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Research Article

Positive Solutions of the One-Dimensional p-Laplacian with Nonlinearity Defined on a Finite Interval

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We use the quadrature method to show the existence and multiplicity of positive solutions of the boundary value problems involving one-dimensional p-Laplacian $(|u'(t)|^{p-2}u'(t))'+\lambda f(u(t))=0, t\in(0,1), u(0)=u(1)=0$, where $p\in(1,2]$, $\lambda\in(0,\infty)$ is a parameter, $f\in C^1([0,r),[0,\infty))$ for some constant $r>0$, $f(s)>0$ in $(0,r)$, and $\lim_{s\to r^-}(r-s)^{p-1}f(s)=+\infty$.

1. Introduction and the Main Results

Let $a:[0,1]\to[0,\infty)$ be continuous and $a(t)\not\equiv 0$ on any subset of $[0,1]$, and let $f:[0,\infty)\to\mathbb{R}$ be a continuous function. Wang [1] proved the existence of positive solutions of nonlinear boundary value problems

$$\begin{gathered} u''(t)+a(t)f(u(t))=0, \quad t\in(0,1), \\ u(0)=u(1)=0, \end{gathered} \tag{1}$$

under the following assumptions:

$$f_0:=\lim_{s\to 0}\frac{f(s)}{s}=0, \qquad f_\infty:=\lim_{s\to+\infty}\frac{f(s)}{s}=\infty. \tag{2}$$

Since then, the existence and multiplicity of positive solutions of (1) and its generalized forms have been extensively studied via the fixed point theorem in cones. For example, Ge [2] showed a series of results on the existence and multiplicity of solutions of nonlinear ordinary differential equation of second order/higher order subjected with diverse boundary conditions via topological degree and fixed point theorem in cones; Wang [3] use fixed point theorem in cones to study the existence of positive solutions for the one dimensional p-Laplacian. For other recent results along this line, see [4–11] and the bibliographies in [2]. For the special case $a(t)\equiv 1$, beautiful results have been obtained via the quadrature method; see Fink et al. [12], Brown and Budin [13], Addou and Wang [14], Cheng and Shao [15], Karátson and Simon [16], and the references therein.

The nonlinearity $f(u)$ that appeared in the above previously papers is assumed to be well defined in $[0,\infty)$ or $(-\infty,\infty)$. Of course, natural question is what would happen if $f(u)$ is only well defined in a finite interval $[0,r)$, where r is a positive constant; that is, what would happen if (2) is replaced with the following limit f_r :

$$f_r:=\lim_{s\to r^-}\frac{f(s)}{s^{p-1}}=+\infty. \tag{3}$$

It is worth remarking that the fixed point theorem in cones method in [1–3] cannot be used to deal with the existence of positive solutions of the problem

$$\begin{gathered} \left(\left|u'(t)\right|^{p-2}u'(t)\right)'+\lambda f(u(t))=0, \quad t\in(0,1), \\ u(0)=u(1)=0, \end{gathered} \tag{4}$$

under the restriction (3) any more since the appearance of singularity of f at r. The purpose of this paper is to use the quadrature method to show the existence and multiplicity of positive solutions of (4), in which $\lambda\in(0,\infty)$ is a parameter, and f satisfies the following assumptions:

(H1) $f\in C^1([0,r),[0,\infty))$;

(H2) $f(s)>0$ in $(0,r)$;

(H3) $\lim_{s\to r^-}(r-s)^{p-1}f(s)=+\infty$.

Let $f_0:=\lim_{s\to 0^+}(f(s)/s^{p-1})$, $F(s)=\int_0^s f(v)dv$. The main result of the paper is the following.

Theorem 1. *Let $p \in (1, 2]$, and let (H1), (H2), and (H3) hold. Then,*

(i) *if $f_0 = +\infty$, then there exist $\lambda_* > 0$, such that (4) has at least two positive solutions for $\lambda \in (0, \lambda_*)$, has at least one positive solution for $\lambda = \lambda_*$, and has no positive solution for $\lambda > \lambda_*$;*

(ii) *if $f_0 = 0$, then (4) has at least one positive solution for $\lambda \in (0, \infty)$;*

(iii) *if $f_0 \in (0, \infty)$, then (4), has at least one positive solution for $\lambda \in (0, \pi_p^p / f_0)$, where*

$$\pi_p := 2 \int_0^{(p-1)^{1/p}} \frac{1}{\left(1 - s^p/(p-1)\right)^{1/p}} ds. \tag{5}$$

The proof of our main result is motivated by Laetsch [17] in which the existence and multiplicity of positive solutions of (4) with $p = 2$ were studied via the quadrature method. Since then, there are plenty of research papers on the study of exact multiplicity of positive solutions of the p-Laplacian problem with general $p > 1$ and some more special nonlinearities; see [18, 19] and the references therein. To find the exact number of positive solutions, the nonlinearity f needs to satisfy some restrictive conditions, such as the monotonic condition or convex condition . . .. Our conditions (H1)–(H3) are not strong enough to guarantee the problem exact number of positive solutions.

The rest of the paper is arranged as follows. In Section 2, we state and prove some preliminary results. Finally in Section 3, we give the proof of Theorem 1.

2. Preliminaries

To prove our main results, we will use the uniqueness results due to Reichel and Walter [20] on the initial value problem

$$\begin{gathered} \left(\left|u'(t)\right|^{p-2} u'(t)\right)' + \lambda f(u(t)) = 0, \\ u(a) = b, \qquad u'(a) = d, \end{gathered} \tag{6}$$

where $a \in [0, 1]$, and $b, d \in \mathbb{R}$.

Lemma 2. *Let (H1) and (H2) hold. Then,*

(a) (6) *with $d \neq 0$ has a unique local solution, and, the extension $u(t)$ remains unique as long as $u'(t) \neq 0$;*

(b) (6) *with $b \in (0, r)$ and $d = 0$ has a unique local solution.*

(c) (6) *with $b = 0$ and $d = 0$ has a unique local solution $u \equiv 0$.*

Proof. (a) It is an immediate consequence of Reichel and Walter [20, Theorem 2].

(b) (H1) implies that f is local Lipschitz continuous. Combining this with the fact that $f(b) \neq 0$ and using [20, (iii) and (v) in the case (β) of Theorem 4], it follows that (6) with $b \in (0, r)$ and $d = 0$ has a unique solution in some neighborhood of a.

(c) Define

$$\tilde{f}(s) = \begin{cases} f(s), & 0 \le s < r, \\ -f(-s), & 0 \le -r < s < 0, \end{cases} \tag{7}$$

and consider the auxiliary problem

$$\begin{gathered} \left(\left|u'(t)\right|^{p-2} u'(t)\right)' + \lambda \tilde{f}(u(t)) = 0, \\ u(a) = u'(a) = 0. \end{gathered} \tag{8}$$

It follows from [20, (i) in the case (δ) of Theorem 4] that (8) has a unique local solution $u \equiv 0$ in some neighborhood of a, and consequently, (6) with $b = d = 0$ has a unique local solution $u \equiv 0$ in some neighborhood of a. □

Lemma 3. *Let (H1) and (H2) hold. Let (λ, u) be a solution of*

$$\begin{gathered} \left(\left|u'(t)\right|^{p-2} u'(t)\right)' + \lambda f(u(t)) = 0, \quad t \in (0, 1), \\ u(0) = u(1) = 0, \end{gathered} \tag{9}$$

with $\|u\|_\infty = \rho < r$. Let $x_0 \in (0, 1)$ be such that $u(x_0) = \|u\|_\infty$. Then, $u(t) > 0$ on $(0, 1)$, $u'(t) > 0$ on $(0, x_0)$, and $u'(t) < 0$ on $(x_0, 1)$, and

$$u(x_0 - t) = u(x_0 + t), \quad t \in [0, \min\{x_0, 1 - x_0\}]. \tag{10}$$

Proof. Since $f(s) \ge 0$ for $s \in [0, \rho]$, it follows from (9) that

$$\left(\left|u'(t)\right|^{p-2} u'(t)\right)' \le 0, \quad t \in [0, 1]. \tag{11}$$

This, together with the fact that $u'(x_0) = 0$, implies that u is nondecreasing in $[0, x_0]$, and u is nonincreasing in $[x_0, 1]$.

We claim that

$$u(t) > 0, \quad t \in (0, 1). \tag{12}$$

In fact, suppose on the contrary that there exists $\tau \in (0, x_0)$ such that $u(\tau) = 0$; then,

$$\hat{\tau} = \max\left\{\tau \in (0, x_0) \mid u(\tau) = 0\right\} \tag{13}$$

is well defined. Moreover,

$$u(t) \equiv 0, \quad t \in (0, \hat{\tau}]. \tag{14}$$

By Lemma 2 (c),

$$u(t) \equiv 0, \quad t \in (0, \hat{\tau} + \eta] \tag{15}$$

for some $\eta > 0$. However, this contradicts the definition of $\hat{\tau}$, see (13). Therefore, $u(t) > 0$ in $(0, x_0]$. Similarly, we may show that $u(t) > 0$ in $[x_0, 1)$.

Notice that (H2), (12), and (9) yield that

$$\begin{gathered} u'(t) > 0, \quad t \in (0, x_0), \\ u'(t) < 0, \quad t \in (x_0, 1). \end{gathered} \tag{16}$$

Now, since f is independent of t, both $u(x_0-t)$ and $u(x_0+t)$ satisfy the initial value problem

$$\left(\left|w'(t)\right|^{p-2}w'(t)\right)' + \lambda f(w(t)) = 0, \qquad w(0) = u(x_0), \quad w'(0) = 0. \tag{17}$$

From (16) and Lemma 2(a), it follows that both $u(x_0-t)$ and $u(x_0+t)$ can be uniquely extended to $[0, \min\{x_0, 1-x_0\}]$. Thus, we have from Lemma 2(b) that (17) has a unique solution $u \equiv 0$ on $[0, \min\{x_0, 1-x_0\}]$, and, accordingly, (10) is true. □

Lemma 4. *Let (H1) and (H2) hold. Assume that (λ, u) is a positive solution of the problem (9) with $\|u\|_\infty = \rho < r$ and $\lambda > 0$. Let $x_0 \in (0,1)$ be such that $u(x_0) = \|u\|_\infty$. Then,*

(a) $x_0 = 1/2$;

(b) x_0 *is the unique point on which u attains its maximum;*

(c) $u'(t) > 0, t \in (0, 1/2)$.

Proof. (a) Suppose on the contrary that $x_0 \neq 1/2$, and say that $x_0 \in (1/2, 1)$; then, $2x_0 - 1 \in (0,1)$, and

$$0 = u(1) = u(2x_0 - 1). \tag{18}$$

However, this contradicts (12). Therefore, $x_0 = 1/2$.

Also (b) and (c) can be easily deduced from (16). □

3. Proof of the Main Result

To prove Theorem 1, we need the following quadrature method.

Lemma 5. *For any $\rho < r$, there exists a unique $\lambda > 0$ such that*

$$\left(\left|u'(t)\right|^{p-2}u'(t)\right)' + \lambda f(u(t)) = 0, \qquad t \in (0,1), \qquad u(0) = u(1) = 0, \tag{19}$$

has a positive solution (λ, u) with $\|u\| = \rho$. Moreover, $\rho \to \lambda(\rho)$ is a continuous function on $[0, r)$.

Proof. By Lemma 4, (λ, u) is a positive solution of (19), if and only if (λ, u) is a positive solution of

$$\left(\left|u'(t)\right|^{p-2}u'(t)\right)' + \lambda f(u(t)) = 0, \quad t \in (0,1), \tag{20}$$

$$u(0) = u'\left(\frac{1}{2}\right) = 0. \tag{21}$$

Suppose that (λ, u) is a solution of (20), with $\|u\| = \rho$. Then,

$$\left(u'(t)\right)^p = \lambda \frac{p}{p-1}\left[F(\rho) - F(u(t))\right], \quad t \in \left[0, \frac{1}{2}\right], \tag{22}$$

and so

$$t\left(\frac{p}{p-1}\lambda\right)^{1/p} = \int_0^{u(t)} \left(F(\rho) - F(s)\right)^{-1/p} ds, \quad t \in \left[0, \frac{1}{2}\right]. \tag{23}$$

Putting $t = 1/2$, we obtain

$$\lambda^{1/p} = 2\left(\frac{p-1}{p}\right)^{1/p} \int_0^\rho \left(F(\rho) - F(s)\right)^{-1/p} ds. \tag{24}$$

Hence, λ (if exists) is uniquely determined by ρ.

If $\rho < r$, we define $\lambda(\rho)$ by (24) and $u(t)$ by (23); it is straightforward to verify that u is twice differentiable, u satisfies (21), $u > 0$ in $(0,1)$, and $u(1/2) = \rho$. The continuity of $\lambda(\cdot)$ is implied by (24), and this completes the proof. □

Lemma 6. *Let (H3) hold, and let $p \in (1,2)$. Then*

$$\lim_{\rho \to r^-} \lambda(\rho) = 0. \tag{25}$$

Proof. By (H3), there are positive numbers $R < r$ and k such that

$$f(s) \geq \frac{k^p}{(r-s)^{p-1}}, \qquad R \leq s < r. \tag{26}$$

Thus, if $R < \rho < r$, (24) implies that

$$\begin{aligned}
\{\lambda(\rho)\}^{1/p} &= 2\left(\frac{p-1}{p}\right)^{1/p} \int_0^\rho \frac{1}{\left(F(\rho) - F(s)\right)^{1/p}} ds \\
&= 2\left(\frac{p-1}{p}\right)^{1/p} \\
&\quad \times \left[\int_0^R \frac{1}{\left(F(\rho) - F(s)\right)^{1/p}} ds + \int_R^\rho \frac{1}{\left(F(\rho) - F(s)\right)^{1/p}} ds\right] \\
&= 2\left(\frac{p-1}{p}\right)^{1/p} \\
&\quad \times \left[\int_0^R \frac{1}{\left(\int_s^\rho f(w)\,dw\right)^{1/p}} ds + \int_R^\rho \frac{1}{\left(\int_s^\rho f(w)\,dw\right)^{1/p}} ds\right]
\end{aligned}$$

$$\leq \frac{2}{k}\left(\frac{p-1}{p}\right)^{1/p}(2-p)^{1/p}$$
$$\times \left\{\int_0^R \frac{1}{\left[(r-s)^{2-p}-(r-\rho)^{2-p}\right]^{1/p}}ds\right.$$
$$\left.+\int_R^\rho \frac{1}{\left[(r-s)^{2-p}-(r-\rho)^{2-p}\right]^{1/p}}ds\right\}$$
$$\text{by (26)}$$
$$\leq \frac{2}{k}\left(\frac{p-1}{p}\right)^{1/p}(2-p)^{1/p}$$
$$\times \left\{R[\xi(\rho)]^{-1/p}+\frac{1}{2-p}\int_0^{\xi(\rho)} x^{-1/p}\right.$$
$$\left.\times\left[x+(r-\rho)^{2-p}\right]^{(p-1)/(2-p)}dx\right\}\longrightarrow 0, \tag{27}$$

as $\rho \to r^-$, where

$$\xi(\rho) := (r-R)^{2-p}-(r-\rho)^{2-p},$$
$$x =: (r-s)^{2-p}-(r-\rho)^{2-p}. \tag{28}$$

If $\rho \to r^-$, then

$$\xi(\rho) := (r-R)^{2-p}-(r-\rho)^{2-p} \longrightarrow (r-R)^{2-p},$$
$$\frac{1}{2-p}\int_0^{\xi(\rho)} x^{-1/p}\left[x+(r-\rho)^{2-p}\right]^{(p-1)/(2-p)}dx$$
$$\longrightarrow \frac{1}{2-p}\int_0^{(r-R)^{2-p}} x^{(p^2-2)/p(2-p)}dx$$
$$= \frac{p}{2(p-1)}(r-R)^{2(p-1)/p}. \tag{29}$$

□

Lemma 7. *Let (H1) and (H2) hold, and let $p=2$. Then,*

$$\lim_{\rho\to r^-}\lambda(\rho)=0. \tag{30}$$

Proof. There are positive numbers $R<r$ and k such that

$$f(s) \geq \frac{k^2}{r-s}, \quad R\leq s<r. \tag{31}$$

Thus, if $R<\rho<r$, (24) implies that

$$\left\{\frac{1}{2}\lambda(\rho)\right\}^{1/2} = \int_0^\rho \frac{1}{\sqrt{F(\rho)-F(s)}}ds$$
$$= \int_0^R \frac{ds}{\sqrt{F(\rho)-F(s)}}+\int_R^\rho \frac{ds}{\sqrt{F(\rho)-F(s)}}$$
$$= \int_0^R \frac{ds}{\sqrt{\int_s^\rho f(w)\,dw}}+\int_R^\rho \frac{ds}{\sqrt{\int_s^\rho f(w)\,dw}}$$
$$\leq \int_0^R\left(k^2\ln\frac{r-R}{r-\rho}\right)^{-1/2}ds$$
$$+\int_R^\rho\left(k^2\ln\frac{r-s}{r-\rho}\right)^{-1/2}ds \quad \text{by (31)}$$
$$\leq \frac{1}{k}\left\{R[\xi(\rho)]^{-1/2}+(r-\rho)\right.$$
$$\left.\times\int_0^{\xi(\rho)} x^{-1/2}e^x dx\right\}\longrightarrow 0, \tag{32}$$

as $\rho \to r^-$, where

$$\xi(\rho) := \ln\frac{r-R}{r-\rho}, \qquad x =: \ln\frac{r-s}{r-\rho}. \tag{33}$$

□

Lemma 8. *Let (H1), (H2), and (H3) hold, and assume that $p \in (1,+\infty)$. Then,*

(a) *if $f_0=\infty$, then $\lim_{\rho\to 0^+}\lambda(\rho)=0$;*

(b) *if $f_0=0$, then $\lim_{\rho\to 0^+}\lambda(\rho)=+\infty$;*

(c) *if $f_0\in(0,\infty)$, then $\lim_{\rho\to 0^+}\lambda(\rho)=\pi_p^p/f_0$, where*

$$\pi_p := 2\int_0^{(p-1)^{1/p}} \frac{1}{\left(1-\left(s^p/(p-1)\right)\right)^{1/p}}ds. \tag{34}$$

Proof. (a) If $f_0=\infty$, then for any positive constant k, there exists $R\in(0,r)$ such that

$$f(s)\geq k^p s^{p-1}, 0<s<R. \tag{35}$$

Thus, if $0 < \rho < R$, (24) implies that

$$\begin{aligned}\{\lambda(\rho)\}^{1/p} &= 2\left(\frac{p-1}{p}\right)^{1/p}\int_0^\rho \frac{1}{(F(\rho)-F(s))^{1/p}}ds \\ &= 2\left(\frac{p-1}{p}\right)^{1/p}\int_0^\rho \frac{1}{\left(\int_s^\rho f(v)\,dv\right)^{1/p}}ds \\ &\le 2\left(\frac{p-1}{p}\right)^{1/p}\frac{p^{1/p}}{k}\int_0^\rho \frac{1}{(\rho^p-s^p)^{1/p}}ds \quad \text{by (35)} \\ &= 2\frac{(p-1)^{1/p}}{k}\int_0^1 \frac{1}{(1-s^p)^{1/p}}ds \\ &= \frac{2}{k}\int_0^{(p-1)^{1/p}} \frac{1}{(1-s^p/(p-1))^{1/p}}ds \\ &= \frac{1}{k}\pi_p,\end{aligned} \tag{36}$$

which implies that $\lim_{\rho\to 0^+}\lambda(\rho) = 0$.

(b) If $f_0 = 0$, then for any $\varepsilon > 0$, there exists $\delta \in (0, r)$ such that

$$f(s) \le \varepsilon^p s^{p-1}, \quad 0 < s < \delta. \tag{37}$$

Thus, if $0 < \rho < \delta$, (24) implies that

$$\begin{aligned}\{\lambda(\rho)\}^{1/p} &= 2\left(\frac{p-1}{p}\right)^{1/p}\int_0^\rho \frac{1}{(F(\rho)-F(s))^{1/p}}ds \\ &= 2\left(\frac{p-1}{p}\right)^{1/p}\int_0^\rho \frac{1}{\left(\int_s^\rho f(v)\,dv\right)^{1/p}}ds \\ &\ge 2\left(\frac{p-1}{p}\right)^{1/p}\frac{p^{1/p}}{\varepsilon}\int_0^\rho \frac{1}{(\rho^p-s^p)^{1/p}}ds \quad \text{by (37)} \\ &= 2\frac{(p-1)^{1/p}}{\varepsilon}\int_0^1 \frac{1}{(1-s^p)^{1/p}}ds \\ &= \frac{2}{\varepsilon}\int_0^{(p-1)^{1/p}} \frac{1}{(1-s^p/(p-1))^{1/p}}ds \\ &= \frac{1}{\varepsilon}\pi_p,\end{aligned} \tag{38}$$

which implies that $\lim_{\rho\to 0^+}\lambda(\rho) = +\infty$.

(c) If $f_0 \in (0, \infty)$, then for any $\varepsilon \in (0, f_0/2)$, there exists $\delta \in (0, r)$ such that

$$f_0 - \varepsilon \le \frac{f(s)}{s^{p-1}} \le f_0 + \varepsilon, \quad 0 < s < \delta. \tag{39}$$

Thus, if $0 < \rho < \delta$, (24) and the second part of (39) imply that

$$\begin{aligned}\{\lambda(\rho)\}^{1/p} &= 2\left(\frac{p-1}{p}\right)^{1/p}\int_0^\rho \frac{1}{(F(\rho)-F(s))^{1/p}}ds \\ &= 2\left(\frac{p-1}{p}\right)^{1/p}\int_0^\rho \frac{1}{\left(\int_s^\rho f(v)\,dv\right)^{1/p}}ds \\ &\ge 2\left(\frac{p-1}{p}\right)^{1/p}\left(\frac{p}{f_0+\varepsilon}\right)^{1/p}\int_0^\rho \frac{1}{(\rho^p-s^p)^{1/p}}ds \\ &= 2\left(\frac{p-1}{f_0+\varepsilon}\right)^{1/p}\int_0^1 \frac{1}{(1-s^p)^{1/p}}ds \\ &= \frac{2}{(f_0+\varepsilon)^{1/p}}\int_0^{(p-1)^{1/p}} \frac{1}{(1-s^p/(p-1))^{1/p}}ds \\ &= \frac{1}{(f_0+\varepsilon)^{1/p}}\pi_p,\end{aligned} \tag{40}$$

which implies that

$$\lim_{\rho\to 0^+}\lambda(\rho) \ge \frac{\pi_p^p}{f_0}. \tag{41}$$

Similarly,

$$\begin{aligned}\{\lambda(\rho)\}^{1/p} &= 2\left(\frac{p-1}{p}\right)^{1/p}\int_0^\rho \frac{1}{(F(\rho)-F(s))^{1/p}}ds \\ &= 2\left(\frac{p-1}{p}\right)^{1/p}\int_0^\rho \frac{1}{\left(\int_s^\rho f(v)\,dv\right)^{1/p}}ds \\ &\ge 2\left(\frac{p-1}{p}\right)^{1/p}\left(\frac{p}{f_0-\varepsilon}\right)^{1/p}\int_0^\rho \frac{1}{(\rho^p-s^p)^{1/p}}ds \\ &= 2\left(\frac{p-1}{f_0-\varepsilon}\right)^{1/p}\int_0^1 \frac{1}{(1-s^p)^{1/p}}ds \\ &= \frac{2}{(f_0-\varepsilon)^{1/p}}\int_0^{(p-1)^{1/p}} \frac{1}{(1-s^p/(p-1))^{1/p}}ds \\ &= \frac{1}{(f_0-\varepsilon)^{1/p}}\pi_p,\end{aligned} \tag{42}$$

which implies that if $0 < \rho < \delta$, (24) and the first part of (39) imply that

$$\lim_{\rho\to 0^+}\lambda(\rho) \le \frac{\pi_p^p}{f_0}. \tag{43}$$

Combining (41) and (43), it follows that

$$\lim_{\rho\to 0^+}\lambda(\rho) = \frac{\pi_p^p}{f_0}. \tag{44}$$

□

Proof of Theorem 1. (i) It is from Lemma 8(a) that

$$\lim_{\rho \to 0^+} \lambda(\rho) = 0. \tag{45}$$

From Lemma 6 or Lemma 7, we have that

$$\lim_{\rho \to r^-} \lambda(\rho) = 0. \tag{46}$$

Combining this with (45) and using the facts that $\lambda(\rho) > 0$ and $\lambda(\rho)$ is continuous, it concludes that there exists $\lambda_* > 0$ such that (4) has at least two positive solutions for $\lambda \in (0, \lambda_*)$, has at least one positive solution for $\lambda = \lambda_*$, and has no positive solution for $\lambda > \lambda_*$.

(ii) It is an immediate consequence of Lemma 6, and Lemma 7, and Lemma 8(b).

(iii) It is an immediate consequence of Lemma 6, and Lemma 7, and Lemma 8(c).

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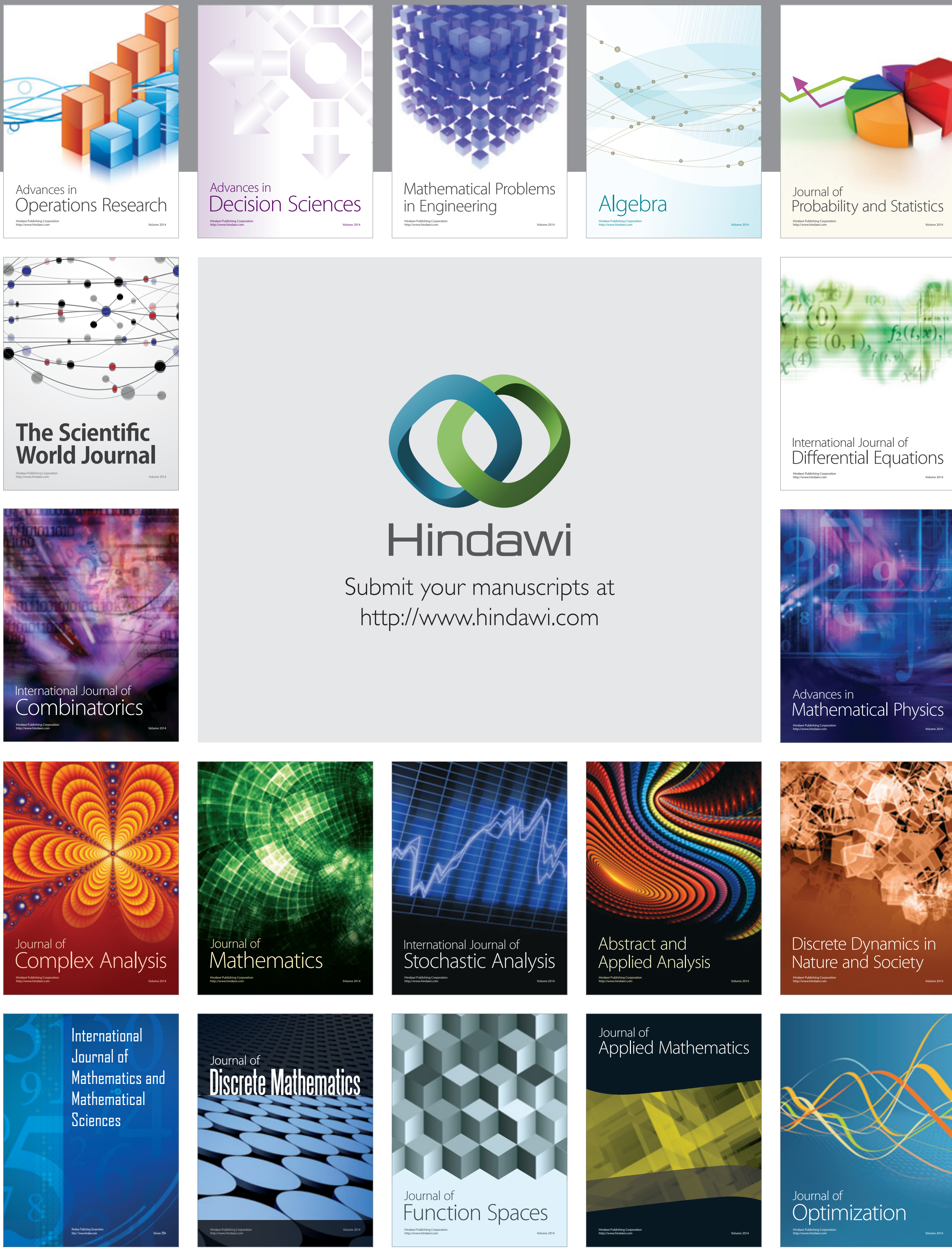
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