LOCAL CONNECTIVITY AND MAPS ONTO NON-METRIZABLE ARCS

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ABSTRACT. Three classes of locally connected continua which admit sufficiently many maps onto non-metric arcs are investigated. It is proved that all continua in those classes are continuous images of arcs and, therefore, have other quite nice properties.

KEY WORDS AND PHRASES: arc, locally connected continuum, monotonically normal, rim-countable, rim-finite, rim-metrizable, rim-scattered

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INTRODUCTION

Let $C$ denote the class of all Hausdorff continuous images of ordered continua. In the last three decades the class $C$ has been studied extensively by a number of authors (see e.g. [2], [4], [6–8], [11–13], [16–22], [26] and [27]). Two results from this study have suggested that the investigation could naturally be extended to the larger class $\mathcal{R}_M$ of all rim-metrizable, locally connected continua. Namely, (1) in [8] in 1967 Mardešić proved that each element of $C$ has a basis of open $F_\sigma$-sets with metrizable boundaries, and (2) in [4] in 1991 Grispolakis, Nikiel, Simone and Tymchatyn showed that if a set $P$ is irreducible with respect to the property of being a compact set which separates the element $X$ of $C$, then $P$ is metrizable.
In his 1989 thesis [23] and two subsequent papers [24] and [25] Tuncali began an investigation of the class $\mathcal{R}_M$ and continuous images of elements of that class. He showed that Treybig's product theorem of [18] which holds in $C$ is no longer valid in $\mathcal{R}_M$. However, he proved that Mardešić's theorem for $C$ on preservation of weight by light mappings is true in $\mathcal{R}_M$, [25]. He also considered the class $\mathcal{R}_S$ of all rim-scattered, locally connected continua, and the class $\mathcal{R}_C$ of all rim-countable, locally connected continua. Later, Nikiel, Tuncali and Tymchatyn gave an example to show that $\mathcal{R}_C$ is not a subclass of $C$, [15]. Then, recently the authors of this paper showed the the continuous image of an element of $\mathcal{R}_M$ need not be in $\mathcal{R}_M$, [14]. Furthermore, Drozdovsky and Filippov proved in [3] that $\mathcal{R}_S$ is a larger class of spaces than $\mathcal{R}_C$.

Also, in 1973 Heath, Lutzer and Zenor, [5], showed that every linearly ordered ordered topological space and each of its Hausdorff continuous and closed images are monotonically normal. In [10] in 1986 Nikiel asked if every monotonically normal compactum is the continuous image of a compact ordered space. That problem still remains open. In what follows we let $\mathcal{R}_{MN}$ denote the class of monotonically normal, locally connected continua. Our first result is the following:

**THEOREM 1.** If $X \in \mathcal{R}_M \cup \mathcal{R}_S \cup \mathcal{R}_{MN}$ and for each pair of points $a, b \in X$ there exists a continuous onto map $f: X \to [c, d]$ such that $f(a) = c, f(b) = d$ and $[c, d]$ is a non-metrizable arc, then $X \notin C$.

We note that a large class of examples satisfying the properties of $X$ above can be constructed as follows: In [1] in 1945 Arens studied the class $\mathcal{L}$ of linear homogeneous continua, that is the class of arcs which are order isomorphic to each of their subarcs. Arens showed, that up to a homeomorphism, there exist at least $\aleph_1$ members of $\mathcal{L}$, including the real numbers interval $[0, 1]$. Thus, some spaces $X$ as in Theorem 1 could be obtained by pasting together copies of any $Z \in \mathcal{L}$.

If a subset $B$ of a space $P$ contains no dense-in-itself, non-empty subset, we say that $B$ is scattered.

In this paper the definition of monotone normality we use is an equivalent one given in Lemma 2.2 (a) of [5]. It says that a space $P$ is monotonically normal provided there is an operator $G$ which assigns to each ordered pair $(S, T)$ of mutually separated subsets of $P$ an open set $G(S, T)$ such that

(i) $S \subset G(S, T) \subset \text{cl}(G(S, T)) \subset P - T$, and

(ii) if $(S', T')$ is also a pair of mutually separated sets such that $S \subset S'$ and $T' \subset T$, then $G(S, T) \subset G(S', T')$.

**PROOF OF THEOREM 1.** Suppose that $X$ is not hereditarily locally connected. Then, there exists a subcontinuum $C$ of $X$ such that $C$ fails to be connected im kleinen at the point $p$. Utilizing the ideas in Theorem 11, p. 90, of Moore [9], there exists a connected open set $U$ containing $p$, a sequence $R_1, R_2, R_3, \ldots$ of connected open in $X$ sets containing $p$, and a sequence
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\(G_1, G_2, G_3, \ldots\) of continua such that

1. \(U \supseteq \overline{R_1} \supseteq R_1 \supseteq \overline{R_2} \supseteq R_2 \supseteq \cdots\);
2. \(G_n \cap R_n \neq \emptyset\) and \(G_n \cap R_{n+1} = \emptyset\) for \(n = 1, 2, 3, \ldots\);
3. each \(G_n\) is a component of \(\overline{U} \cap C\) and \(G_n \cap \text{bd}(U) \neq \emptyset\) for \(n = 1, 2, 3, \ldots\); and
4. \(G_n \cap G_m = \emptyset\) if \(n \neq m\), and there exist mutually exclusive open sets \(V_1, V_2, V_3, \ldots\) such that \(G_n \subset V_n\) for \(n = 1, 2, 3, \ldots\).

For each positive integer \(n\) let \(H_n\) be a component of \(G_n - R_1\) which intersects \(\text{bd}(R_1)\) and \(\text{bd}(U)\), and let \(s_n \in H_n \cap \text{bd}(R_1)\) and \(t_n \in H_n \cap \text{bd}(U)\). Let \(H_0\) denote the limiting set of the sequence \(H_1, H_2, H_3, \ldots\), which by definition is the set of all \(x\) such that every open set containing \(x\) intersects infinitely many sets \(H_n\).

Let \(L_1\) (resp. \(L_2\)) denote the limiting set of \(\{s_1\}, \{s_2\}, \{s_3\}, \ldots\) (resp. \(\{t_1\}, \{t_2\}, \{t_3\}, \ldots\)). There exists \((s, t) \in L_1 \times L_2\) so that if \(V\) is a neighborhood of \(s\) and \(W\) is a neighborhood of \(t\), then \((s_n, t_n)\) belongs to \(V \times W\) for infinitely many \(n\).

We shall show that some component of \(H_0\) contains \((s, t)\). If not, then \(H_0\) is the union of two mutually separated sets \(S\) and \(T\) such that \(s \in S\) and \(t \in T\). There exist disjoint open sets \(V\) and \(W\) so that \(S \subset V\) and \(T \subset W\). Then \((s_n, t_n)\) belongs to \(V \times W\) for infinitely many \(n\). Since each \(H_n\) is a continuum, \(H_n \cap (X - (V \cup W)) \neq \emptyset\) for infinitely many \(n\). It follows that some point of \(H_0\) lies in \(X - (V \cup W)\), a contradiction.

Let \(f : X \rightarrow [c, d]\) be a continuous map onto a non-metrizable arc \([c, d]\), where \(f(s) = c\) and \(f(t) = d\). There is an increasing sequence \(n_1, n_2, n_3, \ldots\) of positive integers such that

1. \(f(s_{n_i}) \geq f(s_{n_{i+1}})\) and \(f(t_{n_i}) \leq f(t_{n_{i+1}})\) for \(i = 1, 2, \ldots\);
2. \(f(s_{n_i}) \rightarrow c\) and \(f(t_{n_i}) \rightarrow d\); and
3. \([f(s_{n_i}), f(t_{n_i})]\) is not metrizable for \(i = 1, 2, \ldots\).

Let \(c' = f(s_{n_1})\) and \(d' = f(t_{n_1})\).

Our proof now divides into three cases.

CASE 1. \(X \in \mathcal{R}_M\). For each \(n \geq 2\) let \(M_n\) denote a metrizable closed set lying in \(X - \bigcup_{k=1}^{n} H_k\) such that if \(1 \leq i < j \leq n\), then \(H_i\) and \(H_j\) are separated in \(X\) by \(M_n\). Let \(D_n\) denote a countable set dense in \(M_n\) for \(n = 2, 3, \ldots\) We intend to show that \(f\left(\bigcup_{k=1}^{\infty} D_k\right)\) is dense in \([c, d]\), which would mean that \([c, d]\) is separable, and therefore metric, a contradiction.

Let \(x \in [c, d]\) and let \(c < u < x < v < d\) in the natural ordering of \([c, d]\). The components of \(f^{-1}([u, v])\) which have limit points in both \(f^{-1}(u)\) and \(f^{-1}(v)\) can be labeled \(P_1, P_2, \ldots, P_{n_0}\). Let \(N_0\) be an integer such that if \(i \geq N_0\) then \(s_n, t_n \in f^{-1}([u, v])\). There exist two of \(N_0, N_0 + 1, \ldots, N_0 + n_0, n_0\), say \(i\) and \(j\), such that \(H_i\) and \(H_j\) both intersect the same \(P_t\), which must then intersect some \(D_m\). Therefore, \(\bigcup_{k=2}^{\infty} f(D_k)\) intersects \([u, v]\).

CASE 2. \(X \in \mathcal{R}_{MN}\). For each \(i = 1, 2, \ldots\) let \(Q_i\) denote a component of \(H_n \cap f^{-1}([c', d')]\) which intersects \(f^{-1}(c')\) and \(f^{-1}(d')\), and let \(Q_0\) denote the limiting set of \(Q_1, Q_2, Q_3, \ldots\). We note that some component of \(Q_0\) intersects both \(f^{-1}(c')\) and \(f^{-1}(d')\) since every map onto an arc is weakly confluent.
By Remark 2.3 (c) of [5], \( Z = \bigcup_{n=0}^{\infty} Q_n \) is monotonically normal; so let \( G \) be a monotone normality operator on \( Z \) as in the earlier definition. For each closed set \( F \) in \([c',d']\) let \( Q_F = \{ x : f(x) \in F \text{ and } x \in Z - Q_0 \} \), and let \( R_F = \{ x : f(x) \in [c',d'] - F \text{ and } x \in Q_0 \} \). Now, \( Q_F \) and \( R_F \) are mutually separated subsets of \( Z \); so for each positive integer \( n \), let 
\[ T(F,n) = \{ y \in [c',d'] : y = f(x) \text{ for some } x \in Q_n \cap G(Q_F,R_F) \}. \]
It can be shown that \( T \) is a stratification for \([c',d']\). Since each stratifiable compact space is metrizable, \([c',d']\) is metrizable, a contradiction.

**CASE 3.** \( X \in \mathcal{R}_G \). For each \( i = 1, 2, 3, \ldots \) let \( K_i \) denote a component of \( H_n \cap f^{-1}([c',d']) \) which intersects \( f^{-1}(c') \) and \( f^{-1}(d') \).

We have to consider some subcases.

**CASE 3A.** \([c',d']\) contains uncountably many mutually exclusive open sets.

**CASE 3A1.** \([c',d']\) does not satisfy the first axiom of countability. Thus, without loss of generality, assume that there is a subset \( \{ d_\alpha : \alpha < \omega_1 \} \) of \([c',d']\) such that \( \alpha_1 < \alpha_2 \) implies that \( d_\alpha_1 < d_\alpha_2 \) in \([c',d']\), and \( d_\alpha \rightarrow d' \).

Let \( K_0 \) denote the limiting set of \( K_1, K_2, K_3, \ldots \). Let \( Q \) denote a component of \( K_0 \) which intersects both \( f^{-1}(c') \) and \( f^{-1}(d') \). For each \( \alpha < \omega_1 \) let \( W_\alpha \) denote a connected open set such that \( W_\alpha \) contains a point \( x_\alpha \) of \( Q \cap f^{-1}(\{d_\alpha, d_{\alpha+1}\}) \), and \( \overline{W_\alpha} \subset f^{-1}(\{d_\alpha, d_{\alpha+1}\}) \).

There exists a positive integer \( n_0 \) and a cofinal subsequence \( \{ d_{\alpha_\gamma} \} \) of \( d_\alpha \) such that \( W_{\alpha_\gamma} \cap K_{n_0} \neq \emptyset \) for all \( \alpha_\gamma \). For each \( \gamma < \omega_1 \) let \( L_\gamma \) denote the closure of the set \( \bigcup_{\beta < \gamma} W_{\alpha_\beta} \). Let \( L = \bigcap_{\gamma < \omega_1} L_\gamma \). Observe that if \( y \in L \), then each open neighborhood of \( y \) intersects uncountably many sets \( W_{\alpha_\gamma} \). Let \( W \) be a component of \( L \). Note that \( W \cap K_{n_0} \neq \emptyset \neq Q \cap W \) and \( W \subset f^{-1}(d') \). Thus, \( W \) is a non-degenerate continuum.

Let \( M_0 \) and \( M_1 \) be connected open sets such that \( \overline{M_0} \cap \overline{M_1} = \emptyset \) and \( M_i \cap W \neq \emptyset \) for \( i = 0, 1 \). Let \( \mathcal{G}_1 = \{ M_0, M_1 \} \).

Now suppose that \( \mathcal{G}_n \) has been chosen and consists of \( 2^n \) mutually exclusive connected open sets such that if \( G, G' \in \mathcal{G}_n \) and \( G \neq G' \), then \( \overline{G} \cap \overline{G'} = \emptyset \) and \( G \cap W \neq \emptyset \neq G' \cap W \). For each \( G' \in \mathcal{G}_n \) let \( G'_0 \) and \( G'_1 \) be mutually exclusive connected open sets such that \( \overline{G'_0} \cap \overline{G'_1} = \emptyset \), \( \overline{G'_0} \cup \overline{G'_1} \subset G' \) and \( G'_0 \cap W \neq \emptyset \neq G'_1 \cap W \). Let \( \mathcal{G}_{n+1} = \{ F : F = G'_0 \text{ or } F = G'_1 \text{ for some } G' \in \mathcal{G}_n \} \). For each \( n \) let \( H_n = \bigcup \mathcal{G}_n \) and let \( H = \bigcap_{n=1}^{\infty} H_n \).

There exists \( \delta_0 < \omega_1 \) such that \( G' \cap f^{-1}(d_{\delta_0}) \neq \emptyset \) for each \( G' \in \bigcup_{j=1}^{\infty} \mathcal{G}_j \). There exists a closed scattered set \( S \) in \( X \) which separates \( f^{-1}([c,d_n]) \) from \( f^{-1}(d') \). However, \( S \cap H \) contains a perfect set because \( S \cap H \) can be mapped onto a Cantor set, and it is well known that a scattered set cannot be mapped continuously onto a perfect set. This is a contradiction.

**CASE 3A2.** \([c',d']\) satisfies the first axiom of countability at each point. Let \( \{ [c_{\alpha}, d_{\alpha}] : \alpha < \omega_1 \} \) denote an uncountable collection of mutually exclusive open intervals in \([c',d']\). Using the local connectivity of \( X \) we find that for each \( \alpha \) there exists only a finite number, say \( n_\alpha \), of components of \( f^{-1}([c_{\alpha}, d_{\alpha}]) \) which have limit points in both \( f^{-1}(c_{\alpha}) \) and \( f^{-1}(d_{\alpha}) \). Some integer \( N_0 = n_\alpha \) repeats for uncountably many \( \alpha \)'s; so we may suppose without loss of generality that
There exists a closed scattered set $S$ such that $S$ separates $K_i$ from $K_j$ for each pair $i,j$ such that $1 \leq i < j \leq N_0 + 1$. Thus, since for each $\alpha$, each set $K_i$ where $1 \leq i \leq N_0 + 1$ has the property that some component of $K_i \cap f^{-1}([c_\alpha, d_\alpha])$ has limit points in both $f^{-1}(c_\alpha)$ and $f^{-1}(d_\alpha)$, it follows that $S$ must intersect each $f^{-1}([c_\alpha, d_\alpha])$.

Since $[c', d']$ is first countable, there exist collections $G_1, G_2, G_3, \ldots$ such that (1) each $G_n$ consists of $2^n$ mutually exclusive closed intervals in $[c', d']$, and (2) each element of each $G_n$ contains exactly two elements of $G_{n+1}$ and contains uncountably many elements of $\{[c_\alpha, d_\alpha]\}$. For each positive integer $n$ let $L_n = \bigcup G_n$, and let $L' = \bigcap_{n=1}^\infty L_n$. We find that $S \cap f^{-1}(L')$ contains a perfect set, a contradiction.

CASE 3B. $[c', d']$ is not metrizable and does not contain uncountably many mutually exclusive open sets (i.e., it is a Souslin line). Thus, $[c', d']$ satisfies the first axiom of countability. If there exists a collection of metrizable open intervals whose union is dense in $[c', d']$, we find that $[c', d']$ is metrizable since it is separable. Hence, without loss of generality we may assume that $[c', d']$ contains no metrizable subinterval.

Similarly as above, for each $x, y \in [c', d']$ we let $n_{xy}$ denote the number of components of $f^{-1}([x, y])$ with limit points in both $f^{-1}(x)$ and $f^{-1}(y)$.

CASE 3B. Suppose there exists a positive integer $N_0$ and a subinterval $[x, y] \subset [c', d']$ such that if $x \leq z < w \leq y$, then $n_{zw} \leq N_0$. Let $S$ be a closed scattered set such that if $1 \leq i < j \leq N_0 + 1$, then $S$ separates $K_i$ from $K_j$. Using the ideas from Case 3A we find that if $x \leq z < w \leq y$, then $S \cap f^{-1}([x, y]) \neq \emptyset$. Therefore, $f(S) \supset [x, y]$, which contradicts the well-known fact that a scattered compactum can not be mapped onto a perfect set.

CASE 3B. Assume that for every $x, y \in [c', d']$ there exists an interval $[z, w] \subset [x, y]$ such that $n_{zw} > n_{xy}$. For each positive integer $n$ let $G_n$ be maximal relative to the property of being a collection of mutually exclusive open intervals lying in $[c', d']$ such that if $[x, y] \in G_n$ then $n_{xy} = n$. Note that each $G_n$ is at most countable. Let $S_n$ denote the set of all end-points of intervals which belong to $G_n$. We are going to show that $\bigcup_{n=1}^\infty S_n$ is dense in $[c', d']$, and thus obtain a contradiction.

Let $[x, y] \subset [c', d']$. There exists $[z, w] \subset [x, y]$ such that $n_{zw} > n_{xy}$. Thus, $x \neq z$ or $y \neq w$. By maximality of $G_{n_{zw}}$, there exists $[s, t] \in G_{n_{zw}}$ such that $[s, t] \cap [x, w] \neq \emptyset$. But $[s, t] \cap [z, y] = \emptyset$, and so $s \in [x, y]$ or $t \in [x, y]$. Therefore, the set $\bigcup_{n=1}^\infty S_n$ is dense in $[c', d']$, a contradiction.

The consideration of subcases 1, 2 and 3 is concluded and we return now to the main proof. Since $X$ is hereditarily locally connected, it is the continuous image of an arc by [12].

**THEOREM 2.** If $X$ is as in Theorem 1, then

(a) $X$ is rim-finite,

(b) every subcontinuum $G$ of $X$ has the property that some point or a pair of points separates $G$, and
(c) each closed set irreducible with respect to the property of being a compact set which separates $X$ is metrizable.

**Proof.** The claims (a), (b) and (c) follow from [19], [18] and [4], respectively, because $X$ contains no non-degenerate metric continuum.

Given a locally connected continuum $X$, for each pair of distinct points $a, b$ of $X$ let $[X, a, b]$ denote the class of all continuous maps $f : X \to P$ such that $P = f(X)$ is a non-metric arc with end-points $c$ and $d$ and $f(a) = c$ and $f(b) = d$. Also, introduce a relation $\sim$ on $X$ in the following way: $a \sim b$ if and only if $a = b$ or $[X, a, b] \neq \emptyset$.

**Theorem 3.** Suppose that $X$ is a locally connected continuum. Then $\sim$ is an equivalence relation on $X$, and if $X$ also satisfies the first axiom of countability, then equivalence classes of $\sim$ are closed and the set $E$ of equivalence classes of $\sim$ is upper semi-continuous.

**Proof.** $\sim$ is easily seen to be reflexive and symmetric, so suppose that $a \sim b$ and $b \sim c$ hold, but that there exists $f \in [X, a, c]$ such that $f(X)$ is a non-metric arc $[d, e]$ with $f(a) = d$ and $f(c) = e$.

**Case 1.** $f(b) = d$. Then $f \in [X, b, c]$, a contradiction.

**Case 2.** $f(b) = e$ analogous to Case 1.

**Case 3.** $d < f(b) < e$. Then one of the arcs $[d, f(b)]$ and $[f(b), e]$ is non-metric, so suppose $[d, f(b)]$ is non-metric. Define $r : [d, e] \to [d, f(b)]$ so that $r(x) = x$ if $x \in [d, f(b)]$ and $r(x) = f(b)$ if $x \in [f(b), e]$. Clearly, $r \circ f \in [X, a, b]$, a contradiction.

Let us now show that each equivalence class $G \in E$ is closed if $X$ is first countable. Let $G \in E$ and suppose that $x \in \overline{G} - G$. There exists a countable basis $U_1, U_2, \ldots$ of open neighborhoods of $x$ in $X$ and a sequence $z_1, z_2, \ldots$ of points of $G$ such that $z_i \in U_i$, for $i = 1, 2, \ldots$. Let $f : X \to [c, d]$ be a continuous map onto a non-metric arc $[c, d]$, where $f(z_1) = c$ and $f(z) = d$. Since each $[f(z_1), f(z_i)]$ is a metric subarc of $[c, d]$, it follows that $[c, d]$ is the closure of a countable union of metric arcs. Consequently, $[c, d]$ is separable, and therefore metrizable, a contradiction. Thus $G$ is closed in $X$.

It remains to show that $E$ is upper semi-continuous if $X$ is first countable. Let the element $G$ of $E$ be a subset of an open set $U$. Suppose that for each open set $V$ such that $G \subset V \subset U$, there is an element $G_V$ of $E$ so that $V \cap G_V \neq \emptyset$ and $G_V \not\subset U$. Thus, for some point $x$ of $G$ there is a countable basis $U_1, U_2, \ldots$ of open neighborhoods of $x$ such that for each $U_i$ there is an element $G_i$ of $E$ with the property that $G_i \cap U_i \neq \emptyset \neq G_i \cap (X - U)$.

There is a point $y$ of $X - U$ so that every neighborhood of $y$ intersects $G_i$ for infinitely many $i$. We may assume without loss of generality that there exists $y_i \in G_i \cap (X - U)$ for each $i$, and that the points $y_i$ converge to $y$. Let $z_i \in U_i \cap G_i$, for $i = 1, 2, \ldots$.

There exists $f \in [X, z, y]$ such that $f : X \to [c, d]$, where $[c, d]$ is a non-metric arc, $f(x) = c$ and $f(y) = d$. Since the points $f(z_i)$ converge to $c$, and the points $f(y_i)$ converge to $d$, and each arc $[f(z_i), f(y_i)]$ is metric, we find that $[c, d]$ is metric -- a contradiction.
THEOREM 4. Suppose that $X \in \mathcal{R}_M \cup \mathcal{R}_S \cup \mathcal{R}_MN$ and $X$ is first countable. Let $\mathcal{H}$ be the family of all components of sets in $\mathcal{E}$. Then $X/\mathcal{H}$ is the continuous image of an arc.

PROOF. Since $\mathcal{E}$ is upper semi-continuous, $\mathcal{H}$ is upper semi-continuous as well (see e.g. [28]). Thus, $\mathcal{H}$ is an upper semi-continuous decomposition of $X$ into closed sets and the quotient space $X/\mathcal{H}$ is a locally connected continuum.

If $X/\mathcal{H}$ is hereditarily locally connected, we apply the main result of [12] to obtain the desired conclusion.

Otherwise, in $X/\mathcal{H}$ there is a subcontinuum $C$ such that $C$ fails to be connected im kleinen at a point $P$. There is thus an open set $W$ in $X/\mathcal{H}$ such that $P \in W$ but the component of $W \cap C$ containing $P$ contains no relatively open subset of $C$ containing $P$. Let $Q$ denote the element of $\mathcal{E}$ containing $P$. There is a closed subset $S$ of $X$ such that $S \subseteq \bigcup W - Q$ and $S$ separates $P$ from $\text{bd}(\bigcup W)$ in $X$. Let $\phi : X \to X/\mathcal{H}$ denote the natural map and let $B = \phi(S)$.

Let $U$ denote the component of $X/\mathcal{H} - B$ which contains $P$. Using the facts that is upper semi-continuous and that $Q \cap S = \emptyset$, we let $R_1, R_2, \ldots; G_1, G_2, \ldots; V_1, V_2 \ldots$ be subsets of $X/\mathcal{H}$ similarly as in the proof of Theorem 1, except for the additional condition that no element of $\mathcal{E}$ intersects $\text{cl}(\bigcup R_i)$ and $\text{bd}(\bigcup U)$.

Now, let $s_1, s_2, \ldots$ and $t_1, t_2, \ldots$ be such that $s_i \in (\bigcup G_i) \cap (\bigcup \text{bd}(R_i))$ and $t_i \in (\bigcup G_i) \cap (\bigcup \text{bd}(U))$ for $i = 1, 2, \ldots$. Since $X$ is first countable, we may assume without loss of generality that the points $s_i$ converge to some point $s$, and the points $t_i$ converge to some point $t$, and the limiting set $L$ of $\bigcup G_1, \bigcup G_2, \bigcup G_3, \ldots$ is a continuum containing $s$ and $t$.

There is an $f \in [X, s, t]$ such that $f(X)$ is a non-metric arc $[c, d]$ with $f(s) = c$ and $f(t) = d$. We may now obtain a contradiction as in the proof of Theorem 1.

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