Journal for Geometry and Graphics
Volume 3 (1999), No. 1, 67–76

A Method for Generating Compound Spring Element Curves in Contact with Cylindrical Surfaces

Mitchel J. Keil, Jorge Rodriguez

Department of Industrial and Manufacturing Engineering
Western Michigan University, Kalamazoo, MI 49006, USA
email: mitchel.keil@wmich.edu

Abstract. This paper describes a method for enhancing the initial design process, as well as the transfer of data, for the geometry of compound contact springs. A force-length constrained model is developed around a cantilever beam section which has short (less than 10% of the beam's length) structural elements to facilitate proper positioning. The short elements are often considered insignificant in deflection analysis, but are shown to contribute an additional 26% to the structure's deflection. Once this is done, a mount position constrained model is developed which must be solved iteratively. The resulting geometry is smooth, precise, and can be readily transferred to CAD to complete a robust design process.

Key Words: springs, compound curves, cylindrical contact

1. Introduction

Developing compound contact springs (i.e., several segments) is a process which appears simple on the surface but can be laden with sources of potential error. The problems result from the steps which must be taken to define the final geometry of the spring. A basic understanding of mechanics of materials is necessary to begin; and for simple cantilever designs, results can come quite easily [5]. However, adding a short length with a bend to facilitate positioning can make even this task quite daunting. The tendency is for the designer to ignore these short segments, but this can result in errors of 20% or greater in predicted load or deflection [1].

Several factors need to be addressed during the design process. Loads are critical in contact design. This is especially true when metal slides on metal. This type of contact must be set heavy enough to break through incidental oxides and avoid electrical arcing. It must also be set light enough to avoid undue wear and the resulting debris. High loads can severely affect contact life [4]. Contact deflection is very important in that it must allow for assembly tolerances [6]. Cylindrical mating geometry is very common in the case of rotating contact assemblies and serves to compound the problem. Decreased mounting distance causes

ISSN 1433-8157/\$ 2.50

a change in contact angle which also results in a reduced effective beam length. The shorter beam is much stiffer causing forces to rise rapidly. In addition, neighboring structure can become points for electrical shorts if contact tips rise too high. The final problem is one of data transfer. Once the theoretical geometry has been determined, a working CAD drawing must be developed. If this transfer is not clean, proper fabrication is impossible.

The proposed methodology starts with the design constraints such as contact force, stiffness, and material. Classic beam theory is then used to determine the geometry for a straight section of the contact beam. The maximum stress condition of the beam is then imposed on the rest of the short structural elements to determine their deflected shapes. Once the shapes have been determined they are assembled to form the complete structure. Since circular arc elements are involved, they must be sized at assembly. The result is a nonlinear equation that must be solved iteratively to obtain the final deflected geometry.

The deflected geometry is useful for visualization and associated interference checking, but has little use for fabrication. To obtain the fabrication geometry, the stressed geometry is relaxed to a stress-free state [2]. Additional forms can then be determined to facilitate intermediate steps of formation. Finally, all geometry can be formatted to integrate into a CAD drawn file where it serves as the base for assembly and manufacturing tasks.

2. Initial analysis

The design of cantilever springs seems almost mundane at times. One needs only to pick up any book dealing with strengths of materials and there is the familiar equation [3]:

$$y = \frac{FL^3}{3EI} \tag{1}$$

where:

$y \equiv$ deflection
$F \equiv$ force
$L \equiv$ beam length
$E \equiv$ Young's Modulus
$I \equiv$ area moment of inertia

which is illustrated in the diagram of Fig. 1.

Adding the constraint that the end of the beam lie tangent to a cylinder makes the problem slightly more interesting since now the slope of the end must also be considered as shown in Fig. 2. Here the slope of the beam end (y') reasonably approximates the angle (θ_e) when the deflection (y) is less than one tenth of the length (L). This angle represents the angular deviation from vertical at which the tangent end touches the cylinder.

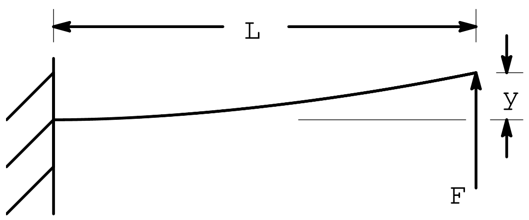

Figure 1: A typical cantilever beam diagram

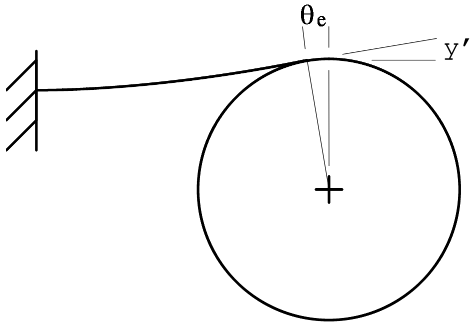

Figure 2: Deflected cantilever tangent to a cylinder

It is fairly common practice to make contact springs from beryllium copper wire which has been insert molded into a plastic block. This often involves forming fixtures which create additional features (sub-elements) used to facilitate a desired mounting position. One configuration is shown in Fig. 3 with the resulting sub-elements labeled for clarity.

Designers frequently analyze this spring system by considering only the straight contact member and arguing that the short base and transition arc contribute little to the deflection characteristics of the system. They are considered too short to be significant, but they are also in the zone of maximum bending moment.

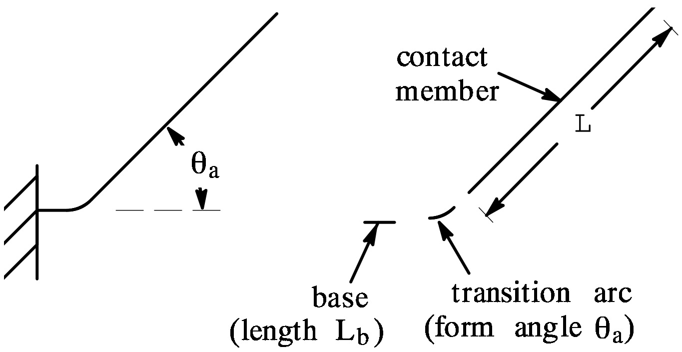

Figure 3: Formed spring with labeled sub-elements

The rest of this paper will study the significance of these "short" elements in the total deflection of the compound spring system. Further, a study of a mount point constraint approach will be developed to show the challenge that this presents to the designer. Finally, the role of a CAD integrated design approach will be presented along with an argument for error minimization.

3. Short element contribution to deflection

The study of the significance of these short elements begins by specifying material and geometry. Beryllium copper wire with a diameter (d) of 0.020 inches is selected. The straight contact member is 1.000 inches[1] long (L). The base element is 0.050 inches long (L_b). The

[1] *Editor's note:* 1 inch equals 25.4 mm (exactly), and 1 pound-mass (1 lbm avoirdupois) is defined in most English-speaking countries as 0.453592370 kg. One pound-force (lbf avoirdupois) equals 4.448222 N.

transition arc has a radius (r_a) of 0.050 inches and a form angle (θ_a) of 45°. A contact force of 0.0224 pounds is desired. The cylinder radius (R) is 1.500 inches.

The deflection of the spring system is then determined. First, the deflection of the contact member is found using equation (2) to analyze the structure of Fig. 1. The deflection of the contact member is then:

$$\begin{aligned} y = \frac{FL^3}{3EI} &= \frac{(0.0224 \text{ lb})\ (1.000 \text{ in})^3}{3\,(19.0 \times 10^6 \text{ lb/in}^2)\ (\pi\ (0.020 \text{ in})^4/64)} \\ &= 0.050 \text{ in} \end{aligned} \tag{2}$$

The moment (M) applied to the other members is:

$$\begin{aligned} M = FL &= (0.0224 \text{ lb})\ (1.000 \text{ in}) \\ &= 0.0224 \text{ in-lb} \end{aligned} \tag{3}$$

The equation for curvature can be used to find the radius (r_b) of the stressed base element:

$$\begin{aligned} r_b = \frac{EI}{M} &= \frac{(19.0 \times 10^6 \text{ lb/in}^2)\ (\pi\ (0.020 \text{ in})^4/64)}{0.0224 \text{ in-lb}} \\ &= 6.662 \text{ in} \end{aligned} \tag{4}$$

Knowing the radius and arc length (L_b), the angular deflection (θ_b) can be found:

$$\theta_b = \frac{L_b}{r_b} = \frac{0.050}{6.662} = 7.51 \times 10^{-3} \text{ radians} \tag{5}$$

The deflection of the transition arc is best handled by considering the curvature (ρ) which is simply the inverse of the radius or:

$$\rho = \frac{1}{r} = \frac{M}{EI} \tag{6}$$

Given the initial radius (r_a) of 0.050 in, the stressed curvature (ρ_s) is:

$$\begin{aligned} \rho_s = \frac{1}{r_a} + \frac{M}{EI} &= \frac{1}{r_a} + \frac{1}{r_b} \\ &= \frac{1}{0.050} + \frac{1}{6.662} = 20.15 \text{ in}^{-1} \end{aligned} \tag{7}$$

Thus, the stressed radius (r_s) is:

$$r_s = \frac{1}{\rho_s} = \frac{1}{20.15} = 0.0496 \text{ in} \tag{8}$$

The angle subtended by the stressed arc (θ_s) has also changed. Of course, the arc length has remained constant. Thus, θ_s is found by equating stressed and unstressed arc lengths:

$$r_s\ \theta_s = r_a\ \theta_a$$

therefore

$$\theta_s = \frac{r_a\ \theta_a}{r_s} = \frac{(0.050)\ (45^\circ)}{0.496} = 45.338^\circ \tag{9}$$

The angular change due to stress ($\Delta\theta$) is:

$$\Delta\theta = \theta_s - \theta_a = 0.338^\circ \tag{10}$$

or

$$(0.338^\circ)\ \frac{\pi}{180^\circ} \quad = \quad 5.89 \times 10^{-3} \text{ radians}$$

The additional rotation (θ_c) of the contact member is:

$$\begin{aligned} \theta_c = \Delta\theta + \theta_b &= 7.38 \times 10^{-3} + 5.89 \times 10^{-3} \\ &= 13.27 \times 10^{-3} \text{ radians} \end{aligned} \tag{11}$$

This also adds a deflection (Δy) to the contact member:

$$\begin{aligned} \Delta y = L\ \theta_c &= (1.000 \text{ in})\ (13.27 \times 10^{-3}) \\ &= 0.01327 \text{ in} \end{aligned} \tag{12}$$

Comparing this additional deflection to that predicted by equation (2), gives:

$$\frac{0.01327}{0.050} \times 100\% = 26.5\%$$

Even though the combined length of these sections is less than a tenth the length of the contact member, they contribute significantly to the total end deflection. Of course, the location of the fixed point of this compound spring system relative to the cylinder it touches still needs to be determined.

4. Mount location

Locating the fixed point begins by constructing the diagram in Fig. 2. The end rotation (θ_e) is found from the equation:

$$\begin{aligned} \theta_e = \frac{FL^2}{2EI} &= \frac{(0.0224 \text{ lb})\ (1.000 \text{ in})^2}{2\ (19.0 \times 10^6 \text{ lb/in}^2)\ (\pi\ (0.020 \text{ in})^4/64)} \\ &= 0.0751 \text{ radians} \qquad \mathit{or} \qquad 4.300^\circ \end{aligned} \tag{13}$$

This places the end of the deflected beam tangent to the cylinder. The beam is then rotated $\theta_s = 45.338^\circ$ to mate up with the stressed transition arc. Finally this assembly is rotated θ_b for an additional 0.438°. The total end rotation (θ_T) is now:

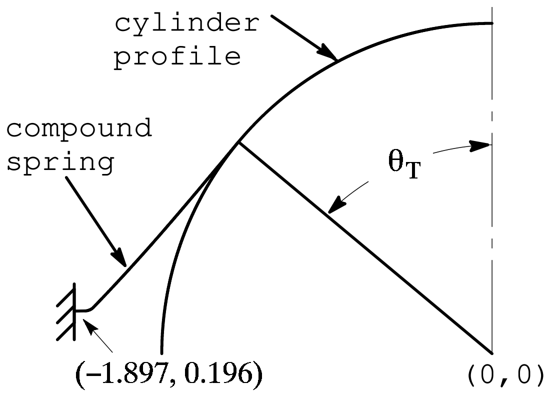

Figure 4: Compound spring positioned on cylinder

$$\theta_T = \theta_e + \theta_b + \theta_s = 50.068^\circ \tag{14}$$

The result of these manipulations is shown in Fig. 4 with a mount point at $x = -1.897$ inches and $y = 0.916$ inches relative to the center of the cylinder.

Failure to take into account the contribution of the short elements alters θ_T to an angle θ_{T1} described by:

$$\theta_{T1} = \theta_T - \theta_e - \Delta\theta = 45.292^\circ$$

The result is shown dashed in Fig. 5 as an overlay on the original system. This may not appear to be significant to the casual observer, but it can have severe consequences if the geometry is being used to design an electrical contact. Furthermore, designers will often compensate in other ways with unpredictable results.

The end rotations being the greatest contributor to the error shown are often not considered at all. In fact, the end rotation of the contact member is often not considered as well. This error is even more dramatic as shown in Fig. 6.

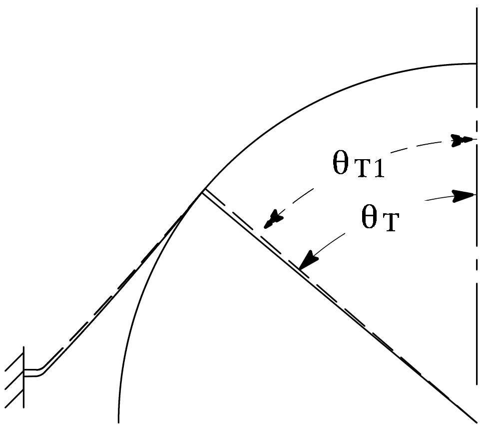

Figure 5: Position error from failure to consider short elements

The reasons for these omissions are familiar to most design organizations. Engineers perform the complex calculations and design draftsmen convert the results into graphics for fabrication drawings. Many designers are not familiar with the spline functions necessary to draw the deflected beam shapes. Another complication is that the design approach taken here does not represent a typical design problem. Here the length of the contact member was used to find a suitable mount position. Typically, the mount position is known. The length of the base element and the radius of the transition arc are determined from fixturing, but the length of the transition arc and the contact member are unknown.

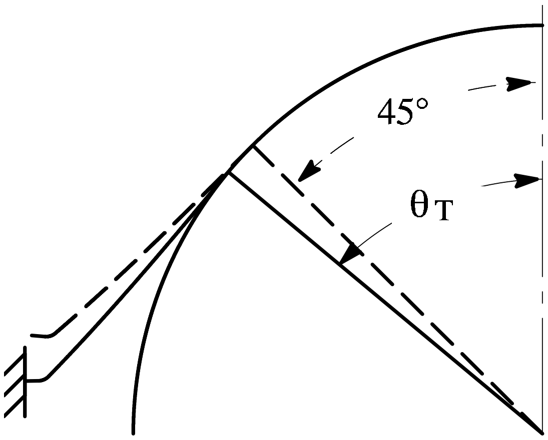

Figure 6: Position error from failure to consider contact member end rotation and sub-element rotations

5. Position constrained design

A designer is usually given an envelope and told to develop his design. For this problem, he starts with a sketch such as that of Fig. 7. The mount point is known — given here by a position vector ($\overline{V}_1$).

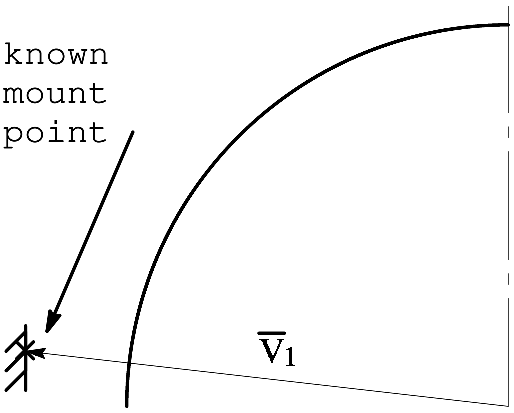

Figure 7: Constraints given to the designer

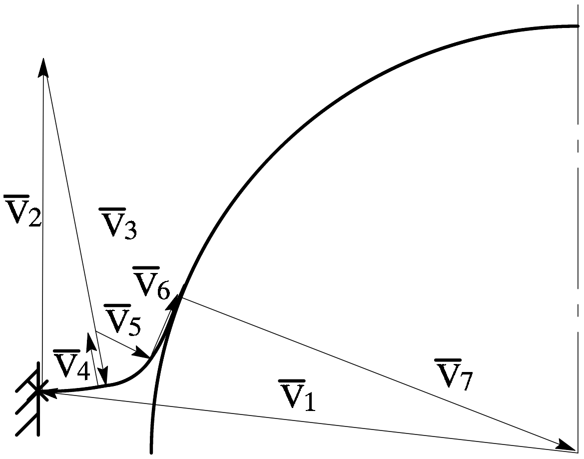

Figure 8: Vector loop

Distorting the geometry, as in Fig. 8, helps to explain the analysis with vectors. Vector $\overline{V}_2$ locates the stressed center of curvature of the base element directly above the mount point. Vector $\overline{V}_3$ locates the deflected end of the base element. Vector $\overline{V}_4$ locates the stressed transition arc center which is colinear but opposite in direction with $\overline{V}_3$. Vector $\overline{V}_5$ connects to the other end of the arc, and vector $\overline{V}_6$ spans the ends of the contact member. Finally, vector $\overline{V}_7$ closes the loop. This can be expressed as a vector equation:

$$\overline{V}_1 + \overline{V}_2 + \overline{V}_3 + \overline{V}_4 + \overline{V}_5 + \overline{V}_6 + \overline{V}_7 = 0 \tag{15}$$

All of these vectors can be expressed as functions of the contact force and the contact beam length. Since the force is known, only the length (L), and the angle between $\overline{V}_4$ and $\overline{V}_5$ is unknown. Thus, there are two equations (the $x-$ and the $y-$ component equations) which can be used to solve for the length L and the angle θ_s. Each equation contains second and third order polynomials as well as trigonometric functions, so an iterative method can be exploited. This can be done in two steps. First, guess a value for L and solve for θ_s directly to satisfy the $y-$ component of the vector loop equation. Then, a Newton-Raphson method can be used to solve for L as follows:

$$L_{i+1} = L_i - \frac{f(L)}{f'(L)} \tag{16}$$

where:

$$f(L) = \overline{V}_{1x} + \overline{V}_{3x} + \overline{V}_{4x} + \overline{V}_{5x} + \overline{V}_{6x} + \overline{V}_{7x} = 0 \tag{17}$$

Note that the $\overline{V}_2$ term was omitted since $\overline{V}_{2x} = 0$. In terms of variables already discussed, this expands to:

$$\begin{aligned} f(L) \quad = \quad & \overline{V}_{1x} + r_b \ \sin(\theta_b) - r_s \ \sin(\theta_b) + r_s \ \sin(\theta_b + \theta_s) + \\ & [L_m \ \cos(\theta_b + \theta_s) - y \ \sin(\theta_b + \theta_s)] + R \ \sin(\theta_T) = 0 \end{aligned} \tag{18}$$

Fortunately, the function has proven to be well-behaved given a reasonable first guess for L. A guess that has shown itself reasonable has been to strike a tangent line from the base element to the cylinder as shown in Fig. 9. This gives a vector:

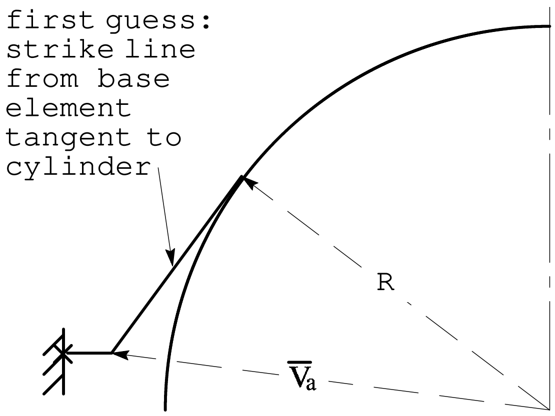

Figure 9: Approximation of the contact member

$$\overline{V}_a = (-1.897 + 0.050,\ 0.192) \tag{19}$$

with magnitude:

$$\left|\overline{V}_a\right| = 1.957 \text{ inches} \tag{20}$$

This results in an estimated length for the contact member (L_0) of:

$$L_0 = \sqrt{\left|\overline{V}_a\right|^2 - R^2} = \sqrt{1.957^2 - 1.500^2} = 1.257 \text{ inches} \tag{21}$$

Starting with this value, equation (16) converges to $L = 1.000$ inches as expected since the mount point was derived from this value.

6. Integration with CAD

Transforming the equations presented here into properly scaled and annotated drawings is a daunting task for the average designer. Much of the drudgery of calculation can be handled with computer programs, but this is not enough. The designer needs accurately scaled geometry. The deflected shape of the compound system involves the precise placement of arcs and polynomial curves. This problem is overcome by integrating the design program with a given CAD package through its native graphics interface. The geometry can also be used to facilitate production. By "relaxing" the stressed shapes, the unstressed geometry can be extracted, through the same interface, to develop forming and inspection fixtures.

Finally, the curves that are generated using this method are smooth (i.e., cusp free), and precise. One might argue that Finite Element Analysis (FEA) could achieve the same results, but this only adds another layer of difficulty. The FEA geometry is often rich with cusped approximations, and the resulting graphics is not a precise extractable representation. Transferring accurate geometry to CAD is important. Doing so gives to the designer and the production team the confidence that parameters such as force, mount location, and others have not been compromised for the sake of approximate geometry.

References

[1] A. Blake: *Practical Stress Analysis in Engineering Design*. 1st ed., Marcel Dekker, Inc., New York 1982.

[2] H. Carlson: *Spring Designer's Handbook*. 1st ed., Marcel Dekker, Inc., New York 1978.

[3] J.M. Gere, S.P. Timoshenko: *Mechanics of Materials*. 4th ed., PWS Publishing Co., Boston 1997.

[4] R. Holm: Electrical Contacts Handbook. Springer-Verlag OHG, Berlin 1958.

[5] E.P. Popov: *Mechanics of Materials*. 2nd ed., Prentice-Hall, Inc., Englewood Cliffs, NJ, 1976.

[6] Society of Automotive Engineers (SAE): *Spring Design Manual*. 2nd ed., SAE. Warrendale, PA, 1996.

Received August 14, 1998