

Remark 20. Corollary 14 generalizes Theorem 10 (with $f = g = T = S$) of Turkoglu and Ozturk [27]. Corollary 18 improves Theorem 1 of Altun et al. [4] by assuming that φ is not continuous.

3. Examples

We give in this section some examples making effective our obtained results.

Example 21. Let $X = [0, 1]$ and $p(x, y) = \max\{x, y\}$ for all $x, y \in X$. Then (X, p) is a complete partial metric space. Consider $T : X \to X$ defined by

$$T x = \frac{x^2}{1 + x}. \qquad (77)$$

Take $\psi(t) = t$ and $\phi(t) = t/(1 + t)$ for all $t \ge 0$. Note that $\psi \in \Psi$ and $\phi \in \Phi$. Take $x \le y$, then

$$\psi \left(p \left(T x, T y \right) \right)$$

$$= \frac{y^2}{1 + y} = y - \frac{y}{1 + y}$$

$$= \psi \left(M \left(x, y \right) \right) - \phi \left(M \left(x, y \right) \right)$$

$$\left(\text{since } M \left(x, y \right) = y \right)$$

$$\le \psi \left(M \left(x, y \right) \right) - \phi \left(M \left(x, y \right) \right)$$

$$+ L \min \left\{ d_m^p \left(x, T x \right), d_m^p \left(y, T y \right), \right.$$

$$\left. d_m^p \left(x, T y \right), d_m^p \left(y, T x \right) \right\}$$

$$(78)$$

for all $L \ge 0$. Thus, (9) holds. Applying Theorem 12, T has a unique fixed point, which is $u = 0$.

Example 22. Let $X = \{0, 1, 2, 3, 4\}$ and $p(x, y) = \max\{x, y\}$. Let $T : X \to X$ be defined as follows:

$$T0 = 0, \qquad T1 = T3 = 2, \qquad T4 = T2 = 1. \qquad (79)$$

By simple calculation, we get that

$$p(T2, T2) = p(T4, T4) = p(T2, T0)$$

$$= p(T4, T0) = p(T2, T4) = 1,$$

$$p(T0, T0) = 0,$$

$$p(T1, T1) = p(T3, T3) = p(T1, T0) \tag{80}$$

$$= p(T3, T0) = p(T1, T3) = 2,$$

$$p(T4, T1) = p(T4, T3) = p(T2, T1)$$

$$= p(T2, T3) = 2.$$

Hence, we derive that

$$M(T1, T4) = M(T2, T4) = M(T3, T4)$$

$$= M(T4, T4) = M(T0, T4) = 4,$$

$$M(T1, T3) = M(T2, T3)$$

$$= M(T0, T3) = M(T3, T3) = 3,$$

$$M(T0, T0) = 0,$$

$$M(T1, T2) = M(T2, T0) = M(T1, T1)$$

$$= M(T0, T1) = M(T2, T2) = 2,$$

$$N(T1, T4) = N(T2, T1) = N(T1, T0) = N(T2, T0)$$

$$= N(T3, T0) = N(T4, T0) = N(T0, T0) = 0,$$

$$N(T1, T1) = N(T1, T3) = N(T2, T3)$$

$$= N(T3, T3) = N(T2, T2) = N(T2, T4) = 1,$$

$$N(T4, T3) = 2, \qquad N(T4, T4) = 3. \tag{81}$$

For $\psi(t) = t/3$, $\phi(t) = t/6$ and $L \geq 1/5$ all conditions of Theorem 12 are satisfied. Notice that 0 is the unique fixed point of T.

Example 23. Let $X = [0, 2]$ and $p : X \times X \to \mathbb{R}^+$ be defined by $p(x, y) = \max\{x, y\}$. Define $T : X \to X$ by

$$T(x) = \begin{cases} \dfrac{x^2}{x+1}, & \text{if } x \in [0, 1[, \\ 0, & \text{if } x \in [1, 2[, \\ \dfrac{4}{3} & \text{if } x = 2 \end{cases} \tag{82}$$

and let $\varphi : [0, +\infty[\to [0, +\infty[$ defined by

$$\varphi(t) = \frac{t^2}{t+1}. \tag{83}$$

By induction, we have $\varphi^n(t) \leq t(t/(1+t))^n$ for all $n \geq 1$, so it is clear that φ is a (C)-comparison function. Now we show

that (59) is satisfied for all $x, y \in X$. It suffices to prove it for $x \leq y$. Consider the following six cases.

Case 1. Let $x, y \in [0, 1[$, then

$$p(Tx, Ty) = \frac{y^2}{y+1}$$

$$= \varphi(p(x, y)) \tag{84}$$

$$\leq \varphi(M(x, y)).$$

Case 2. Let $x, y \in [1, 2[$, then

$$p(Tx, Ty) = p(0, 0) = 0$$

$$\leq \varphi(M(x, y)). \tag{85}$$

Case 3. Let $x = y = 2$, then

$$p(Tx, Ty) = p\left(\frac{4}{3}, \frac{4}{3}\right) = \frac{4}{3}$$

$$= \varphi(2) \tag{86}$$

$$\leq \varphi(M(x, y)).$$

Case 4. Let $x \in [0, 1[$ and $y \in [1, 2[$ then

$$p(Tx, Ty) = p\left(\frac{x^2}{x+1}, 0\right) = \frac{x^2}{x+1}$$

$$\leq \frac{y^2}{y+1} \tag{87}$$

$$= \varphi(p(x, y))$$

$$\leq \varphi(M(x, y)).$$

Case 5. Let $x \in [0, 1[$ and $y = 2$, then

$$p(Tx, Ty) = p\left(\frac{x^2}{x+1}, \frac{4}{3}\right) = \frac{4}{3}$$

$$= \varphi(2) \tag{88}$$

$$= \varphi(p(x, y))$$

$$\leq \varphi(M(x, y)).$$

Case 6. Let $x \in [1, 2[$ and $y = 2$ then

$$p(Tx, Ty) = p\left(0, \frac{4}{3}\right) = \frac{4}{3}$$

$$= \varphi(2) \tag{89}$$

$$= \varphi(p(x, y))$$

$$\leq \varphi(M(x, y)).$$

Since, for all $x, y \in X$

$$L \min\{d_m^p(x, Tx), d_m^p(y, Ty), d_m^p(x, Ty), d_m^p(y, Tx)\} \geq 0 \tag{90}$$

then (59) is verified. Applying Theorem 17, T has a unique fixed point, which is $u = 0$.

All presented theorems involve generalized almost contractive mappings which have a unique fixed point. But, one of the main features of Berinde contractions is the fact that they do possess more that one fixed point. In this direction, Altun and Acar [35] proved the following result.

Theorem 24. *Let* (X, p) *a complete partial metric space. Given* $T : X \rightarrow X$ *satisfying*

$$there\ exist\ k \in [0, 1), \quad L \geq 0$$
$$such\ that\ p(Tx, Ty) \leq kp(x, y) + Ld_m^p(x, Ty), \tag{91}$$

for all $x, y \in X$. *Then,* T *has a fixed point.*

The following example illustrates Theorem 24 where we have two fixed points.

Example 25. Let $X = \{0, 1, 2\}$. A partial metric $p : X \times X \rightarrow \mathbb{R}^+$ is defined by

$$p(0, 0) = p(1, 1) = 0, \qquad p(2, 2) = \frac{1}{4},$$
$$p(0, 1) = p(1, 0) = \frac{1}{3},$$
$$p(0, 2) = p(2, 0) = \frac{11}{24}, \tag{92}$$
$$p(1, 2) = p(2, 1) = \frac{1}{2}.$$

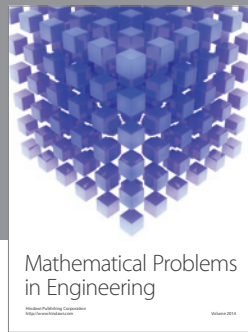Define the mapping $T : X \rightarrow X$ by

$$T0 = T2 = 0, \qquad T1 = 1. \tag{93}$$

It is easy to show that (91) is satisfied. Applying Theorem 24, T has a fixed point. Note that T has two fixed points which are 0 and 1.

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