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Research Article

Nonexistence of Homoclinic Solutions for a Class of Discrete Hamiltonian Systems

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We give several sufficient conditions under which the first-order nonlinear discrete Hamiltonian system $\Delta x(n) = \alpha(n)x(n+1) + \beta(n)|y(n)|^{\mu-2}y(n), \Delta y(n) = -\gamma(n)|x(n+1)|^{\nu-2}x(n+1) - \alpha(n)y(n)$ has no solution $(x(n), y(n))$ satisfying condition $0 < \sum_{n=-\infty}^{+\infty}[|x(n)|^{\nu} + (1+\beta(n))|y(n)|^{\mu}] < +\infty$, where $\mu, \nu > 1$ and $1/\mu + 1/\nu = 1$ and $\alpha(n), \beta(n)$, and $\gamma(n)$ are real-valued functions defined on $\mathbb{Z}$.

1. Introduction

In 1907, Lyapunov [1] established the first so-called Lyapunov inequality:

$$(b-a)\int_a^b q(t)\,dt > 4, \tag{1}$$

if Hill's equation

$$x''(t) + q(t)x(t) = 0 \tag{2}$$

has a real solution $x(t)$ such that

$$x(a) = x(b) = 0, \quad x(t) \not\equiv 0, \quad t \in [a,b], \tag{3}$$

and the constant 4 in (1) cannot be replaced by a larger number, where $q(t)$ is a piecewise continuous and nonnegative function defined on $\mathbb{R}$. Since this result has found applications in the study of various properties of solutions such as oscillation theory, disconjugacy, and eigenvalue problems of (2), a large number of Lyapunov-type inequalities were established in the literature which generalized or improved (1); see [1–20].

In 1983, Cheng [3] first obtained the discrete analogy of Lyapunov inequality (1) for the second-order difference equation:

$$\Delta^2 x(n) + q(n)x(n+1) = 0, \tag{4}$$

where, and in the sequel, Δ denotes the forward difference operator defined by $\Delta x(n) = x(n+1) - x(n)$.

When $a = -\infty$ and $b = +\infty$, that is, system (4) has a solution $x(n)$ satisfying $\lim_{|n|\to\infty} x(n) = 0$, which is called homoclinic solution, whether one can obtain Lyapunov-type inequalities for (4)? To the best of our knowledge, there are no results.

In 2003, Sh. Guseinov and Kaymakçalan [7] partly generalized the Cheng's result to the discrete linear Hamiltonian system:

$$\begin{aligned}\Delta x(n) &= \alpha(n)x(n+1) + \beta(n)y(n),\\ \Delta y(n) &= -\gamma(n)x(n+1) - \alpha(n)y(n),\end{aligned} \tag{5}$$

where $\alpha(n)$, $\beta(n)$, and $\gamma(n)$ are real-valued functions defined on $\mathbb{Z}$ and a and b are not necessarily usual zeros, but rather, generalized zeros. Later, some better Lyapunov-type inequalities for system (5) were obtained in [19, 20].

Very recently, He and Zhang [10] further generalized the result in [19] to the following first-order nonlinear difference system:

$$\begin{aligned}\Delta x(n) &= \alpha(n)x(n+1) + \beta(n)|y(n)|^{\mu-2}y(n),\\ \Delta y(n) &= -\gamma(n)|x(n+1)|^{\nu-2}x(n+1) - \alpha(n)y(n),\end{aligned} \tag{6}$$

where $\mu, \nu > 1$ and $1/\mu + 1/\nu = 1$ and $\alpha(n)$, $\beta(n)$, and $\gamma(n)$ are real-valued functions defined on $\mathbb{Z}$.

When $\mu = \nu = 2$, system (6) reduces to (5). In addition, the special forms of system (6) contain many well-known difference equations which have been studied extensively and have much applications in the literature [21–23], such as the second-order linear difference equation:

$$\Delta [p(n) \Delta x(n)] + q(n) x(n+1) = 0, \quad (7)$$

and the second-order half-linear difference equation:

$$\Delta \left[p(n) |\Delta x(n)|^{r-2} \Delta x(n)\right] + q(n) |x(n+1)|^{r-2} x(n+1) = 0, \quad (8)$$

where $r > 1$, $p(n)$ and $q(n)$ are real-valued functions defined on $\mathbb{Z}$ and $p(n) > 0$. Let

$$y(n) = p(n) |\Delta x(n)|^{r-2} \Delta x(n), \quad (9)$$

then (8) can be written as the form of (6):

$$\begin{aligned} \Delta x(n) &= [p(n)]^{1/(1-r)} |y(n)|^{(2-r)/(r-1)} y(n), \\ \Delta y(n) &= -q(n) |x(n+1)|^{r-2} x(n+1), \end{aligned} \quad (10)$$

where $\mu = r/(r-1)$, $\nu = r$ and $\alpha(n) = 0$, $\beta(n) = [p(n)]^{1/(1-r)}$ and $\gamma(n) = q(n)$.

In this paper, we will establish several Lyapunov-type inequalities for systems (5) and (6) if they have a solution $(x(n), y(n))$ satisfying conditions

$$0 < \sum_{-\infty}^{+\infty} \left[|x(n)|^2 + (1+\beta(n)) |y(n)|^2\right] < +\infty, \quad (11)$$

$$0 < \sum_{-\infty}^{+\infty} \left[|x(n)|^\nu + (1+\beta(n)) |y(n)|^\mu\right] < +\infty, \quad (12)$$

respectively. Taking advantage of these Lyapunov-type inequalities, we are able to establish some criteria for nonexistence of homoclinic solutions of systems (5) and (6). As we know, there are no results on non-existence of homoclinic solutions for Hamiltonian systems in previous literature.

2. Lyapunov-Type Inequalities for System (6)

In this section, we shall establish some Lyapunov-type inequalities for system (6). For the sake of convenience, we list some assumptions on $\alpha(n)$ and $\beta(n)$ as follows:

(A0) $\alpha(n) < 1$, for all $n \in \mathbb{Z}$, $\prod_{s=-\infty}^{+\infty} [1-\alpha(s)]^{-1} < \infty$;

(A1) $\alpha(n) < 1$, for all $n \in \mathbb{Z}$, $\sum_{s=-\infty}^{+\infty} |\alpha(s)| < +\infty$;

(B0) $\beta(n) \geq (\not\equiv)\ 0$, for all $n \in \mathbb{Z}$;

(B1) $\sum_{\tau=-\infty}^{0} \beta(\tau) \prod_{s=\tau}^{0} [1-\alpha(s)]^{-\mu} + \sum_{\tau=1}^{+\infty} \beta(\tau) \prod_{s=0}^{\tau-1} [1-\alpha(s)]^{\mu} < +\infty$.

Denote

$$\begin{aligned} \zeta(n) &:= \left[\sum_{\tau=-\infty}^{n} \beta(\tau) \prod_{s=\tau}^{n} [1-\alpha(s)]^{-\mu}\right]^{\nu/\mu}, \\ \eta(n) &:= \left[\sum_{\tau=n+1}^{+\infty} \beta(\tau) \prod_{s=n+1}^{\tau-1} [1-\alpha(s)]^{\mu}\right]^{\nu/\mu}. \end{aligned} \quad (13)$$

Theorem 1. *Suppose that hypotheses (A0), (B0), and (B1) are satisfied. If system* (6) *has a solution* $(x(n), y(n))$ *satisfying*

$$0 < \sum_{n=-\infty}^{+\infty} \left[|x(n)|^\nu + (1+\beta(n)) |y(n)|^\mu\right] < +\infty, \quad (14)$$

then one has the following inequality:

$$\sum_{n=-\infty}^{+\infty} \frac{\zeta(n)\eta(n)}{\zeta(n)+\eta(n)} \gamma^+(n) \geq 1, \quad (15)$$

where $\gamma^+(n) = \max\{\gamma(n), 0\}$.

Proof. Hypothesis (B1) implies that functions $\zeta(n)$ and $\eta(n)$ are well defined on $\mathbb{Z}$. Without loss of generality, we can assume that

$$\sum_{n=-\infty}^{+\infty} \frac{\zeta(n)\eta(n)}{\zeta(n)+\eta(n)} \gamma^+(n) < +\infty. \quad (16)$$

From (14) and (B0), one has

$$\lim_{|n|\to\infty} |x(n)| = \lim_{|n|\to\infty} |y(n)| = 0, \quad (17)$$

$$\sum_{\tau=-\infty}^{+\infty} \beta(\tau) |y(\tau)|^\mu < +\infty. \quad (18)$$

It follows from (13), (18), and the Hölder inequality that

$$\begin{aligned} &\sum_{\tau=-\infty}^{n} \beta(\tau) |y(\tau)|^{\mu-1} \prod_{s=\tau}^{n} [1-\alpha(s)]^{-1} \\ &\quad \leq \left[\sum_{\tau=-\infty}^{n} \beta(\tau) \prod_{s=\tau}^{n} [1-\alpha(s)]^{-\mu}\right]^{1/\mu} \left[\sum_{\tau=-\infty}^{n} \beta(\tau) |y(\tau)|^\mu\right]^{1/\nu} \\ &\quad = [\zeta(n)]^{1/\nu} \left[\sum_{\tau=-\infty}^{n} \beta(\tau) |y(\tau)|^\mu\right]^{1/\nu} \\ &\quad < +\infty, \quad \forall n \in \mathbb{Z}, \end{aligned} \quad (19)$$

$$\begin{aligned} &\sum_{\tau=n+1}^{+\infty} \beta(\tau) |y(\tau)|^{\mu-1} \prod_{s=n+1}^{\tau-1} [1-\alpha(s)] \\ &\quad \leq \left[\sum_{\tau=n+1}^{+\infty} \beta(\tau) \prod_{s=n+1}^{\tau-1} [1-\alpha(s)]^{\mu}\right]^{1/\mu} \left[\sum_{\tau=n+1}^{+\infty} \beta(\tau) |y(\tau)|^\mu\right]^{1/\nu} \\ &\quad = [\eta(n)]^{1/\nu} \left[\sum_{\tau=n+1}^{+\infty} \beta(\tau) |y(\tau)|^\mu\right]^{1/\nu} \\ &\quad < +\infty, \quad \forall n \in \mathbb{Z}. \end{aligned} \quad (20)$$

From (A0), (17), (19), (20), and the first equation of system (6), we have

$$x(n+1)=\sum_{\tau=-\infty}^{n}\beta(\tau)\left|y(\tau)\right|^{\mu-2}y(\tau)\prod_{s=\tau}^{n}[1-\alpha(s)]^{-1},\quad \forall n\in\mathbb{Z}, \tag{21}$$

$$x(n+1)=-\sum_{\tau=n+1}^{+\infty}\beta(\tau)\left|y(\tau)\right|^{\mu-2}y(\tau)\prod_{s=n+1}^{\tau-1}[1-\alpha(s)],\quad \forall n\in\mathbb{Z}. \tag{22}$$

Combining (19) with (21), one has

$$\begin{aligned}|x(n+1)|^{\nu}&=\left|\sum_{\tau=-\infty}^{n}\beta(\tau)\left|y(\tau)\right|^{\mu-2}y(\tau)\prod_{s=\tau}^{n}[1-\alpha(s)]^{-1}\right|^{\nu}\\&\le\zeta(n)\sum_{\tau=-\infty}^{n}\beta(\tau)\left|y(\tau)\right|^{\mu},\quad \forall n\in\mathbb{Z}.\end{aligned} \tag{23}$$

Similarly, it follows from (20) and (22) that

$$\begin{aligned}|x(n+1)|^{\nu}&=\left|\sum_{\tau=n+1}^{+\infty}\beta(\tau)\left|y(\tau)\right|^{\mu-2}y(\tau)\prod_{s=n+1}^{\tau-1}[1-\alpha(s)]\right|^{\nu}\\&\le\eta(n)\sum_{\tau=n+1}^{+\infty}\beta(\tau)\left|y(\tau)\right|^{\mu},\quad \forall n\in\mathbb{Z}.\end{aligned} \tag{24}$$

Combining (23) with (24), one has

$$|x(n+1)|^{\nu}\le\frac{\zeta(n)\eta(n)}{\zeta(n)+\eta(n)}\sum_{\tau=-\infty}^{+\infty}\beta(\tau)\left|y(\tau)\right|^{\mu},\quad \forall n\in\mathbb{Z}. \tag{25}$$

Now, it follows from (16), (18), and (25) that

$$\begin{aligned}&\sum_{n=-\infty}^{+\infty}\gamma^{+}(n)|x(n+1)|^{\nu}\\&\quad\le\left[\sum_{n=-\infty}^{+\infty}\frac{\zeta(n)\eta(n)}{\zeta(n)+\eta(n)}\gamma^{+}(n)\right]\\&\quad\quad\times\sum_{n=-\infty}^{+\infty}\beta(n)\left|y(n)\right|^{\mu}<+\infty.\end{aligned} \tag{26}$$

By (6), we obtain

$$\Delta\left(x(n)y(n)\right)=\beta(n)\left|y(n)\right|^{\mu}-\gamma(n)|x(n+1)|^{\nu}. \tag{27}$$

Summing the above from $-\infty$ to $+\infty$ and using (17) and (18), we obtain

$$\sum_{n=-\infty}^{+\infty}\gamma(n)|x(n+1)|^{\nu}=\sum_{n=-\infty}^{+\infty}\beta(n)\left|y(n)\right|^{\mu}, \tag{28}$$

which, together with (26), implies that

$$\begin{aligned}&\sum_{n=-\infty}^{+\infty}\gamma^{+}(n)|x(n+1)|^{\nu}\\&\quad\le\left[\sum_{n=-\infty}^{+\infty}\frac{\zeta(n)\eta(n)}{\zeta(n)+\eta(n)}\gamma^{+}(n)\right]\sum_{n=-\infty}^{+\infty}\beta(n)\left|y(n)\right|^{\mu}\\&\quad=\left[\sum_{n=-\infty}^{+\infty}\frac{\zeta(n)\eta(n)}{\zeta(n)+\eta(n)}\gamma^{+}(n)\right]\sum_{n=-\infty}^{+\infty}\gamma(n)|x(n+1)|^{\nu}\\&\quad\le\left[\sum_{n=-\infty}^{+\infty}\frac{\zeta(n)\eta(n)}{\zeta(n)+\eta(n)}\gamma^{+}(n)\right]\sum_{n=-\infty}^{+\infty}\gamma^{+}(n)|x(n+1)|^{\nu}.\end{aligned} \tag{29}$$

We claim that

$$\sum_{n=-\infty}^{+\infty}\gamma^{+}(n)|x(n+1)|^{\nu}>0. \tag{30}$$

If (30) is not true, then

$$\sum_{n=-\infty}^{+\infty}\gamma^{+}(n)|x(n+1)|^{\nu}=0. \tag{31}$$

From (28) and (31), we have

$$\begin{aligned}0&\le\sum_{n=-\infty}^{+\infty}\beta(n)\left|y(n)\right|^{\mu}=\sum_{n=-\infty}^{+\infty}\gamma(n)|x(n+1)|^{\nu}\\&\le\sum_{n=-\infty}^{+\infty}\gamma^{+}(n)|x(n+1)|^{\nu}=0.\end{aligned} \tag{32}$$

It follows that

$$\beta(n)\left|y(n)\right|^{\mu-2}y(n)\equiv0,\quad \forall n\in\mathbb{Z}. \tag{33}$$

Combining (21) with (33), we obtain that

$$x(n)\equiv0,\quad \forall n\in\mathbb{Z}, \tag{34}$$

which, together with the second equation of system (6), implies that

$$\Delta y(n)=-\alpha(n)y(n),\quad \forall n\in\mathbb{Z}. \tag{35}$$

Combining the above with (17), one has

$$y(n)\equiv0,\quad \forall n\in\mathbb{Z}. \tag{36}$$

Both (34) and (36) contradict with (14). Therefore, (30) holds. Hence, it follows from (29) and (30) that (15) holds. □

Corollary 2. *Suppose that hypotheses (A1), (B0), and (B1) are satisfied. If system* (6) *has a solution* $(x(n), y(n))$ *satisfying* (14), *then one has the following inequality:*

$$\sum_{n=-\infty}^{+\infty}\gamma^{+}(n)\left[\sum_{\tau=-\infty}^{n}\beta(\tau)\sum_{\tau=n+1}^{+\infty}\beta(\tau)\right]^{\nu/2\mu}\ge2\prod_{n=-\infty}^{+\infty}\{\Theta[\alpha(n)]\}^{\nu/2}, \tag{37}$$

where and in the sequel,

$$\Theta[\alpha(n)] = \min\left\{1-\alpha^{+}(n), \left[1+\alpha^{-}(n)\right]^{-1}\right\},$$
$$\alpha^{+}(n) = \max\{\alpha(n), 0\}, \qquad \alpha^{-}(n) = \max\{-\alpha(n), 0\}. \tag{38}$$

Proof. Obviously, (A1) implies that

$$0 < \prod_{s=-\infty}^{+\infty}[1-\alpha(s)] < +\infty, \tag{39}$$

and so (A0) holds, and which, together with (B1), implies that $\sum_{\tau=-\infty}^{+\infty}\beta(\tau) < +\infty$. Since

$$\zeta(n) + \eta(n) \geq 2[\zeta(n)\eta(n)]^{1/2}, \tag{40}$$

it follows that

$$\begin{aligned}
1 &\leq \sum_{n=-\infty}^{+\infty}\frac{\zeta(n)\eta(n)}{\zeta(n)+\eta(n)}\gamma^{+}(n) \\
&\leq \frac{1}{2}\sum_{n=-\infty}^{+\infty}[\zeta(n)\eta(n)]^{1/2}\gamma^{+}(n) \\
&= \frac{1}{2}\sum_{n=-\infty}^{+\infty}\gamma^{+}(n)\left\{\sum_{\tau=-\infty}^{n}\beta(\tau)\prod_{s=\tau}^{n}[1-\alpha(s)]^{-\mu}\right. \\
&\qquad \left.\times\sum_{\tau=n+1}^{+\infty}\beta(\tau)\prod_{s=n+1}^{\tau-1}[1-\alpha(s)]^{\mu}\right\}^{\nu/2\mu} \\
&\leq \frac{1}{2}\sum_{n=-\infty}^{+\infty}\gamma^{+}(n)\left\{\sum_{\tau=-\infty}^{n}\beta(\tau)\prod_{s=\tau}^{n}[1-\alpha^{+}(s)]^{-\mu}\right. \\
&\qquad \left.\times\sum_{\tau=n+1}^{+\infty}\beta(\tau)\prod_{s=n+1}^{\tau-1}[1+\alpha^{-}(s)]^{\mu}\right\}^{\nu/2\mu} \\
&\leq \frac{1}{2}\sum_{n=-\infty}^{+\infty}\gamma^{+}(n)\left[\sum_{\tau=-\infty}^{n}\beta(\tau)\sum_{\tau=n+1}^{+\infty}\beta(\tau)\right]^{\nu/2\mu} \\
&\qquad \times\prod_{s=-\infty}^{n}[1-\alpha^{+}(s)]^{-\nu/2}\prod_{s=n+1}^{+\infty}[1+\alpha^{-}(s)]^{\nu/2} \\
&\leq \frac{1}{2}\sum_{n=-\infty}^{+\infty}\gamma^{+}(n)\left[\sum_{\tau=-\infty}^{n}\beta(\tau)\sum_{\tau=n+1}^{b}\beta(\tau)\right]^{\nu/2\mu} \\
&\qquad \times\prod_{s=-\infty}^{+\infty}\{\Theta[\alpha(s)]\}^{-\nu/2},
\end{aligned} \tag{41}$$

which implies that (37) holds. □

Since

$$\left[\sum_{\tau=-\infty}^{n}\beta(\tau)\sum_{\tau=n+1}^{+\infty}\beta(\tau)\right]^{1/2} \leq \frac{1}{2}\sum_{n=-\infty}^{+\infty}\beta(n), \tag{42}$$

then it follows from (37) that the following corollary is true.

Corollary 3. *Suppose that hypotheses (A1), (B0), and (B1) are satisfied. If system* (6) *has a solution* $(x(n), y(n))$ *satisfying* (14), *then*

$$\left(\sum_{n=-\infty}^{+\infty}\beta(n)\right)^{1/\mu}\left(\sum_{n=-\infty}^{+\infty}\gamma^{+}(n)\right)^{1/\nu} \geq 2\prod_{n=-\infty}^{+\infty}\{\Theta[\alpha(n)]\}^{1/2}. \tag{43}$$

Applying Theorem 1 and Corollary 2 to system (8) (i.e., (10)), we have immediately the following two corollaries.

Corollary 4. *Suppose that* $r > 1$ *and* $p(n) > 0$ *for* $n \in \mathbb{Z}$, *and that*

$$\sum_{\tau=-\infty}^{+\infty}\frac{1}{[p(\tau)]^{1/(r-1)}} < +\infty. \tag{44}$$

If (8) *has a solution* $x(n)$ *satisfying*

$$0 < \sum_{n=-\infty}^{+\infty}\left[|x(n)|^{r} + p(n)\left(1+[p(n)]^{1/(r-1)}\right)|\Delta x(n)|^{r}\right] < +\infty, \tag{45}$$

then

$$\begin{aligned}
&\sum_{n=-\infty}^{+\infty}\left(\left\{\sum_{\tau=-\infty}^{n}[p(\tau)]^{-1/(r-1)}\right\}^{r-1}\left\{\sum_{\tau=n+1}^{+\infty}[p(\tau)]^{-1/(r-1)}\right\}^{r-1}\right. \\
&\qquad\times\left(\left\{\sum_{\tau=-\infty}^{n}[p(\tau)]^{-1/(r-1)}\right\}^{r-1}\right. \\
&\qquad\qquad\left.\left.+\left\{\sum_{\tau=n+1}^{+\infty}[p(\tau)]^{-1/(r-1)}\right\}^{r-1}\right)^{-1}\right)q^{+}(n) \geq 1.
\end{aligned} \tag{46}$$

Corollary 5. *Suppose that* $r > 1$ *and* $p(n) > 0$ *for* $n \in \mathbb{Z}$, *and that* (44) *holds. If* (8) *has a solution* $x(n)$ *satisfying* (45), *then*

$$\sum_{n=-\infty}^{+\infty}q^{+}(n)\left\{\sum_{\tau=-\infty}^{n}[p(\tau)]^{-1/(r-1)}\sum_{\tau=n+1}^{+\infty}[p(\tau)]^{-1/(r-1)}\right\}^{(r-1)/2} \geq 2. \tag{47}$$

3. Lyapunov-Type Inequalities for System (5)

When $\mu = \nu = 2$, assumption (B1) reduces the following form:

(B2) $\sum_{\tau=-\infty}^{0}\beta(\tau)\prod_{s=\tau}^{0}[1-\alpha(s)]^{-2} + \sum_{\tau=1}^{+\infty}\beta(\tau)\prod_{s=0}^{\tau-1}[1-\alpha(s)]^{2} < +\infty$.

Applying the results obtained in last section to the first-order linear Hamiltonian system (5), we have immediately the following corollaries.

Corollary 6. *Suppose that hypotheses (A0), (B0), and (B2) are satisfied. If system* (5) *has a solution* $(x(n), y(n))$ *satisfying*

$$0 < \sum_{n=-\infty}^{+\infty} \left[|x(n)|^2 + (1+\beta(n)) |y(n)|^2 \right] < +\infty, \quad (48)$$

then

$$\sum_{n=-\infty}^{+\infty} \left(\left\{ \sum_{\tau=-\infty}^{n} \beta(\tau) \prod_{s=\tau}^{n} [1-\alpha(s)]^{-2} \right\} \times \left\{ \sum_{\tau=n+1}^{+\infty} \beta(\tau) \prod_{s=n+1}^{\tau-1} [1-\alpha(s)]^{2} \right\} \times \left(\sum_{\tau=-\infty}^{n} \beta(\tau) \prod_{s=\tau}^{n} [1-\alpha(s)]^{-2} + \sum_{\tau=n+1}^{+\infty} \beta(\tau) \prod_{s=n+1}^{\tau-1} [1-\alpha(s)]^{2} \right)^{-1} \right) \gamma^{+}(n) \geq 1. \quad (49)$$

Corollary 7. *Suppose that hypotheses (A1), (B0), and (B2) are satisfied. If system* (5) *has a soldution* $(x(n), y(n))$ *satisfying* (48), *then*

$$\sum_{n=-\infty}^{+\infty} \gamma^{+}(n) \left[\sum_{\tau=-\infty}^{n} \beta(\tau) \sum_{\tau=n+1}^{+\infty} \beta(\tau) \right]^{1/2} \geq 2 \prod_{n=-\infty}^{+\infty} \Theta[\alpha(n)]. \quad (50)$$

Corollary 8. *Suppose that* $p(n) > 0$ *for* $n \in \mathbb{Z}$, *and that*

$$\sum_{\tau=-\infty}^{+\infty} \frac{1}{p(\tau)} < +\infty. \quad (51)$$

If (7) *has a solution* $x(n)$ *satisfying*

$$0 < \sum_{n=-\infty}^{+\infty} \left[|x(n)|^2 + p(n)(1+p(n)) |\Delta x(n)|^2 \right] < +\infty, \quad (52)$$

then

$$\sum_{n=-\infty}^{+\infty} q^{+}(n) \left[\sum_{\tau=-\infty}^{n} \frac{1}{p(\tau)} \sum_{\tau=n+1}^{+\infty} \frac{1}{p(\tau)} \right] \geq \sum_{n=-\infty}^{+\infty} \frac{1}{p(n)}. \quad (53)$$

4. Nonexistence of Homoclinic Solutions

Applying the results obtained in Sections 2 and 3, we can drive the following criteria for non-existence of homoclinic solutions of systems (5) and (6) immediately.

Corollary 9. *Suppose that hypotheses (A0), (B0), and (B1) are satisfied. If*

$$\sum_{n=-\infty}^{+\infty} \frac{\zeta(n)\eta(n)}{\zeta(n)+\eta(n)} \gamma^{+}(n) < 1, \quad (54)$$

then system (6) *has no solution* $(x(n), y(n))$ *satisfying*

$$0 < \sum_{n=-\infty}^{+\infty} \left[|x(n)|^{\nu} + (1+\beta(n)) |y(n)|^{\mu} \right] < +\infty. \quad (55)$$

Corollary 10. *Suppose that hypotheses (A1), (B0), and (B1) are satisfied. If*

$$\sum_{n=-\infty}^{+\infty} \gamma^{+}(n) \left[\sum_{\tau=-\infty}^{n} \beta(\tau) \sum_{\tau=n+1}^{+\infty} \beta(\tau) \right]^{\nu/2\mu} < 2 \prod_{n=-\infty}^{+\infty} \{\Theta[\alpha(n)]\}^{\nu/2}, \quad (56)$$

then system (6) *has no solution* $(x(n), y(n))$ *satisfying* (55).

Corollary 11. *Suppose that hypotheses (A1), (B0), and (B1) are satisfied. If*

$$\left(\sum_{n=-\infty}^{+\infty} \beta(n) \right)^{1/\mu} \left(\sum_{n=-\infty}^{+\infty} \gamma^{+}(n) \right)^{1/\nu} < 2 \prod_{n=-\infty}^{+\infty} \{\Theta[\alpha(n)]\}^{1/2}, \quad (57)$$

then system (6) *has no solution* $(x(n), y(n))$ *satisfying* (55).

Corollary 12. *Suppose that hypotheses (A0), (B0), and (B2) are satisfied. If*

$$\sum_{n=-\infty}^{+\infty} \left(\left\{ \sum_{\tau=-\infty}^{n} \beta(\tau) \prod_{s=\tau}^{n} [1-\alpha(s)]^{-2} \right\} \times \left\{ \sum_{\tau=n+1}^{+\infty} \beta(\tau) \prod_{s=n+1}^{\tau-1} [1-\alpha(s)]^{2} \right\} \times \left(\sum_{\tau=-\infty}^{n} \beta(\tau) \prod_{s=\tau}^{n} [1-\alpha(s)]^{-2} + \sum_{\tau=n+1}^{+\infty} \beta(\tau) \prod_{s=n+1}^{\tau-1} [1-\alpha(s)]^{2} \right)^{-1} \right) \gamma^{+}(n) < 1, \quad (58)$$

then system (5) *has no solution* $(x(n), y(n))$ *satisfying*

$$0 < \sum_{n=-\infty}^{+\infty} \left[|x(n)|^2 + (1+\beta(n)) |y(n)|^2 \right] < +\infty. \quad (59)$$

Corollary 13. *Suppose that hypotheses (A1), (B0), and (B2) are satisfied. If*

$$\sum_{n=-\infty}^{+\infty} \gamma^{+}(n) \left[\sum_{\tau=-\infty}^{n} \beta(\tau) \sum_{\tau=n+1}^{+\infty} \beta(\tau) \right]^{1/2} < 2 \prod_{n=-\infty}^{+\infty} \Theta[\alpha(n)], \quad (60)$$

then system (5) *has no solution* $(x(n), y(n))$ *satisfying* (59).

Corollary 14. *Suppose that $p(n) > 0$ for $n \in \mathbb{Z}$, and that* (51) *holds. If*

$$\sum_{n=-\infty}^{+\infty} q^{+}(n) \left(\sum_{\tau=-\infty}^{n} \frac{1}{p(\tau)} \sum_{\tau=n+1}^{+\infty} \frac{1}{p(\tau)} \right) < \sum_{n=-\infty}^{+\infty} \frac{1}{p(n)}, \tag{61}$$

then (7) *has no solution $x(n)$ satisfying* (52).

Example 15. Consider the second-order difference equation:

$$\Delta \left[\left(1 + n^2\right) \Delta x(n) \right] + q(n) x(n+1) = 0, \tag{62}$$

where $q(n)$ is real-valued function defined on $\mathbb{Z}$. In view of Corollary 14, if

$$\sum_{n=-\infty}^{+\infty} \left[\left(\sum_{\tau=-\infty}^{n} \frac{1}{1+\tau^2} \sum_{\tau=n+1}^{+\infty} \frac{1}{1+\tau^2} \right) \right] q^{+}(n) < \sum_{n=-\infty}^{+\infty} \frac{1}{1+n^2}, \tag{63}$$

then (62) has no solution $x(n)$ satisfying

$$0 < \sum_{n=-\infty}^{+\infty} \left[|x(n)|^2 + \left(1+n^2\right)^2 |\Delta x(n)|^2 \right] < +\infty. \tag{64}$$

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