RECAPTURING SEMIGROUP COMPACTIFICATIONS OF A GROUP FROM THOSE OF ITS CLOSED NORMAL SUBGROUPS

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Abstract. We know that if $S$ is a subsemigroup of a semitopological semigroup $T$, and $\mathcal{F}$ stands for one of the spaces $\mathcal{A}, \mathcal{W}, \mathcal{A}, \mathcal{I}, \mathcal{D}$ or $\mathcal{L}$, and $(\epsilon, T^\omega)$ denotes the canonical $\mathcal{F}$-compactification of $T$, where $T$ has the property that $\mathcal{F}(S) = \mathcal{F}(T)|_S$, then $(\epsilon|_S, \epsilon(S))$ is an $\mathcal{F}$-compactification of $S$. In this paper, we try to show the converse of this problem when $T$ is a locally compact group and $S$ is a closed normal subgroup of $T$. In this way we construct various semigroup compactifications of $T$ from the same type compactifications of $S$.

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1. Introduction. For notation and terminology we follow Berglund et al. [2], as much as possible. Thus a topological semigroup is a semigroup $S$ that is a Hausdorff topological space, the multiplication $(s, t) \rightarrow st : S \times S \rightarrow S$ being continuous. $S$ is called a semitopological semigroup if the multiplication is separately continuous, i.e., the maps $\lambda_s : t \rightarrow st$ and $\rho_s : t \rightarrow ts$ from $S$ into $S$ are continuous for each $s \in S$. For $S$ to be right topological only, the maps $\rho_s$ are required to be continuous. Let $G$ denote a locally compact group, and $N$ is a closed normal subgroup of $G$. A semigroup compactification of $G$ is a pair $(\varphi, X)$, where $X$ is a compact right topological semigroup with identity 1, and $\varphi : G \rightarrow X$ is a continuous homomorphism with $\varphi(G) = X$, and $\varphi(G) \subset \Lambda(X) = \{x \in X \mid \lambda_x : X \rightarrow X \text{ is continuous}\}; \Lambda(X)$ is called the topological center of $X$. When there is no risk of confusion we often refer to $(\varphi, X)$, or even to $X$, as a compactification of $G$.

A homomorphism from a compactification $(\psi, X)$ of $S$ to a compactification $(\varphi, Y)$ of $S$ is a continuous function $\theta : X \rightarrow Y$ such that $\theta \circ \psi = \varphi$. Then, $Y$ is called a factor of $X$, and $X$ is an extension of $Y$. A compactification with a given property $P$ (such as that of being a semitopological semigroup or a topological group) is called a $P$-compactification. A universal $P$-compactification of $S$ is a $P$-compactification which is an extension of every $P$-compactification of $S$ (see [1, 2, 3]).

The $C^*$-algebra of all bounded continuous complex-valued functions on $G$ is denoted by $C^*(G)$ with left and right translation operators, $L_s$ and $R_s$, defined for all $s \in G$ by $L_s f = f \circ \lambda_s$ and $R_s f = f \circ \rho_s$. If $\mathcal{A}$ is a $C^*$-subalgebra of $C^*(G)$ containing the constant functions, we denote by $G^\mathcal{A}$ the spectrum of $\mathcal{A}$ furnished with Gelfand topology (i.e., the weak* topology induced from $\mathcal{A}^*$); the natural map $\epsilon : G \rightarrow G^\mathcal{A}$ is defined by $\epsilon(s)f = f(s)$. When $\mathcal{A}$ is left translation invariant (i.e., $L_s f \in \mathcal{A}$ for all $s \in G$ and $f \in \mathcal{A}$) we can define an action of $G$ on $G^\mathcal{A}$ by $(s, \nu) \rightarrow \epsilon(s) \nu$, where $(\epsilon(s) \nu)(f) = \nu(L_s f)$. Right
translation invariance and \( \nu \epsilon(s) \) are analogously defined (see [5, 7]).

A left translation invariant \( C^* \)-subalgebra of \( \mathcal{C}(G) \) containing the constant functions is called left \( m \)-introverted if the function \( s \to (yf)(s) = \nu(L_s f) \) is in \( \mathcal{A} \) for all \( f \in \mathcal{A} \) and \( \nu \in G^\mathcal{A} \); in this situation the product of \( \mu, \nu \in G^\mathcal{A} \) can be defined by \( (\mu \nu)(f) = \mu(\nu f) \). This makes \( (\epsilon, G^\mathcal{A}) \) a semigroup compactification of \( G \). The spaces of almost periodic, weakly almost periodic, left continuous and distal functions, which are denoted by \( \mathcal{A}_\mathcal{P}, \mathcal{W}_\mathcal{A}_\mathcal{P}, \mathcal{L}_\mathcal{P}, \mathcal{A}_\mathcal{P} \), respectively, are left \( m \)-introverted. We refer the reader to [2, 5] for the one-to-one correspondence between compactifications of \( G \) and left \( m \)-introverted \( C^* \)-subalgebras of \( \mathcal{C}(G) \), and also for a discussion of properties \( P \) of compactifications and associated universal mapping properties.

2. Main results. Let \( G \) be a locally compact group with a closed normal subgroup \( N \), and let \( (\varphi, X) \) be a compactification of \( N \). Let \( \sim \) be the equivalence relation on \( G \times X \) with equivalence classes \( \{(sr^{-1}, \varphi(r)x) \mid r \in N \} \). Thus

\[
(s, x) \sim (t, y) \text{ if and only if } t^{-1}s \in N \text{ and } \varphi(t^{-1}s)x = y.
\]

\( \pi : G \times X \to (G \times X) / \sim \) will denote the quotient map. Clearly \( \pi \) is one-to-one on \( \{e\} \times X \), so we can identify \( X \cong \{e\} \times X \) with \( \pi(\{e\} \times X) \). It is important that \( (G \times X) / \sim \) is locally compact and Hausdorff. In this connection we have the following lemmas, which are stated in [6].

**Lemma 2.1.**

(i) The graph of \( \sim \) is closed.

(ii) \( \pi : (G \times X) \to (G \times X) / \sim \) is an open mapping.

(iii) Let \( K \) be a compact subset of \( G \) and let \( L = KN \), then \( \pi(K \times X) = \pi(L \times X) \).

This lemma has the following easy consequences.

**Lemma 2.2.** The quotient space \( (G \times X) / \sim \) is locally compact and Hausdorff.

**Lemma 2.3.** If \( G = KN \) for some compact subset \( K \) of \( G \), then \( (G \times X) / \sim \) is compact.

Let \( \mu : G \to (G \times X) \) be defined by \( \mu(s) = (s, 1) \), where 1 is the identity of \( X \). Then, \( \pi \circ \mu : G \to (G \times X) / \sim \) is continuous as a composition of two continuous functions, and \( \pi \circ \mu(G) = \pi(G \times \varphi(N)) \), since for each \( (s, \varphi(r)) \in G \times \varphi(N) \), \( (s, \varphi(r)) \sim (sr, 1) \), and \( \pi \circ \mu(sr) = \pi(sr, 1) = \pi(s, \varphi(s)) \). Furthermore, if \( \varphi \) is a homeomorphism of \( N \) into \( X \), then \( \pi \circ \mu \) is also a homeomorphism.

We now define \( \sigma_s(r) = s^{-1}rs \) for \( s \in G \) and \( r \in N \); it is obvious that \( \sigma_s : N \to N \) is a surjective homomorphism for each \( s \in G \).

**Definition 2.4.** A \( \mathcal{P} \)-compactification \( (\psi, X) \) of \( N \) is said to be a conjugation invariant \( \mathcal{P} \)-compactification of \( N \) if \( (\psi \circ \sigma_s, X) \) is a \( \mathcal{P} \)-compactification of \( N \) for each \( s \in G \). When we write \( \mathcal{P} \)-compactification instead of \( P \)-compactification, this means that we want to emphasize its conjugation invariance, see Corollary 2.7.

**Remark.** The reader may have noticed that, the definition of \( \mathcal{P} \)-conjugation invariant compactification is different from the compatibility of a compactification in [6], because if \( \mathcal{P} \) is a property of compactifications that is not invariant under homomorphism and \( (\psi, X) \) is a \( \mathcal{P} \)-compactification of \( N \) compatible with \( G \), then \( (\psi \circ \sigma_s, X) \) is a
compactification of $N$ which may not be a $\mathcal{P}$-compactification of $N$, thus $(\psi, X)$ can fail to be a $\mathcal{P}$-conjugation invariant compactification of $N$. On the other hand, if $(\psi, X)$ is a $\mathcal{P}$-conjugation invariant compactification of $N$, i.e., $(\psi \circ \sigma_s, X)$ is a $\mathcal{P}$-compactification of $N$ for each $s \in G$, it is not always true that $\sigma_s$ has an extension from $X$ to $X$.

**Lemma 2.5.** Let $G$ be a locally compact group, $N$ a closed normal subgroup, and $(\varphi, X)$ a conjugation invariant universal $\mathcal{P}$-compactification of $N$, then each $\sigma_s$ can be extended continuously to a mapping from $X$ to $X$.

**Proof.** By conjugation invariance of $(\varphi, X)$, $(\varphi \circ \sigma_s, X)$ is a $\mathcal{P}$-compactification of $N$, and by universality of $(\varphi, X)$ there exists a continuous homomorphism $\nu : X \to X$ such that $\varphi \circ \sigma_s = \nu \circ \varphi$ for each $s \in N$. This $\nu$ is the continuous function extending $\sigma_s$.

It is obvious that if $(\varphi, X)$ is a conjugation invariant universal $\mathcal{P}$-compactification of $N$, then each $\sigma_s$ determines a continuous transformation of $X$, for which we use the same notation $\sigma_s$.

**Corollary 2.6.** Let $N$ be contained in the center of $G$, then each compactification $(\varphi, X)$ of $N$ is conjugation invariant.

**Corollary 2.7.** Let $(\epsilon, N^\varphi)$ denote a universal $\mathcal{P}$-compactification of $N$ and let $\mathcal{P}$ be a purely algebraic property, then $(\epsilon, N^\varphi)$ is a conjugation invariant $\mathcal{P}$-compactification of $N$.

Notice our deviation from the usual notation.

**Corollary 2.8.** Let $(\varphi, X)$ be an $\mathcal{F}$-compactification of $N$, where $\mathcal{F}$ stands for either of the spaces $\mathcal{A}\mathcal{P}$ and $\mathcal{W}\mathcal{A}\mathcal{P}$, then $(\varphi, X)$ is a conjugation invariant $\mathcal{F}$-compactification of $N$.

**Lemma 2.9.** Let $(\varphi, X)$ be a conjugation invariant $\mathcal{P}$-compactification of $N$, then for each $s \in G$, $\sigma_s$ is a continuous automorphism of $X$.

**Proof.** $\sigma_s$ is a homeomorphism of $X$ onto $X$ (since $\sigma_s(N) = N$ and $\sigma_s \sigma_s^{-1} = I$, the identity mapping). Now, we show that $\sigma_s$ is a homomorphism. Obviously,

$$\sigma_s(xy) = \sigma_s(x) \sigma_s(y) \quad \text{for each } x, y \in \varphi(N). \tag{2.2}$$

Since $X$ is a right topological semigroup with $\varphi(N) \subset \Lambda(X)$, we conclude that (2.2) holds for each $x \in \varphi(N)$, $y \in X$. Then it follows that (2.2) holds for all $x, y \in X$, as required.

If $N$ is a closed subgroup of $G$, and $X$ is a conjugation invariant $\mathcal{P}$-compactification of $N$, then we can define a semidirect product structure on $G \times X$ by $(s, x)(t, y) = (st, \sigma_t(x)y)$, where $\sigma_t$ is the conjugation map.

**Lemma 2.10.** Let $G$ be a locally compact group with a closed normal subgroup $N$, and let $(\varphi, X)$ be a conjugation invariant $\mathcal{P}$-compactification of $N$, then $G \times X$ is a right topological semigroup. Furthermore, the map

$$(s, r), (t, y)) \mapsto (st, \varphi(\sigma_t(r)y) : (G \times N) \times (G \times X) \to G \times X \quad \tag{2.3}$$
is continuous, and the equivalence relation \( \sim \) is a congruence on \( G \times X \).

**Proof.** The continuity is an easy conclusion of Ellis theorem. Now, we show that \( \sim \) is a congruence. Suppose \((s, x) \sim (t, y)\) and \((u, z) \in G \times X\), then \(t^{-1}s \in N\) and 
\[
\varphi((t^{-1}s)x) = y, \text{ so } (s, x)(u, z) = (su, \sigma_u(x)z) \text{ and } (t, y)(u, z) = (tu, \sigma_u(y)z).
\]
On the other hand, \((su, \sigma_u(x)z) \sim (tu, \sigma_u(y)z)\) since \((tu)^{-1}su = u^{-1}t^{-1}su \in N\) and 
\[
\varphi((tu)^{-1}su) \sigma_u(x)z = \sigma_u(y)z,
\]
thus 
\[
(s, x)(u, z) \sim (t, y)(u, z).
\]
(2.4)
Similarly 
\[
(u, z)(s, x) \sim (u, z)(t, y).
\]
(2.5)

The following theorem is an easy consequence of the previous corollaries and lemmas.

**Theorem 2.11.** Let \( G \) be a locally compact group with a closed normal subgroup \( N \), and let \((\varphi, X)\) be a conjugation invariant compactification of \( N \). Then \((G \times X)/\sim\) is a locally compact right topological semigroup, and a compactification of \( G \), provided that \( G = KN \) for some compact subset \( K \) of \( G \).

**Theorem 2.12.** The compactification \((\pi \circ \mu, (G \times X)/\sim)\) of \( G \) described in the previous theorem has the following universal property; let \((\varphi, Y)\) be a semigroup compactification of \( G \) such that \( \varphi|_N \) extends to a continuous homomorphism \( \phi : X \to Y \) in such a way that for each \( s \in G \) and \( x \in X \),
\[
\phi(\sigma_s(x)) = \varphi(s^{-1})\phi(x)\varphi(s),
\]
then there exists a (unique) continuous homomorphism \( \theta : (G \times X)/\sim \to Y \) such that \( \theta \circ \pi \circ \mu = \varphi \).

**Proof.** We define \( \theta_0 : G \times X \to Y \) by \( \theta_0(s, x) = \varphi(s)\phi(x) \), then \( \theta_0 \) is a continuous homomorphism which is constant on \( \sim \)-classes of \( G \times X \). Now we take \( \theta = \theta_0 \circ \pi \).

**Theorem 2.13.** Let \( N \) be a closed normal subgroup of \( G \) with \( G = KN \) for some compact subset \( K \) of \( G \). Suppose that \( \mathcal{P} \) is a property of compactifications such that \((\varphi|_N, \varphi(N))\) is a \( \mathcal{P} \)-compactification of \( N \) whenever \((\varphi, \varphi(G))\) is a \( \mathcal{P} \)-compactification of \( G \). Suppose that \((\epsilon, N^\mathcal{P})\) is a conjugation invariant universal \( \mathcal{P} \)-compactification of \( N \). If \((G \times N^\mathcal{P})/\sim \) has the property \( \mathcal{P} \), then \((G \times N^\mathcal{P})/\sim \) is the universal \( \mathcal{P} \)-compactification of \( G \).

**Proof.** We show that \((G \times N^\mathcal{P})/\sim \) is the universal \( \mathcal{P} \)-compactification of \( G \). Let \((\varphi, X)\) be a \( \mathcal{P} \)-compactification of \( G \) such that \((\varphi|_N, \varphi(N))\) is a \( \mathcal{P} \)-compactification of \( N \), by the universal property of \( N^\mathcal{P} \) there exists a continuous homomorphism \( \phi : N^\mathcal{P} \to X \) such that \( \phi \circ \epsilon = \varphi|_N \), and we have \( \phi(\sigma_s(x)) = \varphi(s^{-1})\phi(x)\varphi(s) \) for all \( s \in G \) and \( x \in N^\mathcal{P} \). Notice that we use two different scripts of the same letter to emphasize
their connection. Indeed, for fixed \( s \in G \), both sides represent homomorphisms of \( N^3 \) into \( X \), both sides are continuous in \( x \), and coincide on the dense subspace \( N \). Now the map \( \varphi \times \phi : (G \times N^3) \to X \) defined by \( (\varphi \times \phi)(s, x) = \varphi(s)\phi(x) \) is continuous and a homomorphism, since

\[
(\varphi \times \phi)((s, x)(t, y)) = (\varphi \times \phi)(st, \sigma_t(x)y) = \varphi(st)\phi(\sigma_t(x)y) = \varphi(s)\varphi(t)\phi(\sigma_t(x))\phi(y) = \varphi(s)\phi(x)\phi(t)\phi(y) \tag{2.8}
\]

Also \( \varphi \times \phi \) is constant on \( \sim \)-classes, thus the quotient of \( \varphi \times \phi \) gives a continuous homomorphism from \( (G \times N^3)/\sim \) to \( X \).

**Corollary 2.14.** Let \( N \) be a closed normal subgroup of \( G \) with \( G = KN \) for some compact subset \( K \) of \( G \), then

(i) \( (G \times N^3)/\sim \) is the universal \( \mathcal{L}^\mathcal{E} \)-compactification of \( G \).

(ii) \( (G \times N^3)/\sim \) is the universal \( \mathcal{D} \)-compactification of \( G \).

**Proof.** (i) Since \( (G \times N^3)/\sim \) is a compactification of \( G \), by Theorem 2.13, \( (G \times N^3)/\sim \) is the universal \( \mathcal{L}^\mathcal{E} \)-compactification of \( G \).

(ii) Since \( N^3 \) is a group, \( (G \times N^3)/\sim \), the quotient by a congruence of a semidirect product of groups is also a group, thus by Theorem 2.13 \( (G \times N^3)/\sim \) is the universal \( \mathcal{D} \)-compactification of \( G \).

In some situations, we want to be able to conclude that the right topological semigroup \( (G \times X)/\sim \) of Theorem 2.13 is also left topological. The following lemma can be helpful in this connection.

**Lemma 2.15.** Let \( G \) be a locally compact group with a closed normal subgroup \( N \) and let \( X \) be a universal conjugation invariant compactification of \( N \). Suppose that \( G = KN \) for some compact subset \( K \) of \( G \) and \( s \to \sigma_s(x) : G \times X \to X \) is continuous for all \( x \in X \). Then \( (G \times X)/\sim \) is semitopological.

**Proof.** Since \( (s, x) \sim \sigma_s(x) : G \times X \to X \) is a group action, it is continuous by Ellis theorem, thus \( G \times X \) is semitopological semigroup and also \( (G \times X)/\sim = \pi(G \times X) \).

**Corollary 2.16.** Let \( G \) be a locally compact group with a closed normal subgroup \( N \), \( G = KN \) for some compact subset \( K \) of \( G \) and suppose that \( s \sim \sigma_s(x) : G \to N^3 \) is continuous for all \( x \in N^3 \), then \( (G \times N^3)/\sim \) is the universal semitopological semigroup compactification of \( G \).

**Proof.** Since \( N^3 \) is a semitopological semigroup, by Lemma 2.15, \( (G \times N^3)/\sim \) is semitopological semigroup. Thus by Theorem 2.13, \( (G \times N^3)/\sim \) is the universal semitopological semigroup compactification of \( G \).

A similar argument yields the following corollary.

**Corollary 2.17.** Let \( G \) be a locally compact group with a closed normal subgroup \( N \), \( G = KN \) for some compact subset \( K \) of \( G \) and suppose that \( s \sim \sigma_s(x) : G \to N^3 \) is
continuous for all \( x \in \mathbb{N} \), then \((G \times \mathbb{N}^d) / \sim\) is the universal topological semigroup compactification of \( G \).

**Lemma 2.18.** Let \( N \) be a closed normal subgroup of \( G \) with \( G = KN \) for some compact subset \( K \) of \( G \). Let \( \mathcal{F} \) and \( \mathcal{G} \) be left \( m \)-introverted subalgebras of \( \mathcal{F}(N) \) and \( \mathcal{F}(G) \), respectively. Then \( \mathbb{N}^d \) is a conjugation invariant \( \mathcal{F} \)-compactification of \( N \) if and only if \( \mathcal{G}_N = \mathcal{F} \) and \((G \times \mathbb{N}^d) / \sim\) is the \( \mathcal{G} \)-compactification of \( G \).

**Proof.** Let \( \mathcal{G}_N = \mathcal{F} \), we define \( \sigma_s(x)(f) \) for \( s \in G, x \in \mathbb{N}^d \) and \( f \in \mathcal{F} \) by \( \sigma_s(x)(f) = x(g \circ \sigma_s|_N) \), where \( g \in \mathcal{G}, g|_N = f \). Since every such extension \( g \) yields a \( g \circ \sigma_s \) agreeing with \( f \circ \sigma_s \) on \( N \), \( \sigma_s(x) \) is well defined. So \( \mathbb{N}^d \) is a conjugation invariant \( \mathcal{F} \)-compactification of \( N \).

Conversely, since the quotient map \( \pi : G \times \mathbb{N}^d \to (G \times \mathbb{N}^d) / \sim \) is injective on the compact set \( \mathbb{N}^d \equiv \{e\} \times \mathbb{N}^d \), it gives a topological isomorphism of \( \mathbb{N}^d \) into \( G / \mathcal{G} \). \( \square \)

**Corollary 2.19.** Let \( G \) be a compact group with a closed normal subgroup \( N \), then

(i) \((G \times \mathbb{N}^d) / \sim \cong \mathbb{N}^d \).

(ii) \((G \times \mathbb{N}^d) / \sim \cong \mathbb{N}^d \).

**Corollary 2.20.** Let \( N \) be a closed normal subgroup of a locally compact group \( G \) contained in the center of \( G \), then

\[
\left(G \times \mathbb{N}^d\right) / \sim \cong \mathbb{N}^d .
\] (2.9)

The next example shows that the continuity of \( s \to \sigma_s(x) \) in Corollary 2.14 and Lemma 2.15 is an essential condition.

**Example 2.21.** Let \( G = \mathbb{C} \times \mathbb{T} \) be the Euclidean group of the plane with \((z, w)(z_1, w_1) = (z + wz_1, w w_1)\) and \( N = \mathbb{C} \times \{1\} \), then \( N \) is a closed normal subgroup of \( G \) and \( \mathbb{A}P(G)|_N \) is a proper subset of \( \mathbb{A}P(N) \) \([4, 8]\), so by Lemma 2.15 \((G \times \mathbb{C}^d) / \sim \) is not the universal \( \mathbb{A}P \)-compactification of \( G \). \( \mathbb{C}^d \) is a conjugation invariant compactification of \( N \), so the continuity of \( s \to \sigma_s \) must fail to hold Lemma 2.15. From \([4, 8]\), we can similarly conclude that \((G \times \mathbb{C}^d) / \sim \) is not the universal \( \mathbb{W}A\mathbb{P} \)-compactification of \( G \) and that the continuity of \( s \to \sigma_s \), as required by Corollary 2.14, also fails to hold.

**References**


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