THREE DIMENSIONAL GREEN'S FUNCTION FOR SHIP MOTION AT FORWARD SPEED

MATIUR RAHMAN
Department of Applied Mathematics
Technical University of Nova Scotia
P.O. BOX 1000
Halifax, Nova Scotia
Canada B3J 2X4

(Received December 12, 1988)

ABSTRACT. The Green's function formulation for ship motion at forward speed contains double integrals with singularities in the path of integrations with respect to the wave number. In this study, the double integrals have been replaced by single integrals with the use of complex exponential integrals. It has been found that this analysis provides an efficient way of computing the wave resistance for three dimensional potential problem of ship motion with forward speed.

KEY WORDS AND PHRASES. Ship motion, Green's function, Hydrodynamics, Wave Resistance and Wave Responses.

1. INTRODUCTION.

In ship hydrodynamics, Green's functions play a very important role in predicting the wave resistance, wave induced responses at zero forward speed, and the motions of a vessel advancing in waves. The Green's function formulation for ship motions at forward speed is the most difficult part of the problem partly because it contains double integrals and partly because of the presence of the singularities in the path of integrations with respect to the wave numbers. Nowadays, considerable interest has been paid to evaluate the three dimensional Green's function for ship motions at forward speed.

Many authors including Haskind [1] and Havelock [2] have expressed the Green's function having a constant forward speed as a double integral. This form of Green's function is not suitable for numerical analysis because the detailed computation of the double integral is very expensive. Therefore, in the present study, we have replaced the double integral by a single integral (see Wu and Taylor [3]) involving a complex exponential integral, and it is found that it is more efficient to calculate the Green's function numerically.

2. A FORM OF THE GREEN'S FUNCTION.

Consider the coordinate system oxyz which is moving at constant forward speed U along the x axis and z measured positive upwards from the mean free surface (see Figure 1).
It is assumed that a ship is travelling at a constant forward speed $U$ along the Ox direction and oscillating at a frequency $\omega$ as in the form of $e^{i\omega t}$. Wehausen and Wahl tone [4] in 1960 have shown that the Green's function which satisfies the exact free surface condition can be written as

$$G(x,y,z; \alpha, \beta, \gamma) = \frac{1}{R} - \frac{1}{R_1} + \frac{2g}{\pi} \int_0^\infty d\theta \int_0^\pi F(\theta,k)dk + 2R \int_0^{\pi/2} d\theta \int F(\theta,k)dk$$

$$+ \int_{\pi/2}^\pi d\theta \int F(\theta,k)dk$$

where

$$R = \sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2}$$

is the Rankine source located at $(\alpha, \beta, \gamma)$

$$R_1 = \sqrt{(x-a)^2 + (y-b)^2 + (z+c)^2}$$

is the image about the mean free surface at $(\alpha, \beta, -c)$

$g$ is the acceleration due to gravity

$$F(\theta,k) = \frac{k \epsilon \left(k(z+c) + l(x-a)\cos\theta\right) \cos[k(y-b)\sin\theta]}{gk^2 - (\omega k U \cos \theta)^2}$$

$(x,y,z)$ is the field point and $(\alpha, \beta, \gamma)$ is the source distribution point. The other parameters in Equation (2.1) are defined by

$$\gamma = 0 \quad \text{if} \quad \tau < \frac{1}{4}$$

$$\cos^{-1} \left( \frac{1}{\tau \tau} \right) \quad \text{if} \quad \tau > \frac{1}{4}$$

where $\tau = \frac{\omega U}{g}$ is called the Strouhal number

The contours $L_1$ and $L_2$ are defined as follows:
There are two singular points in $L_1$ and two singular points in the $L_2$ integral of Equation (2.1). These singular points can be obtained as follows:

\[
\gamma_{k_1}, \gamma_{k_3} = \frac{1 - \sqrt{1 - 4\tau \cos \theta}}{2\tau \cos \theta} \nu \quad (2.5)
\]

\[
\gamma_{k_2}, \gamma_{k_4} = \frac{1 + \sqrt{1 - 4\tau \cos \theta}}{2\tau \cos \theta} \nu \quad (2.6)
\]

The alternative forms of these singularities $k_1, k_2, k_3$ and $k_4$ can be written as

\[
k_2, k_1 = \frac{(1 - 2\tau \cos \theta) \pm \sqrt{1 - 4\tau \cos \theta}}{2\tau \cos^2 \theta} \nu \quad (2.8)
\]

for $\pi/2 < \theta < \tau$; and where $\nu = \frac{\omega}{\delta}$.

It should be noted here that $k_1 = k_3$ and $k_2 = k_4$. These singularities are real in the ranges indicated. It is, however, worth mentioning here that in the range $0 < \theta < \gamma$, the singularities $k_1$ and $k_2$ become complex quantities and are either given by

\[
\gamma_{k_2}, \gamma_{k_1} = \frac{1 \pm i\sqrt{4\tau \cos \theta - 1}}{2\tau \cos \theta} \quad (2.9)
\]

or

\[
k_2, k_1 = \frac{1 - 2\tau \cos \theta \pm i\sqrt{4\tau \cos \theta - 1}}{2\tau \cos^2 \theta} \nu \quad (2.10)
\]

Thus the integrand in the integral

\[
\int_0^\infty \left[ \int \gamma F(\theta, k) dk \right] d\theta
\]

contains no real singular points in the path of integration from 0 to $\infty$.

3. EVALUATION OF INTEGRALS.

The Green’s function given in equation (2.1) is difficult to integrate numerically because as we have seen in the previous section, the contours $L_1$ and $L_2$ have singularities at $k_1, k_2, k_3$ and $k_4$. This difficulty can be overcome by introducing the Cauchy Principal Value (PV) integrals.
The first contour integral along the path $L_1$ can be rewritten as

$$G_{L_1} = \frac{2\pi}{\pi} \int_0^{\pi/2} \int_{L_1} \frac{d\theta}{\sqrt{1 - \cos^2 \theta}} F(\theta, k) dk$$

$$= \frac{2\pi}{\pi} \int_0^{\pi/2} \left[ \int_{k_1+\epsilon}^{k_2-\epsilon} + \int_{k_1+\epsilon}^{k_2+\epsilon} \int_{0}^{\pi} F(\theta, k) dk \right]$$

(3.1)

When $\epsilon \to 0$, equation (3.1) can be written as

$$G_{L_1} = \frac{2\pi}{\pi} \int_0^{\pi/2} \int_{0}^{\pi} F(\theta, k) dk$$

where

$$\int_{0}^{\pi} F(\theta, k) dk = \lim_{\epsilon \to 0} \left[ \int_{k_1+\epsilon}^{k_2-\epsilon} \int_{0}^{\pi} F(\theta, k) dk \right]$$

(3.2)

To evaluate the integral along the deformations $\zeta$ and $\zeta'$, we decompose the integral $F(\theta, k)$ in terms of its singularities. We write

$$F(\theta, k) = \frac{1}{2g/\sqrt{1-4\epsilon \cos \theta}} \left[ \frac{k_1 - k_2}{k-k_1-k-k_2} \right] \{ \exp[k((z+c) + i(x-a)\cos \theta)] \cos(k(y-b)\sin \theta) \}$$

(3.3)

which can be put in the following compact form

$$F(\theta, k) = \frac{1}{2g/\sqrt{1-4\epsilon \cos \theta}} \left[ \frac{k_1 - k_2}{k-k_1-k-k_2} \right] [\exp(kx_1) + \exp(kx_2)]$$

(3.4)

where

$$x_1 = (z+c) + i\omega_+, x_2 = (z+c) + i\omega_-$$

(3.5)

We know that $(z+c)\in\mathbb{R}$ so we can redefine $x_1$ and $x_2$ as follows

$$x_1 = (z+c) + i\omega_+, x_2 = (z+c) + i\omega_-.$$  

(3.6)

Thus, equation (3.5) can be rewritten as

$$F(\theta, k) = \frac{1}{2g/\sqrt{1-4\epsilon \cos \theta}} \left[ \frac{k_1 \{ \exp(-kx_1) + \exp(-kx_2) \} - k_2 \{ \exp(-kx_1) + \exp(-kx_2) \}}{k-k_1-k-k_2} \right]$$

(3.7)

The integration along the deformations $\zeta$ and $\zeta'$ in equation (3.2) can be obtained according to the residue theorem. Thus

$$\int_{\zeta} \int_{\zeta'} F(\theta, k) dk$$

$$= \frac{\pi}{2g/\sqrt{1-4\epsilon \cos \theta}} \left( k_1 \{ \exp(-kx_1) + \exp(-kx_2) \} + k_2 \{ \exp(-kx_1) + \exp(-kx_2) \} \right)$$

(3.8)

Thus, equation (3.9) reduces to
In a similar manner the second contour integral along path $L_2$ equation (2.1) can be obtained

$$G_{L_2} = \frac{2\pi}{\gamma} \int_0^{\pi/2} d\theta \int_0^{\infty} F(\theta,k) dk + \frac{2\pi}{\gamma} \int_0^{\pi/2} d\theta \int_0^{\infty} F(\theta,k) dk + \frac{1}{\sqrt{1-4\cos^2 \theta}} \int_0^{\pi/2} \{k_1(e^{-k_1x_1} + e^{-k_1x_2}) + k_2(e^{-k_2x_1} + e^{-k_2x_2})\} d\theta \tag{3.11}$$

Therefore the Greens function in equation (2.1) can be rewritten as follows:

$$G_1(x,y,z; a,b,c) = \frac{1}{R} - \frac{1}{R_1} + \frac{2\pi}{\gamma} \int_0^{\pi/2} d\theta \int_0^{\infty} F(\theta,k) dk + \frac{2\pi}{\gamma} \{\int + \int\} d\theta \int_0^{\infty} F(\theta,k) dk + \frac{1}{\sqrt{1-4\cos^2 \theta}} \int_0^{\pi/2} \{k_1(e^{-k_1x_1} + e^{-k_1x_2}) + k_2(e^{-k_2x_1} + e^{-k_2x_2})\} d\theta \tag{3.12}$$

or,

$$G = G_1 + G_2 + G_3 + G_4 + iG_5 \tag{3.13}$$

where

$$G_1 = \frac{1}{R}, \quad G_2 = -\frac{1}{R_1}$$

$$G_3 = \frac{2\pi}{\gamma} \int_0^{\pi/2} d\theta \int_0^{\infty} F(\theta,k) dk, \quad G_4 = \frac{2\pi}{\gamma} \{\int + \int\} d\theta \int_0^{\infty} F(\theta,k) dk$$

$$G_5 = \frac{\pi/2}{\gamma \sqrt{1-4\cos^2 \theta}} \{k_1(e^{-k_1x_1} + e^{-k_1x_2}) + k_2(e^{-k_2x_1} + e^{-k_2x_2})\} d\theta + \frac{\pi/2}{\gamma \sqrt{1-4\cos^2 \theta}} \{k_3(e^{-k_3x_1} + e^{-k_3x_2}) - k_4(e^{-k_4x_1} + e^{-k_4x_2})\} d\theta \tag{3.14}$$

There are two cases to be considered.

Case I

$$y = 0 \quad \text{if} \quad \tau < \frac{1}{4}$$

and in this case $G_3 = 0$.

Therefore, equation (3.13) becomes

$$G = G_1 + G_2 + G_4 + iG_5 \tag{3.15}$$
Case II

\[ \gamma = \cos^{-1} \frac{1}{4 \tau} \quad \text{if } \tau > \frac{1}{4} \]

and in this case equation (3.13) becomes

\[ G = G_1 + G_2 + G_3 + G_4 + iG_5 \quad (3.16) \]

The double integrals in \( G_3 \) and \( G_4 \) are highly oscillatory at large values of \( k \) because of the imaginary argument of the exponential function. In order to calculate them numerically, at minimum computer cost, these integrals must be reduced to single integrals as suggested by Shen and Farell [5], and Inglis and Price [6]. We shall treat Case I first and evaluate the Cauchy Principal Value (P.V.) integral in \( G_4 \).

The term \( G_4 \) of the Green function can be written as

\[ G_4 = \frac{1}{\pi} \int_{\gamma-\frac{\pi}{2}}^{\gamma+\frac{\pi}{2}} \frac{d\theta}{\sqrt{1-4 \cos \theta}} (I_1 + I_2 - I_3 - I_4) + \frac{1}{\pi} \int_{\gamma-\frac{\pi}{2}}^{\gamma+\frac{\pi}{2}} \frac{d\theta}{\sqrt{1-4 \cos \theta}} (I_5 + I_6 - I_7 - I_8) \quad (3.17) \]

where

\[ I_1 = \text{(P.V.)} \int_{0}^{\frac{\pi}{2}} \frac{k_1 \exp(-kx_1)}{k-k_1} \, dk, \quad I_2 = \text{(P.V.)} \int_{0}^{\frac{\pi}{2}} \frac{k_1 \exp(-kx_2)}{k-k_1} \, dk \quad (3.18) \]

\[ I_3 = \text{(P.V.)} \int_{0}^{\frac{\pi}{2}} \frac{k_2 \exp(-kx_3)}{k-k_2} \, dk, \quad I_4 = \text{(P.V.)} \int_{0}^{\frac{\pi}{2}} \frac{k_2 \exp(-kx_4)}{k-k_2} \, dk \]

for \( \gamma < \theta < \frac{\pi}{2} \)

\[ I_5 = \text{(P.V.)} \int_{0}^{\frac{\pi}{2}} \frac{k_3 \exp(-kx_5)}{k-k_3} \, dk, \quad I_6 = \text{(P.V.)} \int_{0}^{\frac{\pi}{2}} \frac{k_3 \exp(-kx_6)}{k-k_3} \, dk \quad (3.19) \]

\[ I_7 = \text{(P.V.)} \int_{0}^{\frac{\pi}{2}} \frac{k_4 \exp(-kx_7)}{k-k_4} \, dk, \quad I_8 = \text{(P.V.)} \int_{0}^{\frac{\pi}{2}} \frac{k_4 \exp(-kx_8)}{k-k_4} \, dk \]

for \( \frac{\pi}{2} < \theta < \pi \).

To obtain analytic expressions of these integrals, we consider a contour in the \( K = k + ik' \) plane as suggested by Smith et al [7] (see Figure 2) and later used by Chen et al [8].

We impose the condition that

\[ I_m[-Kx_1] = 0 \]

on the integration path 5 which makes an angle \( \alpha \) with the real axis, so that the argument of the exponential can be made real along the ray.

Therefore, we get

\[ I_m[-(k + ik')] (|z+c|-iw+) = 0 \]

which simplifies to yield

\[ k' = \frac{w_+}{|z+c|} \]

and

\[ \alpha = \tan^{-1} \left( \frac{k'}{k} = \frac{1 - \frac{w_+}{|z+c|}}{\frac{w_+}{|z+c|}} \right) \]
Thus with this value of \( \alpha \),

\[
R \left[ -(k + ik') \left( |z+c| -iw_+ \right) \right] = -k \frac{|z+c|^2 + w_+^2}{|z+c|} = -kV < 0
\]

Also, we have

\[
K = k + ik' = \frac{kV}{|z+c|-iw_+}
\]

Integrating along the contour shown in Figure 2,

\[
I_1 = 2\pi i \left( k_1 \exp(-k_1x_1) \right) - \pi i k_1 \exp(-k_1x_1) - \int_5^k \frac{k_1 \exp(-k_1x_1)}{k-k_1} \, dk
\]

\[
= (\pi i) k_1 \exp(-k_1x_1) - \int_5^k \frac{k_1 \exp(-k_1x_1)}{k-k_1} \, dk
\]

Along the path 5

\[
\int_5^k \frac{\exp(-k_1x_1)}{k_1x_1} \, d(kV) = - k_1 \int_0^\infty \frac{\exp(-u)}{u - k_1x_1} \, du
\]

\[
= - k_1 \exp(-k_1x_1) E_1(-k_1x_1)
\]

where

\[
E_1(-z) = \int_{-z}^{\infty} \frac{e^{-t}}{t} \, dt, \quad \text{arg}(z) < \pi = \text{exponential integral}.
\]

Therefore, for \( w_+ > 0 \),

\[
I_1 = k_1 \exp(-k_1x_1) \left( E_1(-k_1x_1) + \pi i \right)
\]

(3.20)

It is to be noted here that for \( w_+ < 0 \) the contour will be as follows:
Thus
\[ I_1 = k_1 \exp(-k_1 x_1) \{E_1(-k_1 x_1) - \pi i\} \] (3.21)

Also, for \( w_+ = 0 \),
\[ I_1 = k_1 \exp(-k_1 x_1) \{E_1(-k_1 x_1) + \pi i\} \] (3.22)

which is obtained using the following definitions (see Abramowitz and Stegun [9] 1965, p. 228).
\[ E_1(-x+i0) = (P.V.) \int_{-\infty}^{x} \frac{e^{-t}}{t} \, dt - \pi i, \quad E_1(-x-i0) = (P.V.) \int_{-\infty}^{x} \frac{e^{-t}}{t} \, dt + \pi i \]

such that for \( w_+ = 0 \)
\[ I_1 = k_1 \exp(-k_2 x_1) \{E_1(-k_1 x_1) + \pi i\} \] (3.23)

Similarly, we can calculate the other integrals. Thus summing up the situation, we get for \( w_+ > 0 \) or \( I_{m}(-k_1 x_1) > 0 \)
\[ I_1 = k_1 \exp(-k_1 x_1) \{E_1(-k_1 x_1) + \pi i\} \]

and for \( w_+ < 0 \) or \( I_{m}(-k_1 x_1) < 0 \)
\[ I_1 = k_1 \exp(-k_1 x_1) \{E_1(-k_1 x_1) - \pi i\} \]

Similarly,
\[ I_2 = k_1 \exp(-k_1 x_2) \{E_1(-k_1 x_2) + \pi i\}, \text{ for } I_{m}(-k_1 x_2) > 0 \]
\[ k_1 \exp(-k_1 x_2) \{E_1(-k_1 x_2) - \pi i\}, \text{ for } I_{m}(-k_1 x_2) < 0 \]
\[ I_3 = k_2 \exp(-k_2 x_1) \{E_1(-k_2 x_1) - \pi i\}, \text{ for } I_{m}(-k_2 x_1) > 0 \]
\[ k_2 \exp(-k_2 x_1) \{E_1(-k_2 x_1) + \pi i\}, \text{ for } I_{m}(-k_2 x_1) < 0 \]
THREE DIMENSIONAL GREEN'S FUNCTION FOR SHIP MOTION AT FORWARD SPEED

\[ I_4 = k_2 \exp(-k_2 x_2) [E_1(-k_2 x_2) - \pi i], \text{ for } I_m(-k_2 x_2) > 0 \]
\[ k_2 \exp(-k_2 x_2) [E_1(-k_2 x_2) + \pi i], \text{ for } I_m(-k_2 x_2) < 0 \]
\[ I_5 = k_3 \exp(-k_3 x_1) [E_1(-k_3 x_1) + \pi i], \text{ for } I_m(-k_3 x_1) > 0 \]
\[ k_3 \exp(-k_3 x_1) [E_1(-k_3 x_1) - \pi i], \text{ for } I_m(-k_3 x_1) < 0 \]
\[ I_6 = k_3 \exp(-k_3 x_2) [E_1(-k_3 x_2) + \pi i], \text{ for } I_m(-k_3 x_2) > 0 \]
\[ k_3 \exp(-k_3 x_2) [E_1(-k_3 x_2) - \pi i], \text{ for } I_m(-k_3 x_2) < 0 \]
\[ I_7 = k_4 \exp(-k_4 x_1) [E_1(-k_4 x_1) + \pi i], \text{ for } I_m(-k_4 x_1) > 0 \]
\[ k_4 \exp(-k_4 x_1) [E_1(-k_4 x_1) - \pi i], \text{ for } I_m(-k_4 x_1) < 0 \]
\[ I_8 = k_4 \exp(-k_4 x_2) [E_1(-k_4 x_2) + \pi i], \text{ for } I_m(-k_4 x_2) > 0 \]
\[ k_4 \exp(-k_4 x_2) [E_1(-k_4 x_2) - \pi i], \text{ for } I_m(-k_4 x_2) < 0 \]  

(3.24)

Now adding the terms in \( G_4 \) and \( G_5 \) given by the equations (3.17) and (3.14), respectively, we obtain

\[
G_4 + iG_5 = \frac{1}{\pi} \int_0^{\pi/2} \frac{1}{\sqrt{1 - 4 \cos \theta}} (I_1 + I_2 - I_3 - I_4 + \pi i (I_{11} + I_{12} + I_{21} + I_{22})) d\theta
\]
\[
+ \frac{1}{\pi} \int_0^{\pi} \frac{1}{\sqrt{1 - 4 \cos \theta}} ((I_5 + I_6 - I_7 - I_8) + \pi i (I_{31} + I_{32} - I_{41} - I_{42})) d\theta
\]

(3.25)

where

\[ I_{ij} = k_j \exp(-k_j x_j), j = 1, 2, 3, 4; j = 1, 2. \]

Thus, if we combine the corresponding integrands of \( G_4 + iG_5 \), we obtain

First Integral = \[
\frac{1}{\pi} \int_0^{\pi/2} \frac{1}{\sqrt{1 - 4 \cos \theta}} [E_1(-k_1 x_1) + 2\pi i] d\theta, \text{ for } I_m(-k_1 x_1) > 0
\]
\[
\frac{1}{\pi} \int_0^{\pi/2} \frac{1}{\sqrt{1 - 4 \cos \theta}} E_1(-k_1 x_1) d\theta, \text{ for } I_m(-k_1 x_1) < 0
\]

Second Integral = \[
\frac{1}{\pi} \int_0^{\pi/2} \frac{1}{\sqrt{1 - 4 \cos \theta}} [E_1(-k_1 x_2) + 2\pi i] d\theta, \text{ for } I_m(-k_1 x_2) > 0
\]
\[
\frac{1}{\pi} \int_0^{\pi/2} \frac{1}{\sqrt{1 - 4 \cos \theta}} E_1(-k_1 x_2) d\theta, \text{ for } I_m(-k_1 x_2) < 0
\]
Therefore, for Case I, we can evaluate the Green's function given in equation (3.15).

To evaluate the Green's function for Case II given in equation (3.16), we need to express the $G_3$ term in exponential integrals as given below:

$$G_3 = \frac{2\pi}{\pi} \int_0^{\pi/2} \int \frac{k e^{k \left[ (z+c) + i(x-a) \cos \theta \right]} \cos[k(y-b) \sin \theta]}{\cos^2 \theta} dk d\theta$$
\[
\frac{1}{\pi i} \int_0^\gamma \frac{1}{\sqrt{4 \cos \theta - 1}} (J_1 + J_2 - J_3 - J_4) d\theta
\]

where

\[
J_1 = \int_0^\infty \frac{k_l}{k - k_1} \exp(-k x_1) dk, \quad J_2 = \int_0^\infty \frac{k_1}{k - k_1} \exp(-k x_2) dk
\]

\[
J_3 = \int_0^\infty \frac{k_2}{k - k_2} \exp(-k x_1) dk, \quad J_4 = \int_0^\infty \frac{k_2}{k - k_2} \exp(-k x_2) dk
\]

and \(k_1\) and \(k_2\) are the complex roots of

\[
gk - (\omega + kU \cos \theta)^2 = 0.
\]

Using the contour in Figure 2, it can be easily shown that

\[
J_1 = k_l \exp(-k_1 x_1) E_1(-k_1 x_1) \quad I_m(-k_1 x_1) > 0 \quad k_1 \exp(-k_1 x_1) [E_1(-k_1 x_1) - 2\pi i] \quad I_m(-k_1 x_1) < 0
\]

\[
J_2 = k_1 \exp(-k_1 x_2) E_1(-k_1 x_2) \quad I_m(-k_1 x_2) > 0 \quad k_1 \exp(-k_1 x_2) [E_1(-k_1 x_2) - 2\pi i] \quad I_m(-k_1 x_2) < 0
\]

\[
J_3 = -k_2 \exp(-k_2 x_1) E_1(-k_2 x_1) \quad I_m(-k_2 x_1) > 0 \quad k_2 \exp(-k_2 x_1) [E_1(-k_2 x_2) + 2\pi i] \quad I_m(-k_2 x_1) < 0
\]

\[
J_4 = k_2 \exp(-k_2 x_2) E_1(-k_2 x_2) \quad I_m(-k_2 x_2) > 0 \quad -k x \ E(-k x)
\]

Thus, with this information, we can evaluate the Green's function for Case II from equation (3.16).

4. RESULTS AND CONCLUSIONS.

The present form of Green's function is equivalent to that used by Wu and Taylor, but in a different form. The terms \(G_1\), \(G_2\) and \(G_3\) are all identical to those used by Chen et al [8]. However, in the present study, we have combined the \(G_4\) and \(G_5\) terms to correspond with the form of Wu and Taylor. It appears that our studies are quite similar to those of Wu and Taylor, and Chen et al.

The double integral arising in the evaluation of Green's function has been replaced by a single integral with the use of complex exponential integrals. The present work has provided an alternative form but similar to that of Wu and Taylor, and has been found to be efficient for the analysis of the three dimensional potential problem of ship motion with forward speed.
ACKNOWLEDGEMENTS. This work has been performed under Contract No. OSC87-00549-(010) with the Defence Research Establishment Atlantic while the author was on Sabbatical Leave from the Technical University of Nova Scotia.

REFERENCES

This subject has been extensively studied in the past years for one-, two-, and three-dimensional space. Additionally, such dynamical systems can exhibit a very important and still unexplained phenomenon, called as the Fermi acceleration phenomenon. Basically, the phenomenon of Fermi acceleration (FA) is a process in which a classical particle can acquire unbounded energy from collisions with a heavy moving wall. This phenomenon was originally proposed by Enrico Fermi in 1949 as a possible explanation of the origin of the large energies of the cosmic particles. His original model was then modified and considered under different approaches and using many versions. Moreover, applications of FA have been of a large broad interest in many different fields of science including plasma physics, astrophysics, atomic physics, optics, and time-dependent billiard problems and they are useful for controlling chaos in Engineering and dynamical systems exhibiting chaos (both conservative and dissipative chaos).

We intend to publish in this special issue papers reporting research on time-dependent billiards. The topic includes both conservative and dissipative dynamics. Papers discussing dynamical properties, statistical and mathematical results, stability investigation of the phase space structure, the phenomenon of Fermi acceleration, conditions for having suppression of Fermi acceleration, and computational and numerical methods for exploring these structures and applications are welcome.

To be acceptable for publication in the special issue of Mathematical Problems in Engineering, papers must make significant, original, and correct contributions to one or more of the topics above mentioned. Mathematical papers regarding the topics above are also welcome.

Authors should follow the Mathematical Problems in Engineering manuscript format described at http://www.hindawi.com/journals/mpe/. Prospective authors should submit an electronic copy of their complete manuscript through the journal Manuscript Tracking System at http://mts.hindawi.com/ according to the following timetable:

<table>
<thead>
<tr>
<th>Manuscript Due</th>
<th>December 1, 2008</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Round of Reviews</td>
<td>March 1, 2009</td>
</tr>
<tr>
<td>Publication Date</td>
<td>June 1, 2009</td>
</tr>
</tbody>
</table>

**Guest Editors**

**Edson Denis Leonel**, Departamento de Estatística, Matemática Aplicada e Computação, Instituto de Geociências e Ciências Exatas, Universidade Estadual Paulista, Avenida 24A, 1515 Bela Vista, 13506-700 Rio Claro, SP, Brazil; edleonel@rc.unesp.br

**Alexander Loskutov**, Physics Faculty, Moscow State University, Vorob’evy Gory, Moscow 119992, Russia; loskutov@chaos.phys.msu.ru