ON THE STRUCTURE Of IONIZING SHOCK WAVES IN MAGNETOFLUIDDYNAMICS

A. AGHAJANI and M. HESAARAKI

Ionizing shock waves in magnetofluiddynamics occur when the coefficient of electrical conductivity is very small ahead of the shock and very large behind it. For planner motion of plasma, the structure of such shock waves are stated in terms of a system of four-dimensional equations. In this paper, we show that for the above electrical conductivity as well as for limiting cases, that is, when this coefficient is zero ahead of the shock and/or is infinity behind it, ionizing fast, slow, switch-on and switch-off shocks admit structure. This means that physically these shocks occur.

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1. Introduction. An ionizing shock is defined as a compressive wave which propagates into a nonionized, nonconducting gas, ionizes it, and thus makes the post-shock gas electrically conducting and capable of interacting with an electromagnetic field. Thus in this type of shock wave the pre-shock of the gas is nonconducting and the post-shock state is ionized and a good electrical conductor. Thus, this type of shock wave is considered as a system of magnetohydrodynamics (MHD) or magnetofluiddynamics (MFD).

From the mathematical point of view, shock waves are discontinuous weak solutions of conservation laws. In order to distinguish physical shock wave solutions of conservation laws, among many of them, one has to apply some criteria. The most widely accepted one is the structure or viscous profile criterion [1, 19, 24, 28].

The question of existence of structure for different types of MFD shock waves in planar motion has been considered by Germain [10], Kulikovskiĭ and Lyubimov [19], Cabannes [1], Conley and Smoller [4, 5], and Mischaikow and Hatori [20]. According to their work, the shock layer equations in MFD is stated in terms of the following four-dimensional system of ordinary differential equations, which is taken from [1]

$$
\begin{aligned}
(\lambda_1 + 2\mu_1)\frac{du}{dx} &= mu + p + \frac{1}{2}\mu H_y^2 - P, \\
\mu_1 \frac{dv}{dx} &= mv - \mu H_x H_y - P_1, \\
\lambda \frac{dT}{dx} &= m\left[\varepsilon - \frac{1}{2}(u^2 + v^2)\right] - \frac{1}{2}\mu H_y^2 + \mu v H_x H_y + uP + vP_1 - E_z H_y - C, \\
\sigma^{-1}\frac{dH_y}{dx} &= E_z + \mu u H_y - \mu v H_x.
\end{aligned}
\tag{1.1}
$$

Here $\mu > 0$ is an electrical constant, (u, v) is the velocity vector of the fluid, ε is the internal energy, p and T are the pressure and the temperature, respectively. The vector (H_x, H_y) is the magnetic field in the xy-plane, and E_z is the electric field in the direction of z-axis, where H_x and E_z are nonnegative constants. This system of equations contains four dissipation coefficients; the two coefficients of viscosities λ_1 and μ_1, the coefficient of thermal conductivity λ and the coefficient of electrical conductivity σ. These coefficients are nonnegative functions of absolute temperature T. Finally, m, P, P_1, and C denote constants of integration. For more details and derivation of these equations the reader is referred to [1].

Now the structure problem for MFD shocks is the above four simultaneous first-order nonlinear differential equations must be integrated between equilibrium points. This problem has been studied before by many authors, when the dissipation coefficients are continuous functions of T [4, 5, 7, 10, 11, 13, 14, 20]. We will describe these works in more details in Section 2. However, in the case of ionizing shock, the electrical conductivity of the gas is assumed to be zero (or very small) in the pre-shock gas and it continues to zero (or very small) until a value $\bar{T}$ is reached by the temperature. At this point in the shock structure the electrical conductivity jumps to infinity (or a high value) which remains the same through the remainder of the shock wave. The analogy with the ignition temperature in flame and detonation problems is evident [9, 12, 26, 27, 29]. In other words, we have

$$\sigma(T) = \begin{cases} \sigma_1(T) & \text{for } T \leq \bar{T}, \\ \sigma_2(T) & \text{for } T > \bar{T}, \end{cases} \tag{1.2}$$

where $0 \leq \sigma_1(T) \ll 1 \ll \sigma_2(T) \leq \infty$, and temperature $\bar{T}$ is given and is assumed to have a value between its upstream and downstream value. We may call $\bar{T}$ the ionizing temperature [1, 2, 12, 18, 25].

The structure problem for ionizing shock wave for the case $\sigma_1 = 0$ and $\sigma_2 > 0$ has been studied by Kulikovskiĭ and Lyubimov, when the gas is perfect and $H_x = \mu_1 = \lambda_1 = 0$ or $H_x = \mu_1 = \lambda = 0$ [18]. Also this problem for perfect gas and $\sigma_1 = 0$, $\sigma_2 > 0$ is studied by Chu, when $H_x = \lambda = 0$ (see [2]). This leaves open the question of the existence of structure of ionizing shock waves in planar motion when $H_x \neq 0$ and none of the viscosity parameters is zero. This general case is studied in this paper.

This paper is organized as follows, in Section 2 we make some observations related to the rest (equilibrium) points of system (1.1) and introduce the problem in detail. In Section 3, we find some general results on the orbits of autonomous system of ordinary differential equations related to the problem. In Section 4, we show the existence of structures when $\sigma_1(T) \equiv 0$ and $\sigma_2(T) \equiv \infty$. The existence of structures in the case $\sigma_1(T) \equiv 0$ and $\sigma_2(T)$ is very high, will be considered in Section 5. In Section 6, we consider the problem for the case $0 < \sigma_1(T) \ll 1 \ll \sigma_2(T) < \infty$.

For an excellent description, experimental, and applications of ionizing shocks the reader is referred to [20, Chapter 4] and [22, Section 5.15].

2. Hypotheses, rest points, and the problem. As we pointed out before, a heteroclinic orbit of system (1.1) is called a structure for an MFD shock wave. In other words,

a structure for an MFD shock wave is a complete orbit of (1.1) connecting two rest points. Thus in the first step we must know the rest points of (1.1). In order to take advantage of some results from previous works in [4, 5, 13], we replace $\lambda_1 + 2\mu_1$, x, H_y, P, μ_1, $v - P_1$, H_x, λ, ε, C, E_z, and σ^{-1} by μ_1, t, x_2, J, μ, x_1, δ, k, e, E, ε, and v. Also, without loss of generality, we may assume $\mu = m = 1$, as m is the mass quantity and μ is an electrical quantity. Then from $u = mV$ [1], we obtain $u = v$, where $V = \rho^{-1}$ and ρ is the density of the fluid. In this way, (1.1) can be written as

$$\mu_1 \dot{V} = \frac{1}{2} x_2^2 + V - J + p(V, T) =: G_1(u),$$

$$\mu \dot{x}_1 = x_1 - \delta x_2 =: G_2(u),$$

$$k \dot{T} = -\frac{1}{2}(x_1^2 - 2\delta x_1 x_2 + V x_2^2) - \varepsilon x_2 - \frac{1}{2} V^2 + JV - E + e(V, T) =: G_3(u),$$

$$v \dot{x}_2 = -\delta x_1 + V x_2 + \varepsilon =: G_4(u),$$

(2.1)

where $u = (V, x_1, T, x_2)^T$ and μ_1, μ, k, and v are functions of T. Notice that for $J \le 0$, this system has no rest point. Hence we assume that $J > 0$. Also note that $\delta \ge 0$ and $\varepsilon \ge 0$.

Let $S(V, T)$ be the entropy of the system. Following the previous works in [4, 5, 11, 13, 14, 15, 16, 21], we consider a general form for thermodynamic state functions (instead of giving a specific expression) and we assume that the functions $p(V, T)$, $e(V, T)$, and $S(V, T)$ satisfy the following hypotheses.

(H_1) If $V, T > 0$, then p, e, and S are positive.

(H_2) For fixed $T > 0$, $p(V, T) \to \infty$ as $V \to 0$.

(H_3) Given $K, V_0 > 0$, there exists $T_0 > 0$ such that if $0 < V \le V_0$ and $T \ge T_0$, then $e(V, T) > K$.

(H_4) On any interval $0 < V \le V_0$, $S(V, T) \to 0$ uniformly in V as $T \to 0$.

(H_5) If we consider p as a function of V and S, then $p_v < 0$, $p_{vv} > 0$ and $p_s > 0$. As an alternative to H_4, we could assume

(H_4') The quantities $S_V (= p_T)$ and S_T are positive whenever $V, T > 0$, and for any fixed V, $S(V, T)$ converges to a limit independent of V as $T \to 0$.

These hypotheses are fairly mild, and have clear thermodynamic interpretations, (see [24, page 516] and [23, pages 125–32]).

We will not use all of these hypotheses directly, but we will take advantage of some results based on them from previous works in [4, 5, 13], specially we will assume the existence of the rest points which is based on the above hypotheses as follows.

Let $0 \le \mu, \mu_1, k, v < \infty$. For fixed $J > 0$, $\delta > 0$, and $\varepsilon > 0$, there are two numbers $E_0 \ge E_1$ such that for $E > E_0$ system (2.1) admits no rest point at all. For $E < E_1$ it admits precisely four rest points, two of these rest points are located in the region $V > \delta^2$, and the other two are located in the region $0 < V < \delta^2$. For $E_1 < E < E_0$, this system admits two rest points, and either both of them lie in $V < \delta^2$ or both of them lie in $V > \delta^2$ [4, 5, 11, 13, 20]. Hereafter, we assume that system (2.1) admits four rest points. We denote them by

$$u_i = (V_i, x_{1i}, T_i, x_{2i}),$$

(2.2)

$0 \le i \le 3$, which are ordered by increasing density. Here (V_i, T_i), for $0 \le i \le 3$, is a solution of algebraic equations

$$
\begin{aligned}
&\frac{1}{2}\varepsilon^2 (V - \delta^2)^{-2} + V - J + p(V, T) = 0, \\
&\frac{1}{2}\varepsilon^2 (V - \delta^2)^{-1} - \frac{1}{2}V^2 + JV - E + e(V, T) = 0,
\end{aligned}
\tag{2.3}
$$

and $x_{1i} = -\varepsilon\delta(V_i - \delta^2)^{-1}$ and $x_{2i} = -\varepsilon(V_i - \delta^2)^{-1}$.

If $0 \le \mu,\ \mu_1, k, \nu < \infty,\ \varepsilon = 0,\ J > 0$, and $\delta > 0$, then for negative large value of E system (2.1) admits four rest points

$$
\bar{u}_i = (\bar{V}_i, \bar{x}_{1i}, \bar{T}_i, \bar{x}_{2i}), \quad 0 \le i \le 3.
\tag{2.4}
$$

Again these rest points are ordered by increasing of density. For $i = 0, 3$, $\bar{u}_i = (\bar{V}_i, 0, \bar{T}_i, 0)$, where $(\bar{V}_i, \bar{T}_i)$ is a solution of (2.2) corresponding to $\varepsilon = 0$. Moreover for $i = 1, 2$, we have $\bar{u}_i = (\delta^2, \delta\bar{x}_{2i}, \bar{T}_i, \bar{x}_{2i})^T$, where $(\bar{T}_i, \bar{x}_{2i})$, $i = 1, 2$, is a solution of the system of equations,

$$
e(\delta^2, T) - \frac{1}{2}\delta^4 + J\delta^2 - E = 0, \qquad \frac{1}{2}x_2^2 + \delta^2 - J + p(\delta^2, T) = 0.
\tag{2.5}
$$

Finally, if $\delta = 0$, then u_2 and u_3 do not exist, and u_0 and u_1 may exist. For more details about the rest points, the reader is referred to [13].

As we mentioned before, the existence of a heteroclinic orbit $u_i \to u_j$ corresponds to the existence of structure for the shock wave between the two states u_i and u_j. This means that, this shock occurs physically. It is shown that if such a heteroclinic orbit exists, then $i < j$. In the following, we explain the concept of different shock waves which may occur.

Shock wave between u_0 and u_1 (u_2 and u_3) is called fast (resp., slow) shock. Physically, this means upstream and downstream states of the shock is super-Alfvénic (resp., sub-Alfvénic). Shock wave between u_i and u_j, $i = 0, 1$ and $j = 2, 3$, is called intermediate shock. The downstream state of this shock is sub-Alfvénic and its upstream is super-Alfvénic.

Shock wave between $\bar{u}_0$ and $\bar{u}_1$ (as well as $\bar{u}_0$ and $\bar{u}_2$) is called switch-on shock, and shock wave between $\bar{u}_2$ and $\bar{u}_3$ (as well as $\bar{u}_1$ and $\bar{u}_3$) is called switch-off shock. This means that $\bar{u}_0$ is super-Alfvénic, $\bar{u}_1$ and $\bar{u}_2$ are Alfvénic and $\bar{u}_3$ is sub-Alfvénic [19, 28].

It is known that for bounded and continuous functions of $\mu_1(T)$, $\mu(T)$, $k(T)$, and $\nu(T)$ fast, slow, switch-on, and switch-off shocks always exist [4, 5, 9, 10, 12]. However, existence of intermediate shocks depend on the values of the above four viscosities [5, 13, 21]. In the present paper, we are concerned with the existence of the ionizing shocks of the above shocks except intermediate shocks, which may be considered in a future work.

3. Some theorems in ODE. In this section, we present some existence theorems which will be used as main tools in the next section.

Consider the autonomous system of ODEs

$$\frac{dx}{dt} = f(x), \quad x = (x_1, x_2, \ldots, x_n)^T, \tag{3.1}$$

on $\mathbb{R}^n$, where f is smooth. We denote by $x \cdot t$ the value of the solution of (3.1) at time t which is x initially. Thus this solution is uniquely defined on an open interval of t containing origin and assumed to be maximal.

The set $S \subset \mathbb{R}^n$ is called invariant with respect to (3.1) if $x \cdot t \in S$, for all $t \in \mathbb{R}$ and $x \in S$. By an orbit we mean a solution of (3.1), and by a complete orbit we mean an orbit which is defined for all values of $t \in \mathbb{R}$. We say that the orbit $y(t)$ is running from x_0 (or running to x_1) if $y(t)$ is defined for $t \leq 0$ (or $t \geq 0$) and $\lim_{t \to -\infty} y(t) = x_0$ (or $\lim_{t \to \infty} y(t) = x_1$). If $y(t)$ is running from x_0 to x_1 and $x_1 \neq x_0$, then this orbit is called a heteroclinic orbit.

System (3.1) is called gradient-like in the open set $U \subset \mathbb{R}^n$, if there is a continuous real-valued function h on U which is strictly increasing on each nonconstant solution of (3.1), lying in U, [3, 6].

The ω-limit (α-limit) set of the orbit $x \cdot t$ is the set of limit points of sequences $x \cdot t_n$, where t_n goes to $+\infty(-\infty)$. If $x \cdot t$ is a complete bounded orbit, then its ω-limit set and α-limit set are nonempty, closed, connected, and invariant. In the case of a gradient-like system, the restriction of h (the gradient-like function) to each of this sets is constant. Therefore each of them consists of rest points [3, 24].

It is known that, the ω-limit set and α-limit set are nonempty and connected if $x \cdot t$ is bounded. In the case of gradient-like flows, the restriction of h to any of these sets is constant. Therefore each of these sets consists of a rest point. For more details the reader is referred to [3, 6, 22].

The following three theorems are modifications of [13, Theorems 2.1.1 and 2.1.2].

THEOREM 3.1. *Suppose that f in (3.1) is locally Lipschitz in a neighborhood of the closure of an open bounded set D which is homomorphic to the semisphere $\{x \in \mathbb{R}^n : |x| < 1, \ x_n > 0\}$, and (3.1) is gradient-like with respect to a real-valued function, h in D. Moreover the following conditions hold.*

(C_1) The set $\{x \in \bar{D} : h(x) = c\}$ corresponds to the set $\{x \in \mathbb{R}^n : |x| < 1, \ x_n = c\}$ under the homomorphism, for all $c \in [0, 1]$.

(C_2) The single point $\{x \in \bar{D} : h(x) = 1\}$ which is denoted by $\tilde{x}$, is a rest point, and this is the only rest point of (3.1) in $\bar{D}$.

(C_3) Let $F = \{x \in \partial D : h(x) > 0\}$. If $p \in \bar{F} \backslash \{\tilde{x}\}$ then $p \cdot t \notin D$ for small positive t and $p \cdot t \notin \partial D$ for small $|t| \neq 0$.

(C_4) For $p \in \partial D \backslash \bar{F}$, $p \cdot t \in D$, for $t > 0$ and small.

Then there is a point $\tilde{p} \in \partial D \backslash \bar{F}$ such that $\tilde{p} \cdot t$ is running to $\tilde{x}$.

PROOF. Let $E = \partial D \backslash \bar{F}$ and $\tilde{E} = \{x \in E : x \cdot (0, \infty) \subset D\}$. We claim that $\tilde{E} \neq \varnothing$. If $\tilde{E} = \varnothing$, then for $p \in \bar{E}$ there is a unique $t(p) \geq 0$ such that $p \cdot t(p) \in \bar{F}$. Now, define $\psi : \bar{E} \to \bar{F} \backslash \{\tilde{x}\}$ by $\psi(p) = p \cdot t(p)$. From continuous dependence of the solution with respect to initial conditions and uniqueness of solution it follows that ψ is a homomorphism from $\bar{E}$ to $\bar{F} \backslash \{\tilde{x}\}$. This is impossible as $\bar{E}$ is closed and $\bar{F} \backslash \{\tilde{x}\}$ is not closed. Thus $\tilde{E} \neq \varnothing$. Let $x \in \tilde{E}$. Then $x \cdot t$ is defined for all $t > 0$ and is lying in D. Since system (3.1) is gradient-like, the ω-limit set of $x \cdot t$ must be $\{\tilde{x}\}$. This completes the proof. $\qquad \square$

For more details of the above proof the reader is referred to the proof of [13, Theorem 2.1.1].

THEOREM 3.2. *Suppose that f and D are the same as in Theorem 3.1, and system (3.1) is gradient-like with respect to a real-valued function g in D. Moreover, the following conditions hold.*

(C_1') The set $\{x \in \bar{D} : g(x) = 1 - c\}$ corresponds to the set $\{x \in \mathbb{R}^n : |x| \leq 1,\ x_n = c\}$ for $0 \leq c \leq 1$ under homomorphism.

(C_2') The set $\{x \in \bar{D} : g(x) = 0\}$ which consists of a single point, say $\tilde{x}$, is a rest point of (3.1), and $\tilde{x}$ is the only rest point of (3.1) in $\bar{D}$.

(C_3') Let $F = \{x \in \partial D : g(x) < 1\}$. For $p \in \bar{F} \setminus \{\tilde{x}\}$, $p \cdot t \notin D$ for small positive t and $p \cdot t \notin \partial D$ for $|t| \neq 0$ and small.

(C_4') For $p \in \partial D \setminus \bar{F}$, $p \cdot t \in D$ for $t < 0$ and $|t|$ small.

Then for $p \in \partial D \setminus \bar{F}$ we must have $\lim_{t \to -\infty} p \cdot t = \tilde{x}$.

PROOF. Since f is Lipschitz on $\bar{D}$ and the flow cannot leave D as t decreases, $p \cdot t$ must be defined for all $p \in \partial D \setminus \bar{F}$ and $t < 0$ and lying in D. Since α-limit set of $p \cdot t$ consists of a rest point, $\lim_{t \to -\infty} p \cdot t = \tilde{x}$. □

As a modification of [13, Theorem 2.1.2] we have the following theorem.

THEOREM 3.3. *Let f, D, h, $\tilde{x}$, and F be the same as in Theorem 3.1. Moreover, the following conditions hold.*

(C_1'') The same as condition (C_1) in Theorem 3.1.

(C_2'') The same as condition (C_2) in Theorem 3.1.

(C_3'') If $p \in \bar{F} \setminus \{\tilde{x}\}$, then $p \cdot t \notin \partial D$ for $|t| \neq 0$ and small.

(C_4'') If $p \in F \setminus \{\tilde{x}\}$ and $p \cdot t \in D$ for $t > 0$ and small, then $p \cdot t \notin \bar{D}$ for $t < 0$ and $|t|$ small.

(C_5'') The set $I = \{p \in \partial D : p \cdot t \in D$ for $t > 0$ and small$\}$ is disconnected.

(C_6'') Let $E = \partial D \setminus \bar{F}$. If $p \in E$, then $p \cdot t \in D$ for $t > 0$ and small.

(C_7'') The connected set E intersects at least with two components of I.

Then there is $x_0 \in E$ such that $\lim_{t \to \infty} x_0 \cdot t = \tilde{x}$.

PROOF. We claim that the set $N = \{x \in E : x \cdot (0, \infty) \subset D\}$ is nonempty. To see this suppose $N = \varnothing$. Then for $p \in E$, there is $t(p) > 0$ such that $p \cdot t(p) \in I$, $p \cdot t \in D$ for $0 < t < t(p)$ and $p \cdot t \notin \bar{D}$ for $t(p) < t$ and $t - t(p)$ small (condition (C_4'')).

Now, define $\varphi : E \to I$ by $\varphi(p) = p \cdot t(p)$. Since the orbit $p \cdot t$ for $p \in E$ intersects ∂D transversely, φ must be continuous and since E is connected $\varphi(E)$ must be connected. On the other hand for $p \in I \cap E$ we have $\varphi(p) = p$. Thus by (C_7''), $\varphi(M)$ must be disconnected. This is a contradiction. Hence $N \neq \varnothing$. If $x_0 \in N$, then, similar to the proof of Theorem 3.1, $\lim_{t \to \infty} x_0 \cdot t = \tilde{x}$. □

The next theorem can be considered as an extension of the continuous dependence of solution of ordinary differential equations on parameters.

THEOREM 3.4. *Let $\{f_m\}$ be a sequence of vector fields in $\mathbb{R}^n$, and let $D \subset \mathbb{R}^n$ be a bounded domain. Suppose for each m, $f_m \in C^1(\bar{D})$, and there is a constant K such that $|f_m(x)| \leq K$ for all m and $x \in \bar{D}$. Suppose $y_m(t)$ is a solution of*

$$\frac{dx}{dt} = f_m(x), \qquad (3.2)$$

which is defined on the interval (a,b) and lies in D for all m. Then $\{y_m(t)\}$ has a uniformly convergent subsequence on compact subsets of (a,b), which converges to a continuous of bounded variations function, say $y(t)$. Moreover, suppose there is a $t_0 \in (a,b)$ and a C^1 vector field, say f, on a neighborhood of $y(t_0)$ such that f_m converges to f uniformly, as $m \to \infty$, in this neighborhood. Then $y(t)$ is a solution of the initial-value problem

$$\frac{dx}{dt} = f(x), \qquad x(t_0) = y(t_0), \tag{3.3}$$

on some neighborhood of t_0.

PROOF. First of all notice that by assumptions $\{f_m\}$ and $\{y_m(t)\}$ are uniformly bounded. It then follows that $\{y_m(t)\}$ is uniformly of bounded variations on (a,b). On the other hand, for $a < t_1 < t_2 < b$, we have

$$\begin{aligned}
|y_m(t_2) - y_m(t_1)| &= \left| \int_{t_1}^{t_2} f_m(y_m(t)) dt \right| \\
&\leq \int_{t_1}^{t_2} |f_m(y_m(t))| dt \\
&\leq \int_{t_1}^{t_2} K\, dt = K|t_2 - t_1|.
\end{aligned} \tag{3.4}$$

Thus $\{y_m(t)\}$ is equicontinuous on (a,b). Let $a < \alpha < \beta < b$. By Arzela-Ascoli theorem there is a subsequence of this sequence which is convergent uniformly on $[\alpha, \beta]$, to a continuous function, say $y(t)$. Hence we may assume $y(t)$ is defined on (a,b) and $\{y_m(t)\}$ converges uniformly to $y(t)$ on each compact subset of (a,b). Moreover, by Helly's first theorem [17], $y(t)$ is of bounded variations too.

Now, suppose for large values of m, in a neighborhood of $y(t_0)$, f_m converges to f uniformly. Since f has continuous first derivatives on the closure of this neighborhood, we may assume that f is uniformly Lipschitz with Lipschitz constant $\lambda > 0$, in this neighborhood. Since $\{y_m(t)\}$ converges uniformly on compact subsets of (a,b), there is $\varepsilon > 0$ such that $y_m(t)$ lies in the above neighborhood for $t \in [t_0 - \varepsilon, t_0 + \varepsilon]$ and all m. Now for $t \in [t_0, t_0 + \varepsilon)$ and n, m, we have

$$\begin{aligned}
|y_m'(t) - y_n'(t)| &= |f_m(y_m(t)) - f_n(y_n(t))| \\
&\leq |f(y_m(t)) - f(y_n(t))| + |f_m(y_m(t)) - f(y_m(t))| \\
&\quad + |f_n(y_n(t)) - f(y_n(t))| \\
&\leq \lambda |y_m(t) - y_n(t)| + \varepsilon_m + \varepsilon_n \\
&\leq \varepsilon_m + \varepsilon_n + \lambda |y_m(t_0) - y_n(t_0)| + \lambda \int_{t_0}^t |y_m'(t) - y_n'(t)| dt,
\end{aligned} \tag{3.5}$$

where "$'$" means d/dt. Therefore by Gronwall's inequality [6], we must have

$$\begin{aligned}
|y_m'(t) - y_n'(t)| &\leq [\varepsilon_m + \varepsilon_n + \lambda |y_m(t_0) - y_n(t_0)|] e^{\lambda(t - t_0)} \\
&\leq [\varepsilon_m + \varepsilon_n + \lambda |y_m(t_0) - y_n(t_0)|] e^{\lambda \varepsilon}.
\end{aligned} \tag{3.6}$$

Similarly, for $t \in (t_0 - \varepsilon, t_0]$ we must have

$$|y'_m(t) - y'_n(t)| \leq [\varepsilon_m + \varepsilon_n + \lambda |y_m(t_0) - y_n(t_0)|]e^{\lambda \varepsilon}. \tag{3.7}$$

Thus $\{y'_m(t)\}$ is uniformly Cauchy on $[t_0 - \varepsilon, t_0 + \varepsilon]$. Therefore, $y'(t)$ exists and $y'(t) = \lim_{m \to \infty} y'_m(t)$ on $(t_0 - \varepsilon, t_0 + \varepsilon)$. Hence on $(t_0 - \varepsilon, t_0 + \varepsilon)$

$$\begin{aligned}
|y'(t) - f(y(t))| &\leq |y'(t) - y'_m(t)| \\
&\quad + |f_m(y_m(t)) - f(y_m(t))| \\
&\quad + |f(y_m(t)) - f(y(t))|.
\end{aligned} \tag{3.8}$$

Thus $y(t)$ is the solution of (3.3) in a neighborhood of t_0. □

Now, in this section, we have the following theorem which can be considered as a continuous dependence of solutions to the parameter for singular situations.

For $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^n$, we consider the following system of equations in $\mathbb{R}^{n+m}$

$$\dot{x} = f(x, y), \qquad \varepsilon \dot{y} = g(x, y), \tag{3.9}$$

where "$\cdot = d/dt$," as before, $f : \mathbb{R}^{n+m} \to \mathbb{R}^n$ and $g : \mathbb{R}^{n+m} \to \mathbb{R}^m$ are continuous functions and $\varepsilon \in \mathbb{R}$ is a parameter.

THEOREM 3.5. *Let $D_1 \subset \mathbb{R}^n$ and $D_2 \subset \mathbb{R}^m$ be bounded domains and f and g on $D_1 \times D_2$ have continuous second derivatives and $(x_0, y_0) \in D_1 \times D_2$ be hyperbolic rest point of system (3.9). Moreover, assume that the following conditions hold.*

(C_1) *$g(x, y) \equiv 0$ if and only if $y = G(x)$, where $G : D_1 \to \mathbb{R}^m$ has continuous second derivatives.*

(C_2) *There exist nonnegative constant integers k_s and k_u, with $k_s + k_u = m$, such that for all $x \in D_1$ the $m \times m$ matrix $\partial g(x, G(x))/\partial y$ has k_s eigenvalues with negative real part and k_u eigenvalues with positive real part, uniformly bounded away from zero for all $x \in D_1$.*

(C_3) *For $\varepsilon = 0$, there is an orbit of system (3.9) say $y_0(t)$ which is running to the rest point (x_0, y_0) and intersects a hypersurface, say Q, at the point $y_0(t_0)$, transversely.*

Then for given $\delta > 0$, there is an $\varepsilon_0 > 0$, such that for each $\varepsilon \in (-\varepsilon_0, \varepsilon_0)$ there exists an orbit of system (3.9) corresponding to ε, say $y_\varepsilon(t)$, which is running to (x_0, y_0) and intersects Q transversely at a point in δ-neighborhood of $y_0(t_0)$.

PROOF. Let $p = (x_0, y_0)$ and $\Gamma^s(p)$ be the stable manifold of system (3.9) corresponding to $\varepsilon = 0$, at the rest point p.

Thus $y_0(t)$ is lying on this manifold. For each $q \in \Gamma^s(p)$, let $F^s(q)$ be the stable manifold of system

$$\dot{x} = 0, \qquad \dot{y} = g(x, y), \tag{3.10}$$

at the rest point q of this system. Now define

$$W^s(p) = \bigcup_{q \in \Gamma^s(p)} F^s(q). \tag{3.11}$$

By [8, Theorem 12.2(ii)], $W^s(p)$ perturbes smoothly to the stable manifold $W^s_\varepsilon(p)$ of the hyperbolic rest point p of system (3.9) for small ε. This means that for $y_0(t_0) \in W^s(p)$ and given $\delta > 0$ there is an $\varepsilon_0 > 0$ such that for every $\varepsilon \in (-\varepsilon_0, \varepsilon_0)$ there is a $p_\varepsilon \in W^s_\varepsilon(p) \cap B_\delta(y_0(t_0))$ such that if $y_\varepsilon(t)$ is the solution of (3.9) corresponding to this ε initiating at p_ε, then $y_\varepsilon(t)$ is running to p, moreover,

$$|y_\varepsilon(t) - y_0(t)| + |\dot{y}_\varepsilon(t) - \dot{y}_0(t)| < \delta, \tag{3.12}$$

for all $t \geq t_0$. Thus for δ small, $y_\varepsilon(t)$ intersects Q, transversely. $\qquad \square$

4. Existence of structure when $\sigma = 0$ for $T \leq \bar{T}$ and $\sigma = \infty$ for $T > \bar{T}$. In this section, we discuss the existence of structure for fast, slow, switch-on, and switch-off shocks when the coefficient of electrical conductivity is zero ahead of the shock and very large behind it. That is in this case we have

$$\sigma(T) = \begin{cases} 0 & \text{for } T \leq \bar{T}, \\ \infty & \text{for } T > \bar{T}, \end{cases} \quad \text{or} \quad \nu(T) = \begin{cases} \infty & \text{for } T \leq \bar{T}, \\ 0 & \text{for } T > \bar{T}. \end{cases} \tag{4.1}$$

Thus for $T \leq \bar{T}$, $\dot{x}_2 \equiv 0$ which means that x_2 is constant. Let $\tilde{x}_2$ be the fourth component of u at the downstream state. Then at this state from $G_i(u) = 0$, $1 \leq i \leq 3$, we obtain $x_1 = \delta \tilde{x}_2$ and

$$\tilde{F}_1(V, T) := \frac{1}{2} \tilde{x}_2^2 + V - J + p(V, T) = 0,$$
$$\tilde{F}_2(V, T) := -\frac{1}{2} \tilde{x}_2^2 (V - \delta^2) - \varepsilon \tilde{x}_2 + JV - E + e(V, T) = 0. \tag{4.2}$$

Now if we let $J_1 = J - (1/2)\tilde{x}_2^2$ and $E_1 = E + \varepsilon \tilde{x}_2 - (1/2)\delta^2 \tilde{x}_2^2$, then this system of algebraic equations is the same as [13, equations (3.2.2)]. Thus from [13, Theorem 3.2.1], we have the following lemma.

LEMMA 4.1. *System (4.2) has a unique solution, say $(\tilde{V}, \tilde{T})$, in the region $0 < T < \bar{T}$, if*

$$\{V : \tilde{F}_1(V, \bar{T}) < 0\} \cap \{V : \tilde{F}_2(V, \bar{T}) > 0\} \neq \varnothing. \tag{4.3}$$

If the point $(\tilde{V}, \tilde{T})$ exists as above, then downstream state of the shock is

$$\tilde{u} = (\tilde{V}, \delta \tilde{x}_2, \tilde{T}, \tilde{x}_2), \tag{4.4}$$

for some $0 < \tilde{T} < \bar{T}$. Notice that also for $0 < T < \bar{T}$, system (2.1) reduces to the following system:

$$\mu_1 \dot{V} = \frac{1}{2} \tilde{x}_2^2 + V - J + p(V, T) =: G_{1\infty}(\hat{u}),$$
$$\mu \dot{x}_1 = x_1 - \delta \tilde{x}_2 =: G_{2\infty}(\hat{u}), \tag{4.5}$$
$$k\dot{T} = -\frac{1}{2} \left(x_1^2 - 2\delta \tilde{x}_2 x_1 + V \tilde{x}_2^2 \right) - \varepsilon \tilde{x}_2 - \frac{1}{2} V^2 + JV - E + e(V, T) = G_{3\infty}(\hat{u}),$$

where $\hat{u} = (V, x_1, T)$.

For $T > \bar{T}$, $\nu(T) = 0$. Thus $G_4(u) \equiv 0$ or $x_2 = V^{-1}(\delta x_1 - \varepsilon)$. If we substitute this value of x_2 into the other equations of system (2.1), then this system reduces to the system

$$\mu_1 \dot{V} = \frac{1}{2} V^{-2}(\delta x_1 - \varepsilon)^2 + V - J + p(V, T) =: G_{10}(\hat{u}),$$

$$\mu \dot{x}_1 = \left(1 - \frac{\delta^2}{V}\right) x_1 + \frac{\delta \varepsilon}{V} =: G_{20}(\hat{u}), \tag{4.6}$$

$$k\dot{T} = -\frac{1}{2}\left(1 - \frac{\delta^2}{V}\right)x_1^2 - \varepsilon \delta V^{-1} x_1 + \frac{1}{2}\varepsilon^2 V^{-1} - \frac{1}{2}V^2 + JV - E + e(V, T) =: G_{30}(\hat{u}),$$

where $\hat{u} = (V, x_1, T)$.

Let T_i be the third component of u_i, $0 \le i \le 3$, which is given by (2.2). If $T_0 < \bar{T} < T_1$, then consider systems (4.5) and (4.6) in the region $V > \delta^2$ and the downstream and upstream states of the shock are $\tilde{u}_0 = (\tilde{V}, \delta\tilde{x}_2, \tilde{T}, \tilde{x}_2)$ and $u_1 = (V_1, x_{11}, T_1, x_{21})$, respectively. This case corresponds to the fast ionizing shock. For $T_2 < \bar{T} < T_3$, these systems must be considered in $0 < V < \delta^2$. In this case the downstream and upstream states of the shocks are $\tilde{u}_2 = (\tilde{V}, \delta\tilde{x}_2, \tilde{T}, \tilde{x}_2)$, and $u_3 = (V_3, x_{13}, T_3, x_{23})$, respectively. In this case the shock between $\tilde{u}_2$ and u_3 corresponds to slow shock. In the following we prove the existence of structures.

4.1. Fast shock. In order to prove the existence of structure for fast shock we define

$$D_f = \{\hat{u} \in \mathbb{R}^3 : G_{10}(\hat{u}) < 0,\ G_{20}(\hat{u}) < 0,\ G_{30}(\hat{u}) > 0,\ V > \delta^2,\ \bar{T} < T < T_1\}. \tag{4.7}$$

Now, if we differentiate $G_{i0}(\hat{u})$, $1 \le i \le 3$, along the orbits of system (4.6) we obtain

$$\frac{dG_{10}(\hat{u})}{dt}\bigg|_{G_{10}(\hat{u})=0} = \delta V^{-2}(\delta x_1 - \varepsilon)\mu^{-1} G_{20}(\hat{u}) + k^{-1} p_T(\hat{u}) G_{30}(\hat{u}),$$

$$\frac{dG_{20}(\hat{u})}{dt}\bigg|_{G_{20}(\hat{u})=0} = \delta V^{-2}(\delta x_1 - \varepsilon)\mu_1^{-1} G_{10}(\hat{u}), \tag{4.8}$$

$$\frac{dG_{30}(\hat{u})}{dt}\bigg|_{G_{30}(\hat{u})=0} = -\mu^{-1}[G_{20}(\hat{u})]^2 - \mu_1^{-1}[G_{10}(\hat{u})]^2 + \mu_1^{-1} T S_V(\hat{u}) G_{10}(\hat{u}),$$

where in the last equality we used the identity $e_V + p = T S_V$. In D_f we have $\delta^2 < V$. It follows, from $G_{20}(\hat{u}) < 0$, that $\delta x_1 - \varepsilon < 0$ in D_f. Then from (4.8) we see that every orbit of system (4.6) is oriented in such a way that as t increases, it must go out of D_f when crossing the boundary of D_f on $G_{i0}(\hat{u}) \equiv 0$, at a point different from the rest point $\hat{u}_1 = (V_1, x_{11}, T_1)$. Also system (4.6) is gradient-like with respect to $h(\hat{u}) = T$, and the hypersurface $T = $ constant, intersects D_f on a set homomorphic to the unit disk. Finally, from

$$\frac{dT}{dt}\bigg|_{T=\bar{T}} = G_{30}(\hat{u})\big|_{T=\bar{T}}, \tag{4.9}$$

we see that every orbit of system (4.6) starting at a point on $\{\hat{u} \in \partial D_f : T = \bar{T}\}$, it must get into D_f as t increases. Therefore, system (4.6) together with D_f, the rest point $\hat{u}_1 = (V_1, x_{11}, T_1)$, and the gradient-like function $h(\hat{u}) = T$ satisfy all the conditions of

Theorem 3.1. Hence, by that theorem, there must be a point $\hat{u}_f$ on $\{\hat{u} \in \partial D_f : T = \bar{T}\}$ such that the orbit of system (4.6), initiating at this point, is lying in D_f and is running to $\hat{u}_1 = (V_1, x_{11}, T_1)$. We denote this orbit by $\hat{u}(t) = (\hat{V}(t), \tilde{x}_1(t), \tilde{T}(t))$, $t \in [0, \infty)$. Then $\hat{u}(0) = \hat{u}_f$ and $\lim_{t \to \infty} \hat{u}(t) = \hat{u}_1$.

Now consider system (4.5) with $\tilde{x}_2$ as $\tilde{x}_{2f} = [\tilde{V}(0)]^{-1}[\delta \tilde{x}_1(0) - \varepsilon]$. Since $\hat{u}_f \in D_f$, we get $G_{10}(\hat{u}_f) < 0$ and $G_{30}(\hat{u}_f) > 0$, then the equalities $G_{10}(\hat{u}_f) = \tilde{F}_1(\tilde{V}(0), \bar{T})$ and $G_{30}(\hat{u}_f) = \tilde{F}_2(\tilde{V}(0), \bar{T}) - (1/2)(\tilde{x}_1(0) - \delta \tilde{x}_2(0))^2$ follow $\tilde{F}_1(\tilde{V}(0), \bar{T}) < 0$ and $\tilde{F}_2(\tilde{V}(0), \bar{T}) > 0$. Thus from Lemma 4.1, this system admits the unique rest point say $\hat{u}_{0f} = (\tilde{V}_f, \tilde{x}_{1f}, \tilde{T}_f)$ in $T < \bar{T}$, where $\tilde{x}_{1f} = \delta \tilde{x}_{2f}$. This rest point is located on the boundary of the set

$$D'_f = \{\hat{u} \in \mathbb{R}^3 : G_{1\infty}(\hat{u}) < 0, \ G_{2\infty}(\hat{u}) < 0, \ G_{3\infty}(\hat{u}) > 0, \ V > \delta^2, \ T < \bar{T}\}. \qquad (4.10)$$

Notice that $\hat{u}_f = \hat{u}(0) \in \partial D'_f \cap \{\hat{u} : T = \bar{T}\}$. If we differentiate T and $G_{i\infty}(\hat{u})$, $0 \le i \le 3$, along the orbits of system (4.5) yield

$$\frac{dG_{1\infty}(\hat{u})}{dt}\bigg|_{G_{1\infty}(\hat{u})=0} = k^{-1} p_T G_{3\infty}(\hat{u}) > 0,$$

$$\frac{dG_{2\infty}(\hat{u})}{dt}\bigg|_{G_{2\infty}(\hat{u})=0} = 0,$$

$$\frac{dG_{3\infty}(\hat{u})}{dt}\bigg|_{G_{3\infty}(\hat{u})=0} = -\mu^{-1}\big[G_{2\infty}(\hat{u})\big]^2 - \mu_1^{-1}\big[G_{1\infty}(\hat{u})\big]^2 + \mu_1^{-1} T S_V(\hat{u}) G_{1\infty}(\hat{u}) < 0,$$

$$\frac{dT}{dt}\bigg|_{T=\bar{T}} = G_{3\infty}(\hat{u})\big|_{T=\bar{T}} > 0,$$

(4.11)

where in the third equality we used the identity $e_V + p = T S_V$ from thermodynamics. Thus the surface $G_{2\infty}(\hat{u}) = 0$ is invariant and the flow of system (4.5) goes out of D'_f on $\partial D'_f \setminus \{\hat{u} : G_{2\infty}(\hat{u}) = 0\}$. Since this system is gradient-like with respect to $h(\hat{u}) = T$, thus every orbit of this system initiating at a point on $\partial D'_f \cap \{\hat{u} : T = \bar{T}\}$ must be lying in D'_f for $t < 0$ and is running to the rest point $\hat{u}_{of}$ as t tends to minus infinity. Let $\tilde{\hat{u}}(t)$ be the orbit initiating at the point $\hat{u}_f$, which is defined for $t \le 0$.

Now define

$$u_f(t) = \begin{cases} (\tilde{\hat{V}}(t), \tilde{\hat{T}}(t), \tilde{\hat{x}}_1, \tilde{x}_{2f}) & \text{for } t \le 0, \\ (\hat{V}(t), \hat{T}(t), \hat{x}_1(t), \hat{x}_2(t)) & \text{for } t \ge 0, \end{cases} \qquad (4.12)$$

where $\hat{V}(t)$, $\hat{T}(t)$, $\hat{x}_1(t)$ are the components of $\hat{u}(t)$, $\hat{x}_2(t) = [\hat{V}(t)]^{-1}[\delta \hat{x}_1(t) - \varepsilon]$, $\tilde{x}_{2f} = \hat{x}_2(0)$, and $\tilde{\hat{V}}(t)$, $\tilde{\hat{T}}(t)$, $\tilde{\hat{x}}_1(t)$ are the components of $\tilde{\hat{u}}(t)$. Finally, notice that the first three components of u_f has continuous first derivative at $t = 0$. Therefore, we have proved the following theorem.

THEOREM 4.2. *Suppose that system (2.1) admits the rest points u_0 and u_1 and $T_0 < \bar{T} < T_1$, where T_0 and T_1 are the temperatures at u_0 and u_1, respectively. Moreover, assume that $\mu_1(T)$, $\mu(T)$, and $k(T)$ are smooth, bounded and bounded away from zero, and $v(T)$ is given by (4.1). Then there is a rest point of (2.1), say $\hat{u}_0$ in $T < \bar{T}$ and a complete orbit of this system corresponding to fast shock which is running from $u_{0f} = (\tilde{V}_f, \tilde{x}_1, \tilde{T}_f, \tilde{x}_{2f})$ to u_1 and the components of this orbit are monotone.*

4.2. Slow shock. Suppose that the rest points $u_i = (V_i, x_{1i}, T_i, x_{2i})$, $i = 2, 3$, exist and in (4.1) we have $T_2 < \bar{T} < T_3$. In order to show the existence of heteroclinic orbit corresponds to slow shock, we define

$$D_s = \{\hat{u} \in \mathbb{R}^3 : G_{10}(\hat{u}) < 0, \ G_{20}(\hat{u}) < 0, \ G_{30}(\hat{u}) > 0, \ 0 < V < \delta^2, \ \bar{T} < T < T_3\}. \quad (4.13)$$

First of all notice that in D_s we have $\delta x_1 - \varepsilon > 0$.

From the second equality in (4.8) it follows that the flow gets into D_s on $G_{20}(\hat{u}) = 0$. Also from third equality of (4.8) we see that the flow goes out of D_s on $G_{30}(\hat{u}) = 0$. Finally the first equality of (4.8) implies that the flow goes out of D_s on $\{\hat{u} \in \partial D_s : G_{10}(\hat{u}) = 0, \ \dot{G}_{10}(\hat{u}) > 0\}$ and gets into D_s on $\{\hat{u} \in \partial D_s : G_{10}(\hat{u}) = 0, \ \dot{G}_{10}(\hat{u}) < 0\}$, where "$\cdot = d/dt$" as before.

Here we wish to show that the flow goes out of D_s on

$$E = \{\hat{u} \in \partial D_s \setminus \{\hat{u}_3\} : G_{10}(\hat{u}) = 0, \ \dot{G}_{10}(\hat{u}) = 0\}. \quad (4.14)$$

In order to do this, it is sufficient to show that the second derivative of $G_{10}(\hat{u})$ is positive on $G_{10}(\hat{u}) = \dot{G}_{10}(\hat{u}) = 0$.

$$\frac{d^2 G_{10}(\hat{u})}{dt^2}\Bigg|_{\substack{G_{10}(\hat{u})=0 \\ \dot{G}_{10}(\hat{u})=0}} = -\frac{G_{20}(\hat{u})}{\mu^2}\left\{ -\frac{\delta^2}{V^2} G_{20}(\hat{u}) - \frac{\delta}{V^2}(\delta x_1 - \varepsilon)\left(1 - \frac{\delta^2}{V}\right) \right.$$

$$-\frac{\delta^2}{V^2} p_{TT}[p_T]^{-2}(\delta x_1 - \varepsilon)^2 G_{20}(\hat{u}) \quad (4.15)$$

$$\left. +\frac{\mu}{k} p_T G_{20}(\hat{u}) + \frac{\mu}{k}\frac{\delta}{V^2}(\delta x_1 - \varepsilon) e_T(\hat{u}) \right\}.$$

This is positive for each of the following cases.

   CASE 1. When $|G_{20}(\hat{u})|$ is small on $G_{10}(\hat{u}) = 0$.
   CASE 2. When $p_{TT} \geq 0$ and μ/k is small.
   CASE 3. When $p_{TT} \geq 0$ and $p_T G_{20} + (\delta/V^2)(\delta x_1 - \varepsilon) e_T(\hat{u}) \geq 0$
   CASE 4. The gas is ideal with $\gamma < 2$.

In order to see Case 4, notice that for the ideal gas we have $p = nRT/V$ and $e = nRT/(\gamma - 1)$. Thus $p_{TT} = 0$ and $e_T - Vp_T = nR(2 - \gamma)/(\gamma - 1)$ which is positive for $\gamma \leq 2$. On the other hand, in D_s we have $\delta x_1 - \varepsilon \geq 0$. Thus

$$p_T G_{20}(\hat{u}) + \frac{\delta}{V^2}(\delta x_1 - \varepsilon) e_T(\hat{u}) = p_T\left[\left(1 - \frac{\delta^2}{V}\right)x_1 + \frac{\delta \varepsilon}{V}\right] + \frac{\delta}{V^2}(\delta x_1 - \varepsilon) e_T$$

$$= \frac{\delta}{V^2}(-Vp_T + e_T)(\delta x_1 - \varepsilon) + x_1 p_T > 0. \quad (4.16)$$

Therefore Case 3 implies Case 4.

Now, system (4.6) together with D_s, the rest point $\hat{u}_3 = (V_3, x_{13}, T_3)$, and the function $h(\hat{u}) = T$ satisfy all conditions of Theorem 3.3. Thus there is an orbit of this system initiating at a point $\hat{u}_s \in \{\hat{u} \in \partial D_s : T = \bar{T}\}$, lying in D_s and is running to $\hat{u}_3$. We denote this orbit by $\hat{u}^+(t) = (V^+(t), x_1^+(t), T^+(t))$, $0 \leq t < \infty$.

Now, consider system (4.5). If we substitute for $\tilde{x}_2$ in this system by $\tilde{x}_{2s} = [V^+(0)]^{-1}[\delta x_1^+(0) - \varepsilon]$, then similar to the fast case this system admits a unique rest point say $\hat{u}_{2s} = (\tilde{V}_s, \tilde{x}_{1s}, \tilde{T}_s)$ in $T < \bar{T}$, where $\tilde{x}_{1s} = \delta \tilde{x}_{2s}$ and $(\tilde{V}, \tilde{T})$ is given in (4.4). This rest point and $\hat{u}_s = \hat{u}^+(0)$ are located on the boundary of

$$D'_s = \{\hat{u} \in \mathbb{R}^3 : G_{1\infty}(\hat{u}) < 0,\ G_{2\infty}(\hat{u}) < 0,\ G_{3\infty}(\hat{u}) > 0,\ 0 < V < \delta^2,\ T < \bar{T}\}. \quad (4.17)$$

Similar to the case of fast shock, the orbit of system (4.5) corresponds to the initial condition $\hat{u}(0) = \hat{u}_s = \hat{u}^+(0)$ is defined for $t < 0$, lies in D'_s and is running from $\hat{u}_{2s}$. Let $u^-(t) = (V^-(t), x_1^-(t), T^-(t))$ be this orbit. Now define

$$u_s(t) = \begin{cases} (V^+(t), x_1^+(t), T^+(t), x_2^+(t)) & \text{for } t \geq 0, \\ (V^-(t), x_1^-(t), T^-(t), \tilde{x}_{2s}) & \text{for } t \leq 0, \end{cases} \quad (4.18)$$

where $x_2^+(t) = [\delta x_1^+(t) - \varepsilon][V^+(t)]^{-1}$ and $\tilde{x}_{2s} = x_2^+(0)$, as before. Thus we have proved the following theorem.

THEOREM 4.3. *Suppose that system (2.1) admits the rest points u_2 and u_3 and $T_2 < \bar{T} < T_3$, where T_i is the temperature at the rest point u_i, $i = 2, 3$, and the viscosity parameters $\mu_1(T)$, $\mu(T)$, $k(T)$, and $\nu(T)$ are as in Theorem 4.2. Moreover, one of the above four cases holds. Then the ionizing slow shock admits structure. Along this structure, density, temperature and the vertical component of velocity are increasing, but the vertical component of the magnetic field is nondecreasing.*

4.3. Switch-on and switch-off shocks. As we mentioned in Section 2, switch-on and switch-off shocks occur if in system (2.1) we have $\varepsilon = 0$ (i.e., in the absence of electric field). Here we obtain the structure for switch-on and switch-off ionizing shocks as limits of structure for fast and slow shocks, respectively. We do this for switch-on shock, the same arguments work for switch-off too.

Suppose $\bar{u}_i$, $i = 0, 1$, the rest point corresponds to the switch-on shock exist and the ionizing temperature $\bar{T}$ is between the temperature at $\bar{u}_0$ and $\bar{u}_1$. Then by [13, Theorem 3.3.1], $u_0(\varepsilon)$ and $u_1(\varepsilon)$, the rest points corresponding to fast shock exist for $\varepsilon > 0$ and small, moreover, $\lim_{\varepsilon \to 0} u_i(\varepsilon) = \bar{u}_i$. Thus $\bar{T}$ is between the temperature at $u_0(\varepsilon)$ and $u_1(\varepsilon)$.

Now choose $\varepsilon_m \to 0$. By Theorem 4.2, for each ε_m there is a heteroclinic orbit, say $\gamma_m(t)$, which is running from a rest point of system (2.1), say u_{0fm} to the rest point $u_1(\varepsilon_m)$. Since $\{u_{0fm}\}$ is bounded, it contains a convergent subsequence. We may assume that $\lim_{m \to \infty} u_{0fm} = \bar{u}_{0n}$.

The sequence $\{\gamma_m(t)\}$ is uniformly bounded and componentwise monotone. Therefore, by Helly's theorem [17], it has a subsequence which is uniformly convergent on the compact subsets of $\mathbb{R}$. We may assume that $\gamma_m(t)$ converges to a continuous function, say $\gamma_0(t)$. Then similar to the fast shock, we can show that $\gamma_0(t)$ is componentwise strictly monotone and intersects the surface $T = \bar{T}$ at a single point, say t_0. Moreover, similar to the proof of Theorem 3.4 we can show that $\gamma_0(t)$ are componentwise differentiable except its last component at t_0 and satisfies system (2.1). Hence we have proved the following theorem.

THEOREM 4.4. *If the rest point $\bar{u}_i$, $i = 0, 1$, exists, then under the same conditions of Theorem 4.2, the ionizing switch-on shock admits structure, which has all of the properties of the ionizing fast shock. Similarly, if $\bar{u}_2$ and $\bar{u}_3$ exist, then under the same conditions of Theorem 4.3 the structure for switch-off shock exists and has the same properties as those of slow shock.*

4.4. Transverse magnetic field shock. In the case of transverse magnetic field in system (2.1), δ must be considered zero. In this case system (2.1) admits two rest points which are the limiting case of the rest points corresponding to fast shock as δ tends to zero. Thus similar to switch-on shock the structure for the related ionizing shock can be found from the structure for ionizing fast shock as δ tends to zero. Also this structure can be found directly by using the same technique which is used for the ionizing fast shock.

5. Existence of structure for $\sigma = 0$ ahead of the shock and very large behind it. In this section, we show that the ionizing fast, slow, switch-on, switch-off admit structure when the electrical conductivity coefficient is zero ahead of the shock and is very large behind it. Thus in this case we have

$$v(T) = \begin{cases} \infty & \text{for } T \leq \bar{T}, \\ v_2(T) & \text{for } T > \bar{T}, \end{cases} \tag{5.1}$$

where $v_2(T)$ as well as $\mu_1(T)$, $\mu(T)$, and $k(T)$ are smooth (i.e., C^1) positive value with $v_2(T) \ll 1$. As we said in the introduction, this case has been discussed in [2, 18] for the case of transverse magnetic field (i.e., $\delta = 0$) and $k(T)$ or $\mu_1(T)$ is assumed to be zero and the gas is taken to be ideal gas. In this section similar to the previous section, we consider the problem for all types of the ionizing shocks as follows.

5.1. Fast shocks. Suppose the rest points u_0 and u_1 exist and $\bar{T}$ is between the temperature at u_0 and u_1. Here we define

$$D_f = \{u \in \mathbb{R}^4 : G_1(u) < 0,\ G_2(u) < 0,\ G_3(u) > 0, \\ G_4(u) < 0,\ V > \delta^2,\ \bar{T} < T < T_1\}, \tag{5.2}$$

where $u = (v, x_1, T, x_2)$ and T_1 is the value of temperature at u_1. First of all, $V > \delta^2$, $G_2(u) < 0$, and $G_4(u) < 0$ imply that $x_2 < 0$ and $x_1 < 0$ in D_f. Now, if we differentiate $G_i(u)$, $0 \leq i \leq 4$, and T along the orbits of system (2.1) we obtain

$$\dot{G}_1(u)\Big|_{G_1(u)=0} = v_2^{-1} x_2 G_4(u) + k^{-1} p_T G_3(u),$$

$$\dot{G}_2(u)\big|_{G_2(u)=0} = -\delta v_2^{-1} G_4(u),$$

$$\dot{G}_3(u)\Big|_{G_3(u)=0} = -\mu^{-1}[G_2(u)]^2 - v_2^{-1}[G_4(u)]^2 - k^{-1}[G_3(u)]^2 + \mu_1^{-1} TS_V G_1(u), \tag{5.3}$$

$$\dot{G}_4(u)\Big|_{G_4(u)=0} = -\delta \mu^{-1} G_2(u) + \mu_1^{-1} x_2 G_1(u),$$

$$\dot{T}\big|_{T=\bar{T}} = G_3(u),$$

where in the third equality we used the identity $e_V = p + TS_V$ which is a result of second law of thermodynamics. Thus the flow goes out of D_f on $\{u \in \partial D_f : T > \bar{T}\}$ and gets into D_f on $\{u \in \partial D_f : T = \bar{T}\}$. Moreover, system (2.1) is gradient-like with respect to $h(u) = T$ in D_f. Similar to the fast shock in the previous section, it follows from Theorem 3.1, there exists a point, say u_f, on $\{u \in \partial D_f : T = \bar{T}\}$ such that the orbit of system (2.1), initiating at this point, is lying in D_f and running to u_1 as t tends to ∞ and its components are monotone. We denote this orbit by $u_f^+(t)$, $t \in [0, \infty)$. Let $\tilde{x}_2$ be the value of x_2 at the point $u_f = u_f^+(0)$. Now, consider system (4.5) for this value of $\tilde{x}_2$. Similar to the fast and slow shocks in Section 4, we can show system (4.5) admits a unique rest point in $T < \bar{T}$, say u_{0f} which is on the surface $x_2 = \tilde{x}_2$ and the orbit of system (4.5) initiating at the point u_f is running from u_{0f}; lying on the surface $x_2 = \tilde{x}_2$, and the other its components are strictly monotone. We call this orbit $u_f^-(t)$. Now define

$$u_f(t) = \begin{cases} u_f^-(t) & \text{for } t \leq 0, \\ u_f^+(t) & \text{for } t \geq 0. \end{cases} \tag{5.4}$$

This orbit is the structure for ionizing fast shock, in this case. Along this orbit, V, x_1, and x_2 are decreasing and T is increasing. Hence we have the following theorem.

THEOREM 5.1. *Let in Theorem 4.2, $v(T)$ be given by (5.1) and the other assumptions remain the same. Then all of its conclusions are valid.*

5.2. Slow shock. The technique we use here to show the existence of structure for slow shock, for $v(T)$ is given by (5.1), is different from the previous one. The reason for using a different approach is that Theorems 3.1, 3.2, and 3.3 cannot be applied in this case. The technique we use here is to obtain the structure for ionizing slow shock, corresponding to $v(T)$ which is given by (5.1), as a perturbation of the structure of ionizing slow shock which is found in Section 4.

Let $x = (V, x_1, T)^T$, $y = x_2$, $v = \varepsilon$, $f(x, y) = (G_1(u), G_2(u), G_3(u))^T$ and $g(x, y) = G_4(u)$, where $u = (V, x_1, T, x_2)^T$, v and G_i, $1 \leq i \leq 4$, are the same as in system (2.1). Then system (2.1) is of the form of system (3.9). We consider this system on the bounded domain

$$D_1 \times D_2 = \left\{ (V, x_1, T) : \frac{V_3}{2} < V < \frac{1}{2}(V_1 + V_2), \ \frac{T_2}{2} < T < 2T_3, \ \frac{x_{13}}{2} < x_1 < 2x_{12} \right\} \\ \times \left\{ x_2 : \frac{x_{23}}{2} < x_2 < 2x_{22} \right\}. \tag{5.5}$$

Thus f and g have continuous second derivative on $D_1 \times D_2$ and u_3 is the only rest point of this system in $D_1 \times D_2$. It is known that this rest point is hyperbolic [10, 11]. For $\varepsilon \neq 0$ its linearized matrix has two positive eigenvalues and two negative. For $\varepsilon = 0$ it has one positive and two negative eigenvalues. For details about the eigenvalues the reader is referred to [13, Section 2.5].

In order to see conditions (C_1) and (C_2) of Theorem 3.5 hold, notice that the equation $g(x, y) = 0$, namely, $-\delta x_1 + V x_2 + \varepsilon = 0$, implies $x_2 = V^{-1}(\delta x_1 - \varepsilon)$. Hence condition (C_1) holds too. Also the one by one matrix $\partial g(x, G(x))/\partial y = V$ has one positive eigenvalue uniformly bounded away from zero. This means that condition (C_2) holds.

Now, consider the orbit $u_s(t)$ which is given by (4.9). For $t \geq 0$ this orbit is a solution of (2.1) corresponding to $\nu = 0$. This orbit intersects the surface $T = \bar{T}$ transversely, runs to u_3 and lies in $D_s = \{u \in \mathbb{R}^4 : G_1(u) < 0,\ G_2(u) < 0,\ G_3(u) > 0,\ G_4(u) = 0\}$. Thus condition (C$_4$) holds too. Therefore by Theorem 3.5 there is $\nu_0 > 0$ such that for each $0 < \nu < \nu_0$ there exists an orbit of system (2.1) corresponding to ν, intersecting the surface $T = \bar{T}$, transversely, running to u_3, and lying in the region $\{u \in \mathbb{R}^4 : G_1(u) < 0,\ G_2(u) < 0,\ G_3(u) > 0,\ T \leq \bar{T}\}$. We denote this orbit by $u_{\nu s}^+(t)$, $t \in [0, \infty)$. Along this orbit $-V$, $-x_1$, and T are increasing.

Let, in system (4.5), $\tilde{x}_2$ be the value of x_2 at $u_{\nu s}(0)$. Then the orbit $u_s(t)$ which is given by (4.9) is a solution of this system for $t < 0$, initiating at $u_{\nu s}(0)$ and is running from a rest point of this system, say u_{2s}, which exists similar to previous cases since $u_{\nu s}(0) \in \{u \in \mathbb{R}^4 : G_1(u) < 0,\ G_2(u) < 0,\ G_3(u) > 0,\ T \leq \bar{T}\}$. Along this orbit $-V$, $-x_1$, and T are increasing. We call this orbit $u_{\nu s}^-(t)$. Now define

$$u_{\nu s}(t) = \begin{cases} u_{\nu s}^+(t) & \text{for } t \geq 0, \\ u_{\nu s}^-(t) & \text{for } t \leq 0, \end{cases} \tag{5.6}$$

which is the structure for ionizing slow shock corresponding to $\nu(T)$ which is given by (5.1). Therefore we have the following theorem.

Theorem 5.2. *Let in Theorem 4.3, $\nu(T)$ be given by (5.1) and the other assumptions be the same. Then all of its conclusions remain valid, except the x_2 component of the structure orbit may not be monotone.*

5.3. Switch-on and Switch-off shocks. In the previous section, by taking advantage from the existence of structure for fast and slow ionizing shocks and componentwise monotonicity of the related orbits, and using Helly's theorem, we were able to prove the existence of structure for switch-on and switch-off ionizing shocks as a limiting of the structure of the above shocks as ε tends to zero. Here in this section we have all of the above situations with one exception for the orbit of the slow shock. This exception is that the x_2 component of this orbit is of bounded variations instead of monotone. In the proof we used monotonicity for applying Helly's theorem. But Helly's theorem works for the class of bounded variations, too [17]. Thus we have the following theorem.

Theorem 5.3. *Let, in Theorem 4.4, $\nu(T)$ be given by (5.1) and the other assumptions remains the same. Then the structure for ionizing switch-on and switch-off exists and has all of the properties the same except that the x_2-component of the structure for switch-off shock is of bounded variations instead of being monotone.*

5.4. Transverse magnetic field shock. In this case, again system (2.1) admits two rest points and by the same argument which is used in the previous section we can show this shock admits structure. Here we should mention that this is the only case which is considered in literature, where at least one of the viscosity parameters, μ_1, μ or k is assumed to be zero.

6. Existence of structure for σ very small ahead of the shock and very large behind it. In order to prove the existence of structure for slow as well as fast shock,

when the electrical conductivity is very small ahead of the shock and very large behind it, we consider a sequence of a system of ordinary differential equations related to system (2.1) as follows:

$$
\begin{aligned}
\dot{V} &= \frac{1}{\mu_1(T)} G_1(u) := H_1(u), \\
\dot{x}_1 &= \frac{1}{\mu(T)} G_2(u) := H_2(u), \\
\dot{T} &= \frac{1}{k(T)} G_3(u) := H_3(u), \\
\dot{x}_2 &= \frac{1}{\nu_m(T)} G_4(u) := H_4(u),
\end{aligned}
\tag{6.1}
$$

where $G_i(u)$, $0 \le i \le 3$ are the same as before. Here we assume that the viscosity parameters $\mu_1(T)$, $\mu(T)$, $k(T)$, and $\nu_m(T)$ are smooth functions of T, bounded and bounded away from zero. Moreover, we assume that

$$
\nu_m(T) = \begin{cases}
\nu_1(T) & \text{for } T < \bar{T} - \dfrac{1}{m+1}, \\
\nu_{0m}(T) & \text{for } \bar{T} - \dfrac{1}{m+1} < T < \bar{T} + \dfrac{1}{m+1}, \\
\nu_2(T) & \text{for } T > \bar{T} + \dfrac{1}{m+1},
\end{cases}
\tag{6.2}
$$

where $\bar{T}$ is the ionizing temperature and $\nu_{0m}(T)$ is a smooth monotone decreasing function such that $\nu_m(T)$ is smooth too. About system (6.2) we know the following facts from [3, 4, 10, 11, 13, 20, 22].

Theorem 6.1. *System (6.1) (as well as system (2.1)) is gradient-like with respect to the real function*

$$
P(u) = T^{-1}\left(-G_3(u) + TS\right),
\tag{6.3}
$$

where $G_3(u)$ and S are the same as before [10].

Theorem 6.2. *For fixed m, there is a unique orbit of system (6.1), say $\gamma_m(t)$, which is running from u_0 to u_1. For all m these orbits are lying in the bounded domain*

$$
D_f = \{u \in \mathbb{R}^4 : G_1(u) < 0,\ G_2(u) < 0,\ G_3(u) > 0,\ G_4(u) < 0\}.
\tag{6.4}
$$

Along these orbits x_1, x_2, and T are decreasing, but $T(t)$ is increasing (see [10] and [13, Theorem 2.2.2]).

Theorem 6.3. *For fixed m, there is an orbit of system (6.1), say $\gamma_m(t)$, which is running from u_2 to u_3. For all m these orbits are lying in a bounded domain*

$$
B \subset \{u \in \mathbb{R}^4 : p(u_2) - a < p(u) < p(u_3) + a\},
\tag{6.5}
$$

for some $a > 0$ and small. (See [3, Theorem 4.1].)

Here we should mention that in [3, Theorem 4.1] proved the existence of heteroclinic orbit between u_2 and u_3 for the six-dimensional system (2.3) in [3]. The authors in

[4, page 435] showed that the above complete orbits must be lying in the subspace $y_1 = y_2 = 0$ (y_1 and y_2 are the fifth and sixth variables in their work, the first four variables are the same as ours). Now, if the substitute $y_1 = y_2 = 0$ in their system (2.4), we obtain our system (6.1).

Another point about Conely and Smoller's work is that in their work they had never assumed that the viscosity parameters are constants, nor they mentioned that the viscosities are functions of T. However, their proofs are organized in such a way that they work even if the viscosities are functions of u, as long as they are smooth, bounded and bounded away from zero. Thus their Theorem 4.1 in [3] implies Theorem 6.3 above.

Now we can prove the existence of structure for ionizing shocks as follows.

6.1. Fast shock. Consider system (6.1), the rest points u_0 and u_1 and assume that $T_0 < \bar{T} < T_1$ where T_i, $i = 0, 1$, is the temperature at u_0 and u_1, respectively.

According to Theorem 6.2 for $m = 1, 2, \ldots$, there is a unique orbit, $y_m(t)$, which is running from u_0 to u_1 and is lying in the bounded domain D_f. Thus by Theorem 6.1, $\{y_m(t)\}$ contains a subsequence which is convergent to a continuous function $y(t)$. Let $y_m(t) = (V_m(t), x_{1m}(t), T_m(t), x_{2m}(t))$ and $y(t) = (V(t), x_1(t), T(t), x_2(t))$. Since $H_1(u)$ is continuous and bounded on $\bar{D}_f$, from the first equation of system (6.1) and Lebesgue dominated convergence theorem we have

$$V(t) = \lim_{m \to \infty} V_m(t) = \lim_{m \to \infty} \left[V_m(0) + \int_0^t H_1(y_m(s))ds \right] = V(0) + \int_0^t H_1[y(s)]ds. \quad (6.6)$$

This means that for $t \in \mathbb{R}$, we have

$$\dot{V}(t) = H_1(y(t)). \quad (6.7)$$

Similarly,

$$\dot{x}_1(t) = H_2(y(t)), \qquad \dot{T}(t) = H_3(y(t)). \quad (6.8)$$

By the proof of [13, Theorem 2.2.2], the flow goes out of D_f on $H_3(u) \equiv 0 \equiv G_3(u)$. Thus $H_3(y(t)) > 0$ for all $t \in \mathbb{R}$. Thus $y(t)$ intersects the surface $\{u \in \mathbb{R}^3 : T = \bar{T}\}$ at a single point. Therefore by Theorem 3.4, $y(t)$ is differentiable for all $t \in \mathbb{R}$ except at a single point, and satisfies system (2.1). On the other hand, $y(t)$ is componentwise monotone and bounded, thus $\lim_{t \to \infty} y(t)$ exists and is a rest point of system (2.1) in $D_f = \{u \in \mathbb{R}^4 : T > \bar{T}\}$. But the only possibility is u_1. That is $\lim_{t \to \infty} y(t) = u_1$. Similarly $\lim_{t \to -\infty} y(t) = u_0$. Finally, in [10], Germain showed that the stable manifold of system (2.1) at u_1 is one-dimensional. Therefore the above heteroclinic orbit is unique. Hence we have proved the following theorem.

THEOREM 6.4. *Let $v_1(T)$ and $v_2(T)$ be smooth, bounded and bounded away from zero and assume that in Theorem 5.1 $v(T)$ is given by*

$$v(T) = \begin{cases} v_1(T) & \text{for } T < \bar{T}, \\ v_2(T) & \text{for } T \geq \bar{T}. \end{cases} \quad (6.9)$$

Then the heteroclinic orbit corresponding to the structure for fast ionizing shock exist. Along this orbit, $\dot{V}(t)$, $\dot{x}_1(t)$, and $\dot{T}(t)$ are continuous and $\dot{x}_2(t)$ has a single jump discontinuity. Moreover, along this orbit $T(t)$ is increasing, and $x_1(t)$, $x_2(t)$, and $V(t)$ are decreasing.

REMARK 6.5. Here we should mention that the above theorem can be proved by the technique which is used in Sections 3 and 4 for fast shock. But the above technique gives more information about the smoothness of the components of the structure of the shock.

6.2. Slow shock. Suppose that u_i, $i = 2, 3$, exist and $T_2 < \bar{T} < T_3$, where T_i is the value of the temperature at the point u_i, as before. According to Theorem 6.3, for each m, there is an orbit of system (6.1) which is lying in a bounded set, say $D_s \subset B \subset \mathbb{R}^4$ and is running from u_2 to u_3.

We denote this orbit by $y_m(t)$. Similar to the fast shock, this sequence contains a subsequence which converges uniformly on compact subsets of $\mathbb{R}$ to a continuous function say $y(t) = (V(t), x_1(t), T(t), x_2(t))$. For $u = (V, x_1, T, x_2) \in D_s$ with $T \neq \bar{T}$ and large value of m, there is a neighborhood of u such that in this neighborhood systems (2.1) and (6.1) coincide to each other. Thus by Theorem 3.4, $y(t)$ is differentiable on $\mathbb{R} \setminus \{t_0 : T(t_0) = \bar{T}\}$. Moreover, for all t, $y(t)$ satisfies the first three equations of system (2.1), and satisfies the last equation of (2.1) for t with $T(t) \neq \bar{T}$, we have

$$\dot{x}_2(t) = \frac{1}{\nu(T(t))} G_4(y(t)). \tag{6.10}$$

On the other hand, $\{x_{2m}(t)\}$ is uniformly bounded and its total variations is uniformly bounded, thus by Helly's first theorem [17], $x_2(t)$ must be of bounded variations. Hence on the set $\{t_0 : T(t_0) = \bar{T}\}$, $T(t)$ is differentiable almost everywhere.

Finally for $\tilde{T} \in \{T : T_2 < T < T_3\} \setminus \{\bar{T}\}$, $T_m(t)$ intersects the plane $T = \bar{T}$ at a point, say t_m, for the first time as t increases. Systems (6.1) and (2.1) are both autonomous, thus we may assume that $t_m = t_0$ for all m and some $t_0 \in \mathbb{R}$. Therefore, $y(t_0) = \lim_{m \to \infty} y_m(t_0)$ is located on the plane $T = \tilde{T}$. If $\tilde{T} < \bar{T}$, then $y(t)$ is differentiable and is lying in the region $\tilde{D}_s = D_s \cap \{u : T < \bar{T}\}$, for all $t \le t_0$. Since by Theorem 6.1, system (2.1) is gradient-like, $\lim_{t \to -\infty} y(t)$ exists and is a rest point of this system in this region. But u_2 is the only possibility. Therefore, $\lim_{t \to -\infty} y(t) = u_2$. Similarly, $\lim_{t \to \infty} y(t) = u_3$. Thus we have proved the following theorem.

THEOREM 6.6. *Under the assumptions of Theorem 5.2, for $\nu(T)$ given by (6.1), the heteroclinic orbit corresponding to slow ionizing shock exists, which is running from u_2 to u_3. Along this orbit $\dot{V}(t)$, $\dot{x}_1(t)$, and $\dot{T}(t)$ are continuous. For $t \notin \{t : T(t) = \bar{T}\}$, $\dot{x}_2(t)$ exists too. On each point of $\{t : T(t) = \bar{T}\}$, $x_2(t)$ has a jump discontinuity. This set of discontinuities has measure zero.*

REMARKS. (1) By using the same technique which is used in Section 5, we can show that the switch-on ionizing shock admits structure. Similarly, the structure for ionizing transverse magnetic field exists.

(2) Although the case

$$\nu(T) = \begin{cases} \nu_1(T) & \text{for } T \le \bar{T}, \\ \infty & \text{for } T > \bar{T}, \end{cases} \tag{6.11}$$

is not considered in the literature, but the cases of ionizing fast, switch-on and the case of transverse magnetic field can be solved with the same technique which is used in this section and the previous sections.

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A. AGHAJANI: INSTITUTE FOR STUDIES IN THEORETICAL PHYSICS AND MATHEMATICS, P.O. BOX 19395-5746, TEHRAN, IRAN

Current address: SCHOOL OF SCIENCES, TARBIAT MODARRES UNIVERSITY, P.O. BOX 14155-4838, TEHRAN, IRAN

E-mail address: aghajani@karun.ipm.ac.ir

M. HESAARAKI: INSTITUTE FOR STUDIES IN THEORETICAL PHYSICS AND MATHEMATICS, P.O. BOX 19395-5746, TEHRAN, IRAN

Current address: DEPARTMENT OF MATHEMATICAL SCIENCES, SHARIF UNIVERSITY OF TECHNOLOGY, P.O. BOX 11365-9415, TEHRAN, IRAN

E-mail address: hesaraki@karun.ipm.ac.ir