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The Taxonomy of Geometry and Graphics

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Abstract. At the Eighth International Conference on Engineering Computer Graphics and Descriptive Geometry (August, 1998) an international panel was organized to discuss the taxonomy of 'Geometry and Graphics' and its relation to the interests of the International Society for Geometry and Graphics (ISGG). This is a summary of this discussion with the panel members J. E. Baker (Australia), L. Cocchiarella (Italy), I. Kalcic (Slovenia), P. I. Nauk (Russia), K. Suzuki (Japan), G. Weiss (Germany), and E. N. Wiebe (USA).

Key Words: Descriptive Geometry, Engineering Graphics, visualization, geometry education

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1 Introduction

"Nothing can be known concerning the things of the world without the power of geometry."[1]
Roger Bacon (1260)

"A picture shows me at a glance what it takes dozens of pages of a book to expound."
Ivan Sergeyevich Turgenev (1862)

ISGG logo

At the Eighth International Conference on Engineering Computer Graphics and Descriptive Geometry (August, 1998), R. Barr and D. Juricic organized an international panel to discuss the taxonomy of 'Geometry and Graphics' and its relation to the interests of the *International Society for Geometry and Graphics* (ISGG). What follows is an attempt to capture the extremely interesting and insightful discussions we had at the conference.

[1] in Robert Belle Burke: *The Opus majus of Roger Bacon: A translation of Robert Belle Burke.* Oxford University Press, London 1928, p. 234.

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2 Summary

This summary will begin with *geometry*, in large part because there seems to be a higher degree of consensus with its definition. Most all of the panel members agree that geometry is a branch of mathematics concerning itself with, as Baker states, "the properties, relationships, and measurements of spatial entities". As Cocchiarella notes, the practice of geometry originally focused on measuring and only later made use of relations and operations. In this way, geometry evolved into a deductive system founded upon agreed axioms and concepts. Geometry is, as Weiss states "... a certain method of thinking ...".

Any taxonomy of geometry must note that it is not a single monolithic field of study, but is divided into *numerous branches*. The panel members note such branches of geometry as: Euclidean, non-Euclidean, projective, descriptive, hyperbolic, topological, fractal, analytic, differential, and so on. Each area will have its own axioms and theorems as its basis and have varying degrees of overlap with each other or with other branches of mathematics. In a similar vein, different professions will apply different branches of geometry in different ways. Kalcic contends that architecture makes unique use of geometry through its melding of artistic, engineering, social and political demands. Baker notes that even though geometry has at its roots the study of spatial entities, it does not mean that these entities must be represented graphically. Extending on this theme, Weiss contends that defining geometry based on n-dimensional space and time lends itself to restricted definitions which depend on 'visual' representations, while purely geometric concepts can be modeled without objects such as points, lines, and planes.

A definition of *graphics* becomes harder to bring into common ground. Cocchiarella traces the term back to an etymological root meaning 'to engrave'. This follows right in line with Suzuki's and Kalcic's contention that graphics are inherently two-dimensional representations. To ground graphics in the physical world even more, a number of the panelists state that graphics is a tool rather than a deductive system or way of thinking.

Still, as a physical representation, it makes use of many technologies in its production. Computers, pencils, pens, knives, lasers, and photosensitive chemicals are all put to use in the creation of graphics that are often classified based on its visual properties. They can be line drawings or shaded images, etchings or photographs, color or monochromatic. Graphics can be further classified based on what subject matter they are representing. It is here that it may be worthwhile beginning a discussion of how *geometry and graphics* relate to one another.

Many of the panelists brought up the inherent synergy of geometry and graphics. As Cocchiarella states "... geometry can be regarded as a guide for graphics, and graphics can be regarded as a tool which allows geometry to be expressed and thought." Suzuki echoed this statement "... geometry is the theoretical basis of graphics ... [while] graphics are useful for understanding geometrical relations in space intuitively." Though geometry does not need to be represented visually, from an educational and communications standpoint, there is great advantage in doing so. Nauk, Weiss, and others noted the importance of a better understanding of both the psychological basis of the perception of graphics and how this knowledge can be used to further research into the use of graphics in education and professional communication. Of particular interest, of course, is the teaching of geometric theory and practice.

This panel certainly did not pretend to come up with the definitive statements on geometry and graphics. Instead we reaffirmed our belief in the interrelationship of these two fields study and the importance of investigations and discussions in these issues if our professions are going to continue to be vital and expanding.

3 Statements

J. Eddie Baker (Sydney, Australia)

Geometry is a valid division of mathematics, one concerned with the properties, relationships and measurements of spatial entities. In its various manifestations, it is a deductive system founded upon agreed axioms and concepts. Felix KLEIN defines it broadly as a space together with a set of transformations of that space. Although we commonly draw upon pictorial aids, the results in geometry are independent of such devices.

Graphics includes the technical use of illustrations as an aid to mathematical calculation or to engineering, architectural or other professional design, but it has wide application in many areas, such as the social sciences, art and commerce. It remains, nevertheless, a tool for manipulating images, a process to engage, inform or amuse the eye, a provider of illusion. Whilst graphical procedures may be subject to some mathematical laws, the objective is to assist in visualization, not to determine consequences of assumed statements.

In the light of the foregoing assertions, it might be concluded that *descriptive geometry* is a form of graphics, its prime purpose being, for example, to assist the mechanical engineer in laying out planar views of machine components. On the contrary, because its rules are strictly in accordance with the tenets of (admittedly relatively simple) projective geometry, it must be considered a special case of this body of analysis.

Geometry has been functioning as our servant and companion throughout recorded history and evolves still. Graphics is a burgeoning field of activity. Each area, for different reasons, independently commands a strong and vigorous contingent of devotees. Their essential characters are such, however, that they are naturally mutually supportive and will be enhanced by a close formal association.

Luigi Cocchiarella (Naples, Italy)

The panel statements and panel discussion show that while each panelist remarked on specific aspects, we generally agree with the basic points of the question.

Geometry has been both regarded as an independent scientific theory and as one of the basic knowledge in several disciplines and for graphic operators. Among geometries, *descriptive geometry* has been particularly regarded as a theory and as a specific tool for graphic representations in several fields.

Graphics has been regarded as the most general theory and praxis of visual representations, based not only on specific kinds of geometry or science, but moreover on the intuitive geometrical thinking of the operator, at least on his more or less high 'cultural' level. The 'analogical', 'analogical-symbolic' and 'symbolic' codes of graphics as well as 'alphabets', are very close to several levels of expression and communication, whether in science, in techniques and arts or in daily life. Computers gave us a powerful, versatile tool which provides its own specific logical structure. This tool may be destined to change and to standardize the codes as well as the ways of thinking that sustain codes: this seems to be a great theme of discussion in the future. About a new 'taxonomy of geometry and graphics', its utility comes from the 'synaptical' use of geometry and graphics in every application, each one by creating specific models of work.

For example, as an architect, I feel I can affirm that the several different aspects of architectural complexity can be integrated, developed and expressed by using several kinds of *geometry and graphics* or several mixes of them. So, without a synaptical coexistence of geometry and graphics, no factual architecture is possible. Similarly in each disciplinary bound, no absolutely pure use of geometry or graphics is possible, because the same human brain and perception has a synaptical structure, and every division can be regarded as a disciplinary and methodological tool. For a general revision of and bringing up to date the 'taxonomy of geometry and graphics' – with all different branches – we need to enlarge our debate in our own countries. In future ISGG forums, we'll detect the points of view and the specific and detailed aspects of the question in every disciplinary field, to open new 'taxonomic ways' for geometry and graphics.

Igor Kalcic (Ljubljana, Slovenia)

I am an architect and my opinion is the opinion of an architect, a creative artist, making projects to build houses and other architectural objects. This component of my work is practical and means making projects to realize architecture. I was at the ISGG conference in Austin as one of the few practicing architects designing projects to build. Because of this work, I have been involved in numerous realizations of architectural objects. From this point of view my definition of 'geometry and graphics' may be quite different from definitions of other, more theoretical members of panel presentation.

Geometry is for me, first and foremost, a way of life, and a way of thinking in the widest meaning of the terminus. Geometry is in a way also a platform for the theoretical basis of graphics. Architecture uses geometry in its unique, very special meaning: it is an elementary platform to create architecture, as a very complicated system of different but equally important variables. *Architecture* is art but also technical; it is engineering and science, and it is a socially and politically conditioned activity. Even more, geometry and descriptive geometry is a way of understanding and presenting spatial relations.

Graphics is for me, from my point of view, being an active architect, a picture, or better a presentation of my artistic, architectural work, a presentation of my project. In other words, graphics means to me a 2D visual representation of 3D objects according to geometric (mathematical) rules. Graphics has many faces as it uses many different ways and technologies of presentation. During the study of architecture, we teach our students: technical drawing, architectural drawing, freehand drawing and freehand sketching, drawing in color techniques, graphics in a classical sense, modeling and computer drawing. All these methods of presentation in architecture – from simple to complicated, from plain to complex – is graphics. I have to point out in this context *computer graphics* as a very important new way of presenting architecture. But this is still only a presentation of architecture, using the computer as a most perfect tool for presentation.

Graphics also means to me a communicative technique between different or equal fields of human activity. Graphics is a *communication tool* between the architect and his clients, a connection between the architect and his collaborators on a project, how to understand and speak with each other during team work.

Pyotr I. Nauk (Tyumen, Russia)

I am glad to continue our discussion after the ISGG conference in Austin, TX, comparing the results of our investigation of the origins and evaluation of geometry and graphics and our very interesting and profound statements of geometry and graphics taxonomy. I am quite sure it is expedient to maintain the classic terminology. On the other hand, it is necessary to expand the notion of graphics and geometry proceeding from the modern achievements of the mankind and the tendency of its evolution.

Graphics is inherent in a great variety of human activities. At one pole is art creation (engraving, etching, illustrative drawing, easel-drawing, lithography and so on), and at the other pole is technical creation (engineering graphics, cartography, computer graphics and so forth). Activities such as architecture, design, and technical aesthetics are between these two poles.

One of the most studied areas of *graphics and geometry* is the function of communicative, cognitive *visualization*: the technique of making a drawing, picture, or engraving using one of the widely accepted sign-symbolic systems. The promising trends of graphics as multifunctional system of activity are:

1. Creating a methodology of developing and diagnosing spatial intellect as the basis of abstracting and self-completing mental images.
2. Psychophysiology of images identification.
3. Developing new technologies of communicative, cognitive visualization.

It is possible to suggest the *classifications of graphics* on the base of homology of subgraphics:

1. According to the structure of a mental geometrical image being formed as a part of concrete sphere of activity (e.g., engineering graphics, cartography, illustrative graphics, etc.)
2. According to the degree of formalization of mental geometrical image: analogue, analogue-sign-symbolic, sign-symbolic graphics.
3. According to the belonging to concrete technology of communicative, cognitive visualization: easel-drawing engraving, computer graphics, etc.

Let us come back to *geometry* as a fundamental science, which is a one-function system of human activity directed to working out the regularities and systematizing objective knowledge of reality in the field of spatial forms and relations. Achievements in geometry are widely used in graphics, which serves different spheres of activity including geometry as a science. The combination of integration and differentiation processes is typical for geometry. That is why geometry is classified according to its scientific trends and numerous scientific schools: topological geometry, Euclidean geometry, analytical geometry, etc.

As the results of investigation confirm, finding solutions to geometrical problems develop man's spatial intellect, forming a methodology for abstracting and self-completing the mental geometrical image. Thus, geometry is a theoretical basis of graphics. At the same time, graphics by means of the function of communicative, cognitive visualization materializes geometrical ideas for the information bearer.

Geometry and graphics are dialectically connected with each other as various examples of a single whole surrounding world imaging. Geometry idealizes surrounding reality, and graphics artificially materializes idealized images of surrounding reality. Geometry and graphics cannot exist without each other. That is why it is impossible to create hierarchical relations between them. At the end of the 20th century, *geometry and graphics education* offers not only modern knowledge and technologies mastering but also development of the intellectual and active person. Graphics training is one of the factors which provides the exposure to individual abilities, the revealing of creative potential, and, on the base of this, the achieving of spatial image thinking of a high level; the content of which is in contradictory unity of the general and individual.

Kenjiro Suzuki (Tokyo, Japan)

My first impression from the ISGG forum was that the answers to the question "What is geometry? What is graphics?" are not so different between the panelists, though they came from different fields and different countries. I would like to add the following comments to Prof. Eric WIEBE's summary to emphasize the importance of graphics.

We, graphics educators, know well from daily experiences in classrooms that it is difficult to recognize three-dimensional objects from their two-dimensional representations, i.e., *graphics*. It should be emphasized that it is not easy for human beings to recognize three-dimensional objects even from real three-dimensional objects. Human beings detect *visual information* through eyes. They are basically two-dimensional detectors.

They are sensitive to detect two-dimensional information, which lies in the plane perpendicular to view direction, but are not sensitive to the information in the view direction. It is, therefore, necessary to select a proper view direction, i.e., to make proper "graphics on his/her retina", for recognizing three-dimensional objects from visual stimuli. It is especially true for metric properties of three-dimensional objects. It is, therefore, not too much to say that human beings can recognize three-dimensional objects only through 'graphics'.

I would like to add a few words about the importance of *line drawings*. It should be noted that human beings could recognize something only through some kind of abstraction. Without abstraction, no recognition! The line drawings are the results of the abstraction of geometric properties of three-dimensional objects. As mentioned above, graphic representations serve as important substitutes for real three-dimensional objects, providing a means of recognition closely related to the nature of the perceptual system of human beings.

Gunter Weiß (Dresden, Germany)

Our discussion on 'geometry and graphics' aims at future developments of the ISGG. Keeping this in mind, any definition of the concepts 'geometry' and 'graphics' in a mathematical sense would be too restrictive or too wide as to be useful. We should use encyclopedic definitions rather as working hypotheses and keep them flexible to future paradigm changes.

In our discussions we agreed that 'graphics' is comparable with writing and letters, while 'geometry' corresponds to the meaning of that writing. Thus *'graphics'* is a means of transportation to geometry. The latter seems to be a matter of reasoning and of handling abstract models of details of our (natural and technical) environment. According to individual abilities there will be different depths of abstractions, and what has a graphic (!) nature for one might be incomprehensible for another.

Geometry deals with forms, relationships and structures. By this, it is inherent in all technical disciplines, but also in every natural science and even, to some extent, in fine arts and philosophy. So the biggest part of 'geometry' is *'applied geometry'*, applied to a certain problem arising from topics of the above mentioned sciences. And it is often the geometric approach, which first leads to a clear formulation of those problems.

For our ISGG this means, in my opinion, that it should keep on being an open forum for all sorts of applied geometry. The corresponding and necessary graphics will occur automatically and need not to be emphasized for its own sake.

In addition, the ISGG and its members should try to influence the public opinion of the necessity of *geometric and graphic education*. There is a worldwide need now for geometry (and graphics). Ignorance of basic geometric facts and methods is a form of illiteracy, which we all agree are necessary to fight against. The ability to produce and interpret graphic geometric images is just as little inborn as the ability to read and to write, but requires learning, training, and teaching!

Conclusion: Our International Society for Geometry and Graphics (ISGG) should be an open forum for theoretical and/or applied geometry and it should stand for more (modernized) geometry at all levels of education.

Eric N. Wiebe (Raleigh, USA)

I would like to add to the ISGG forum members' insightful statements some specific thoughts on the psychological basis of geometry and graphics.

Graphics is a form of visual representation and communication that has two primary psychological components. The *perceptual* component involves the intuitive or automatic interpretation of graphics, which represent elements that are in some way analogical to experiences we have in the real world. The *cognitive* component is the learned, often discipline-specific component that is critical for the higher order understanding of symbolic graphics. While the cognitive component gives graphics extended power when applied in specialized applications, it can also serve as a barrier to those without the appropriate training and experiences to interpret those graphics.

Geometry is a branch of mathematics that uses a deductive system to understand the properties, relationships, and measurements of spatial entities. It has a strong cognitive component built from agreed upon rules. It is also worth noting that geometry does not have to be represented graphically. Understanding the relationship of geometry and graphics can be found through understanding how geometry forms a critical foundation for representing and understanding graphics. Geometry provides support for the scientific and psychological basis for perception of form and motion. In turn, graphics provides support for the communication and understanding of geometry, providing a (potentially) universal language for communication.

For those involved in the research and application of geometry and graphics, one issue, which needs to be regularly addressed, is the *role of geometry and graphics in education*. It is important that individuals understand the unique role each plays throughout the education process. Geometry has wide application as an analytic, problem-solving method; it is a precision tool for developing solutions to problems. Graphics has equally wide application as a communications tool, providing a holistic method for synthesizing information.

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