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Presentation of Visualization Problems Using an Expanded Coded Plan Technique

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Abstract. A teaching technique for presenting visualization problems is discussed which requires students to generate both multiview (orthographic) and pictorial views of simple to complex objects from symbolic information. The technique makes use of an expanded coded plan or base design method for describing objects in space, and transcends the traditional isometric/orthographic, orthographic/isometric translation exercises. The technique can be used in traditional or CAD-based engineering design graphics courses and can be implemented using either sketching or instrument construction methods.

Key Words: Visualization, Coded plan, Teaching techniques, Geometric modeling, Sketching, Mental rotation

1 History of Visualization in Engineering Graphics and the Origin of the Symbolic Code

Visualization has been an integral part of graphics instruction since McGraw-Hill published Thomas French's second edition of Engineering Drawing in 1918. Traditionally, visualization exercises appear as transformation problems for the student where an object is pictured in an isometric view to be converted into a multiview drawing or vice versa. These exercises also lead into "missing view" and "missing lines" problems where the student is required to use projection to solve for the shape of an object when given either two complete principal orthographic views or when given three partially complete principal orthographic views. These problems are troublesome for most engineering and technology students. Added to these traditional problems are the difficulties experienced by students in trying to interpret wire-frame images of objects as typically presented in computer-aided design (CAD) applications. The author has felt for several years that some additional practice and instruction in visualization was necessary for students. The proper vehicle for this practice appears to be freehand sketching. Freehand techniques are one constant that has been a part of engineering design graphics for at least a century, and this same technique is still called for in the CAD environment. The author instituted a sketchbook or journal as a part of the fundamental

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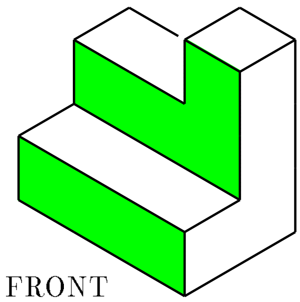
graphics course in 1991, but the student outcomes were not predictable in the sketchbooks, no consistency was evident, and no discernible change in the students' visualization abilities was seen. The author was selected as one of the twenty-five participants at a National Science Foundation sponsored workshop administered by Sheryl SORBY and Beverly BAARTMANS at Michigan Technological University in June, 1995. The symbolic code that is presented here is the outgrowth of the experiences at this workshop.

2 Visualization Practice in Mathematics Education

Mathematics instruction contains modules on visualization as a part of the discussion and practice in various aspects of geometry. A technique for presenting three-dimensional objects for students to sketch is presented in this discipline by a method called "base design". The premise is that an object is to be built using cubes or dice. The plan of a structure is drawn as a series of squares whose heights are indicated by a single digit centered inside each square in the plan. See Fig. 1.

In the figure, the drawing on the left shows a shape. The drawing on the right tells you how many cubes are on each base square. The drawing on the right is called a base design.



2	2	3
1	1	1

Figure 1: Base Design (cf. [2], page 594)

In general, the geometric models presented in mathematics textbooks are limited to objects that can be described by cube stacking in a square matrix, but a part of the exercises presented also involves developments (folded patterns) and mental rotations of similarly shaped objects. Holes, voids, and inclined and oblique surfaces are generally not discussed as a part of geometry in elementary mathematics.

3 The Sorby/Baartmans Model

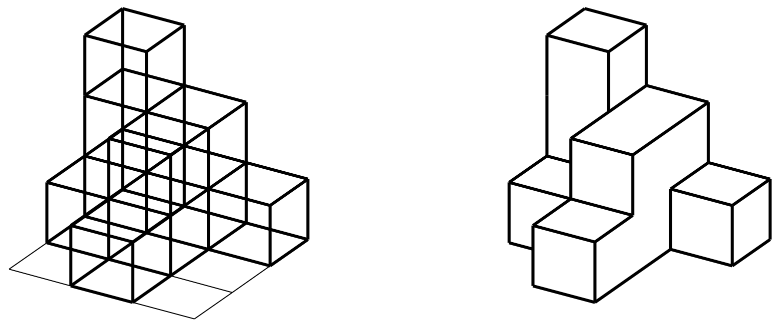
Beverly Gimmestad BAARTMANS and Sheryl A. SORBY use the base design technique in their book [1]. In this publication, the authors refer to the technique as the "coded plan" and take particular care to have the students practice at both orthographic multiview sketches as well as isometric sketches from the plans. In form, these authors follow the pattern established in mathematics textbooks; giving a visual coded plan or multiview images of a structure and having the students sketch the pictorial (isometric) view from these.

4 Standardization of the Coded Plan

For uniformity of communication, the author has developed a technique which may be used to code a simple structure for students to draw using only symbolic rather than visual information. Rather than diagram a geometric structure in a visual form, the information is given strictly as a sequence of numbers separated by commas and semicolons to indicate the relative position of the code in a standardized manner. For purposes of simplicity (for the students) the coded structures are limited to those which can be constructed in a three unit cube matrix. An integer number in the coded plan is used to indicate how many units high each stack in the matrix is. A semicolon is used to indicate the separation of rows of stacks. Stacks are listed in order from left to right; back to front. For stacks which are vacant, the number zero is used; therefore, the pictorial image and coded plan for the structure in Fig. 2 can be written as:

$$3, 2, 1;\ 1, 2, 0;\ 0, 1, 0$$

3	2	1
1	2	0
0	1	0



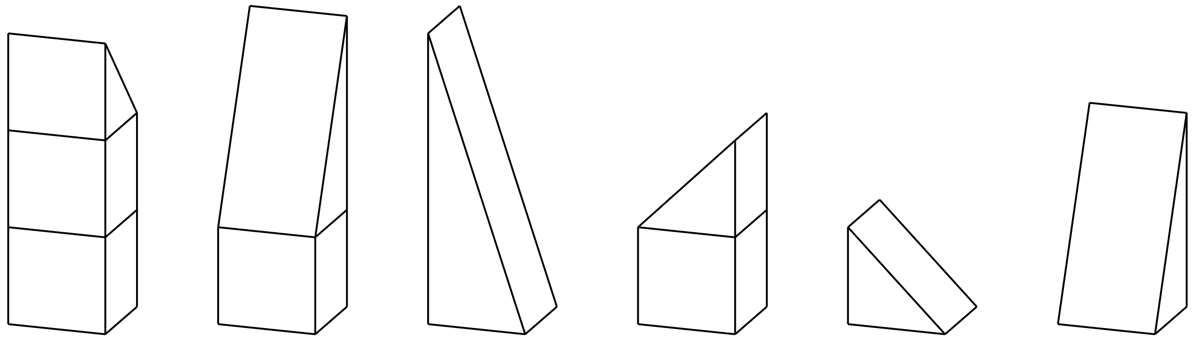
Coded Plan — Wire-Frame — Pictorial

Figure 2: Standardized Base Plan

If more complex shapes are desired, a larger matrix can be used. Using such a code to assign objects for students to sketch not only is a fast technique, but it forces the students to convert from a totally symbolic form of information to a subsequent graphical form. In practice, the author has students provide a total of 11 sketches of each structure that is assigned; three views which correspond roughly to the primary orthographic views and eight pictorial views, one from above each of the four corners of the structure (bird's-eye views) and another four views drawn from points of view that are below the object (worm's-eye views); If the object is tumbled in space, a total of 24 pictorial sketches could be drawn, but 16 of these are redundant. So, even though tumbling the object in space is encouraged, only eight of the pictorial views are required of the students.

5 Symbolizing Inclined Surfaces and Other Features

Mathematics texts and the Sorby/Baartmans model do not proceed beyond objects which only contain normal surfaces, but for purposes of having engineering and technology students develop their visualization skills into more complex shapes, additional coding has been developed by the author to impart the concept of inclined surfaces and holes in objects. Inclined surfaces can be coded by placing two integer digits, which are separated by an arrow, in the position of the code for any stack. The arrow points in the downhill direction, and the numbers indicate the two different heights of the stack at the bottom and top of the inclined surface. Within the limitation of the standardized 3-unit matrix, there are three possible slopes or inclinations: $3 \rightarrow 2$, $3 \rightarrow 1$ and $3 \rightarrow 0$. Four directions are possible for the slopes, and additional surfaces are available at different elevations as $2 \rightarrow 1$, $1 \rightarrow 0$, and $2 \rightarrow 0$ (see Fig. 3).



$3 \uparrow 2$ $3 \downarrow 1$ $3 \rightarrow 0$ $2 \leftarrow 1$ $1 \rightarrow 0$ $2 \downarrow 0$

Figure 3: Inclined Surface Symbols

An assigned structure for sketching could then be:

$$3 \rightarrow 2,\ 2 \rightarrow 1, 1 \rightarrow 0\,;\ 0,1,0\,;\ 0 \leftarrow 3,3,3 \downarrow 0$$

An argument could be made that the object should be coded as:

$$3 \rightarrow 2, 2 \rightarrow 1, 1 \rightarrow 0\,;\ 0,1,0\,;\ 3 \leftarrow 0,3,3 \downarrow 0$$

The difference being an assumption concerning whether it is important to list the heights in order from high to low. In practice, it doesn't seem to make much difference to the students as long as a consistent system is used.

To code the information concerning a hole in an object, the standard dimensioning symbols for diameter and depth can be used. The code $3\,\phi\,\overline{\downarrow}\,2$ indicates that a stack three units high contains a hole two units deep.

6 Multiview Sketches from Symbolic Information

Sketching exercises are assigned even before attendance is taken on the first day of class. As a result, the students have no background concerning formal projection schemes for the multiview portion of each exercise. They are simply instructed to sketch a view straight down from the top, and straight in from the front and one of the sides. The concepts of standard view alignment, hidden lines, center lines, and so on, come later in the course.

7 The Generic Pictorial View

In a similar manner, no instruction is given concerning formal axonometric views. The students are simply instructed to provide four different bird's-eye views and four different worm's-eye views. Gridded paper is not even supplied for the students, so they devise their own techniques for indicating depth. Formal definition of pictorial drawing schemes comes much later in the course, but the students appear to be able to devise reasonable techniques for depth delineation without the constraints of the formal work.

8 The Block Set

As an aid to helping the students get started in visualizing the assigned structures, a set of wooden blocks are available at each desk during laboratory sessions. Each set consists of 18

- 1 unit cubes, 9 - 1 by 1 by 1 inclines, 6 - 1 by 1 by 2 inclines, 6 - 1 by 1 by 3 inclines, and 3 - 1-unit cubes with a hole through them. Use of the block set is completely voluntary. The pattern that seems to develop is that all the students make use of the block set during the first week, but then use of it drops off quickly as the students develop their visualization skills. The universal complaint about the block set is that a structure made using it cannot be picked up to obtain a worm's-eye view of it. A couple of students jokingly threatened to bring glue guns to class, and several others obtained Lego block sets to get around the problem.

9 Mental Rotation to the Eight Points-of-View

In order to get the requisite bird's-eye views, students have to develop some technique for revolving the model of each structure constructed using the block set. Some students actually will reassemble the blocks in a different order to get the different views. More enterprising ones simply construct the model on top of a piece of paper that they can move on the desk to get whichever view they are interested in. Every student commented about the difficulty of the worm's-eye views. At present, no mechanism is provided to allow the students to lift a structure up so that they can look at it from the bottom. In the future, a photo cube will be provided as a part of each set of blocks so that the structure can be assembled inside of it and the whole assembly lifted off the work surface for viewing. A significant percentage of students failed to attempt the worm's-eye view of the assignments. Some gave as their excuse that they didn't have time to complete the sketches, but several honestly confessed to not be able to visualize what the bottom views of the structures would look like. Most of these students simply had trouble trying to draw a simple cube (to represent the matrix) upside down. It helped when the suggestion was made to turn the paper around while sketching the matrix and then complete the sketch with the paper right side up. Within a short time (one week) most of the students were able to perform the rotations mentally instead of having to physically turn the model and by the third week of classes, the block sets started to be used simply as a check on their sketches to see if they had been constructed correctly.

10 Implementation in EDG and CAD Courses

At the first class meeting one structure is assigned. Five additional ones are assigned during the first laboratory meeting. Two to four new structures are assigned each week for the remainder of the semester. These assignments are not trivial. Each sketch with its requisite 11 views will take the students a minimum of 30 to 40 minutes to complete.

11 Conclusions

The premier offering of this material is in a course that is still in session. Early response, particularly from students who started the course previously but who dropped it because of difficulties, is that the symbolic code and the bird's-eye/worm's-eye translations are very helpful in developing their abilities at visualization.

The following structures are presented for readers to test their own or their students' abilities:

11.1 Normal Surfaces:

3, 2, 1 ; 1, 2, 0 ; 0, 1, 0	3, 1, 2 ; 0, 1, 0 ; 0, 3, 0	3, 1, 1 ; 0, 1, 0 ; 2, 2, 0
1, 1, 2 ; 0, 1, 0 ; 3, 2, 3	3, 3, 2 ; 3, 1, 2 ; 0, 0, 1	2, 3, 3 ; 1, 0, 0 ; 1, 2, 0
2, 3, 0 ; 0, 2, 0 ; 3, 2, 2	1, 2, 0 ; 3, 2, 1 ; 2, 0, 0	2, 2, 0 ; 2, 1, 3 ; 2, 0, 0
2, 3, 0 ; 0, 3, 0 ; 1, 2, 2	3, 2, 2 ; 1, 0, 0 ; 2, 2, 1	3, 2, 2 ; 1, 0, 0 ; 2, 2, 1
1, 3, 0 ; 1, 1, 0 ; 2, 1, 3		

11.2 Inclined Surfaces:

$3 \rightarrow 2, 2 \rightarrow 1, 1 \rightarrow 0;\ 0, 1, 0;\ 0 \leftarrow 3, 3, 3 \downarrow 0$
$2 \uparrow 1, 3, 3 \downarrow 2;\ 0, 3, 2 \downarrow 1;\ 2 \downarrow 1, 3, 1 \downarrow 0$
$3, 2, 2;\ 3, 1 \rightarrow 0, 0;\ 3, 1 \rightarrow 0, 0$
$3 \uparrow 1, 2 \rightarrow 0, 0;\ 0 \leftarrow 2, 1, 0;\ 1, 1 \downarrow 0, 3 \rightarrow 0$
$3, 3 \rightarrow 1, 1;\ 3 \downarrow 2, 2 \downarrow 0, 1 \rightarrow 0;\ 2 \rightarrow 0, 0, 0$
$2, 1, 3 \leftarrow 1;\ 3 \downarrow 2, 0, 2 \rightarrow 0;\ 2 \downarrow 0, 0, 0$
$3 \downarrow 2, 2, 3 \downarrow 2;\ 0, 2 \leftarrow 0, 1;\ 0, 2 \leftarrow 0, 1$

11.3 Inclined Surfaces and Circular Holes:

$3, 3, 3;\ 1, 3 \downarrow 2, 1;\ 1 \emptyset \mapsdown 1, 2 \downarrow 1, 1 \emptyset \mapsdown 1$
$1 \emptyset \mapsdown 1, 3 \uparrow 1, 1 \emptyset \mapsdown 1;\ 3 \leftarrow 1, 3, 3 \rightarrow 2;\ 1 \emptyset \mapsdown 1, 3 \downarrow 2, 0$
$2 \leftarrow 1, 2 \emptyset \mapsdown 2, 1 \rightarrow 0;\ 2 \leftarrow 1, 2, 0;\ 1, 0, 0$
$2 \leftarrow 1, 2 \emptyset \mapsdown 2, 1 \rightarrow 0;\ 0, 2, 1 \rightarrow 0;\ 1 \emptyset \mapsdown 1, 2 \downarrow 1, 0$
$3 \rightarrow 2, 2 \rightarrow 1, 1 \rightarrow 0;\ 3 \rightarrow 2, 1 \emptyset \mapsdown 1, 1 \rightarrow 0;\ 0, 1, 1 \rightarrow 0$
$0, 3 \downarrow 2, 3 \downarrow 2;\ 2 \leftarrow 1, 3 \emptyset \mapsdown 3, 2 \downarrow 1;\ 0, 1 \leftarrow 0, 1 \rightarrow 0$
$3 \leftarrow 2, 3 \rightarrow 2, 2 \uparrow 1;\ 3 \leftarrow 2, 3 \emptyset \mapsdown 3, 2 \downarrow 1;\ 2 \downarrow 1, 2 \downarrow 1, 0$

References

[1] B.G. Baartmans, S.A. Sorby: *Introduction to 3-D Spatial Visualization.* Prentice-Hall, Englewood Cliffs 1995.

[2] G.L. Musser, W.F. Burger: *Mathematics for Elementary Teachers: A contemporary Approach.* 3rd ed., Macmillan College Publ. Comp., 1994.

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