Journal for Geometry and Graphics
Volume 4 (2000), No. 1, 71–78.

Fractal Geometry as Design Aid

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Abstract. From Charles JENCKS in England to Itsuko HASEGAWA in Japan, there is discussion in the architectural press of chaos, fractals, complexity theory, and self-organization. Architecture and design should be informed by and express the emerging scientific view that the world around us is more chaotic and complex than previously thought. However, the architectural response has a tendency to be fairly shallow. Twists and folds and waves, jumps in organizing grids, and superposition of different ordering systems are used to express in architectural form the new scientific ideas about complexity. These are moves in the right direction toward connecting architecture with contemporary cosmic concepts. However, knowledge of the mathematics of fractal geometry can provide a path to an even deeper expression.

Key Words: Fractal geometry, architecture.

MSC 1994: 00A06.

1. Introduction

Fractal geometry is the formal study of self-similar structures and is at the conceptual core of understanding nature's complexity. The fractal dimension provides a measure of the complexity of a structure. It is a measure of the mixture of order and surprise in a structure. The measured dimension and the box counting dimension methods provide procedures that can determine the fractal dimension of natural shapes like coast lines or mountain ridges and man made shapes like Frank Lloyd WRIGHT's stained glass windows. Natural rhythms in time are also fractal. Range analysis provides a method of quantifying the fractal dimension of these rhythms. Midpoint displacement methods provide procedures for generating fractal rhythms that can be used to set up complex rhythms in a building design that echo the complexity of nature. Musical rhythms provide easy access to fractal rhythms if a computer is not available. Curdling provides a method of generating fractal fields in two and three dimensions. Iterated function systems provide a means of generating an unlimited diversity of fractal shapes. None of these methods are difficult to use and they provide the designer with the power to get beyond the superficial to a deeper expression and exploration of the cosmic complexity that contemporary science is disclosing.

ISSN 1433-8157/\$ 2.50

2. The Measured Dimension

Mandelbrot [7] presents the measured dimension through a discussion of the length of the coast of England. A coastline is an irregular form that is made up of large and small bays and inlets and rocky points as well as relatively even beaches. The measurement problem is that one has to use a straight measuring device. If the length of the coastline is determined with a mile long measuring device a length will be determined. This length will not include many irregular features that are smaller in scale than one mile. Another measurement can be made with a measuring device that is 100 yards long. This length measurement will include more of the irregularities than the measurement with the mile long device but it will also leave out some irregularities. The problem is that the measurement of coastline length keeps getting longer as the measuring device gets smaller. Mandelbrot [7] suggests that coastline length is thus very poorly defined. He proposes that if the measured length and the length of the measuring device are plotted they form a power law relationship. The exponent of the power law relationship is usually fractional for natural features like coastlines. In the case of the coastline of Britain the number is approximately 1.25 . Mandelbrot [7] then suggested the extraordinary concept that this represents a dimension. The coastline is more than a line with dimension one but less than a surface with dimension two.

3. The Box Counting dimension

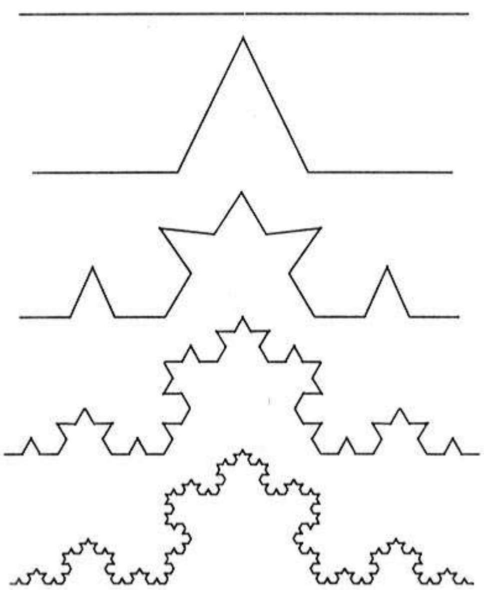

Figure 1: A Koch curve developed through four levels.

The measured dimension method can only be used in the case where the irregularities to be measured are in the form of a continuous line like a coastline. When one would like to include offshore islands in the calculation or do a calculation of the fractal dimension of an irregular group of lines, then the box counting procedure is appropriate. In the box counting method a grid of boxes of a given size (s_1) is superimposed over the drawing to be measured.

The number of boxes with lines in them is counted (n_1). Then a smaller grid of boxes (s_2) is superimposed over the drawing. The number of these smaller boxes with lines from the drawing in them is counted (n_2). The fractal dimension at this scale range is then calculated by subtracting $\log n_1$ from $\log n_2$ and then dividing this by the difference between $\log s_1$ and $\log s_2$ (PEITGEN et al. [8]). This calculation represents the slope of a line on a log-log plot of the points in question. A mathematical fractal like the KOCH curve, shown as Fig. 1, that continues its irregularities to infinity will have the same fractal dimension no matter what box sizes one uses from very large to very small. Natural forms like coastlines tend to have a scale range over which they are fractal.

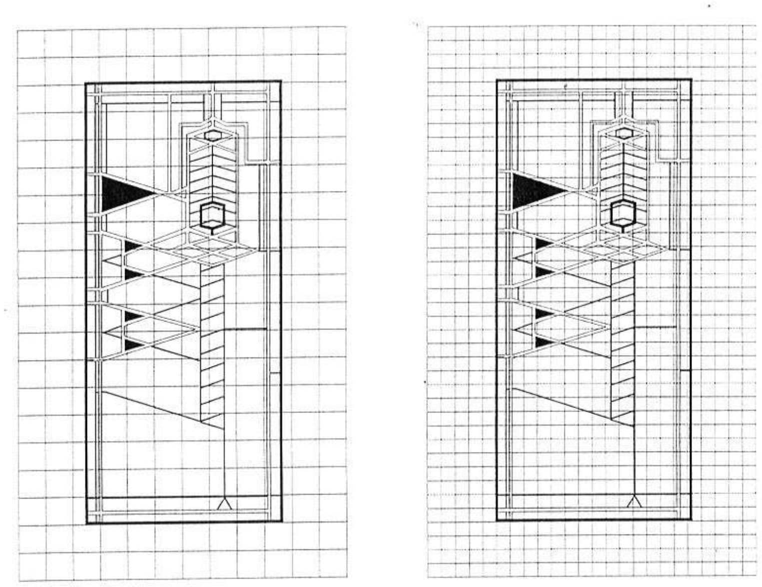

Figure 2: A box counting grid placed over a window from the Robie House.

BOVILL [4] has used the box counting method to explore the fractal characteristics of Frank Lloyd WRIGHT's work. The long elevation of the Robie House displays a fractal dimension that slowly drops from 1.6 toward 1.4 as the measurement scale drops from 24 feet to 3 feet. This range of scale represents the progression of views from across the street to approaching the front door. The fractal dimension of a typical casement window from the Robie House was determined to range from 1.7 to 1.6 through a scale range of six inches down to one and a half inches. WRIGHT kept up the complexity of his designs through a scale range from 24 feet down to one inch. Fig. 2 shows a box counting grid placed over a casement window from the Robie House. This is not surprising since his writings about design keep coming back to observation of the deep structure of nature. Nature as MANDELBROT [7] has shown is fractal. Nature displays a cascade of complexity from the organization of galaxies in the universe (MANDELBROT [7]) to the structure of gold clusters observed under an electron microscope (FEDER [6]). The Villa Savoye subjected to a box counting analysis displays a fractal dimension of 1.4 at the scale range of 8 feet but rapidly descends to a box counting dimension of 1.0, minimal complexity, at the scale range of one foot (BOVILL [4]). Corbu's forms display complexity at large and mid scale ranges but flatten out at the close in scale range of a few feet down to inches.

4. Range Analysis

The box counting dimension looks at the cascade of complexity of line from large scale to small scale. It will not however see with much precision the difference between a very regular cascade of line density, and a cascade of line density that has a complex rhythm structure. To determine the fractal characteristics of rhythm structures one must use range analysis. Range analysis comes from H. E. Hurst who studied natural fluctuations through time (Feder [6]). One of the time rhythms Hurst studied was flood levels on rivers. If one plots flood levels over time the range from the lowest to the highest level can be measured. Range analysis is performed by dividing the time span into equal sized increments (t_1). The average range from high to low value is calculated (r_1). Then the time span is divided into smaller increments (t_2) and the average range from high to low value is calculated (r_2). In general as the time increments get smaller the range gets smaller. The Hurst Exponent is then calculated as

$$\frac{\log r_1 - \log r_2}{\log t_1 - \log t_2}$$

(Bovill [4]). The fractal dimension is equal to two minus the Hurst Exponent.

A central characteristic of vernacular building is the pleasant variation in visual composition that is achieved with a limited pallet of architectural components. Range analysis was applied to this vernacular variation by Bovill [5]. The process was looking for an ordering of the architectural elements that provided the most consistent fractal measurements across a range of scales. The result showed that there was more than one ordering of the architectural elements that produced a consistent fractal calculation across a range of scales and that different orderings produced different fractal measurements. This suggests that the attraction to the complex rhythms of vernacular design may result from the ability to order the composition in multiple overlapping ways. Classical compositions generally provide only one ordering of the composition to the observer.

5. Midpoint Displacement

Midpoint displacement is the most common method of producing fractal rhythms in time. By varying the scaling of the midpoint jogs of a curve in an iterative procedure, rhythms with different fractal dimensions can be produced. Adding a random component to the jogs produces random fractal rhythms that are similar to the random fractal rhythms that Hurst observed in natural fluctuations in time like river flows and temperature variations. Midpoint displacement can be produced by hand at a small scale (Bovill [4]), but to truly explore midpoint displacement a computer is necessary.

6. Music as a Source of Fractal Rhythms

If a computer program is not available to produce fractal rhythms, musical scores can be used. Richard Voss has studied many forms of music and shown that they all show similar fractal characteristics (Barnsley et al. [2]). Range analysis can be used to measure the fractal dimension of a line of sheet music (Bovill [4]).

Musical scores provide a very easy to use method of varying the heights and or widths of a grouping of elements in an architectural composition like a row of townhouses. All that needs to be done is to assign an appropriate range of height or width to the musical scale.

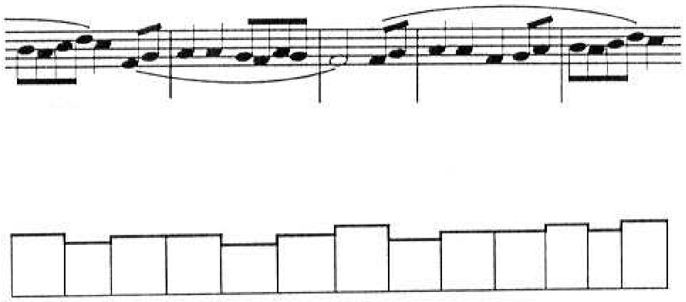

Figure 3: A row of townhouses laid out with the aid of a line of sheet music.

Then the notes on the score pick out a fractal variation of height and or width as shown in Fig. 3. In a similar way architectural elements like window types, or cornice moldings can be chosen in a fractal manner by ordering the architectural elements to be chosen to the musical scale and then letting the notes chose the element (Bovill [4]).

7. Curdling

Mandelbrot [7] gave the name curdling to a procedure that produces a random fractal dust in two dimensions. A fractal dust is a disconnected set of points that displays a clustered characteristic. The Cantor set is an example of a fractal dust, however the Cantor set is regular in its clustering. The stars in the night sky are a good example of a random fractal dust.

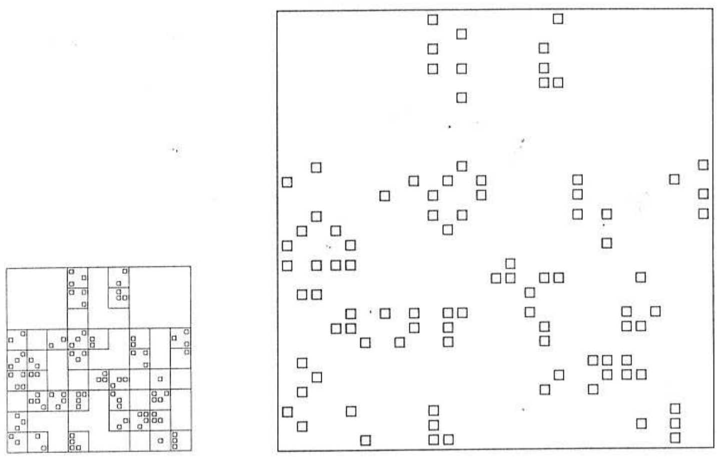

Figure 4: An example of curdling to create a complex layout in plan.

Curdling is very simple. Start with a grid on a piece of paper. Then use a coin, dice, or random generator to decide whether each of the squares of the grid is kept or not. Then subdivide the kept squares again with a grid and repeat the process. This process is repeated until the resolution of the drawing is reached. Fig. 4 shows an example of curdling through three levels. Curdling produces a set of points that have a random clustered structure on to which the observer can project his own interpretation of order. This makes curdling an ideal method of exploring graphical interpretations of the concept of deconstruction.

8. Iterated Function Systems

Iterated function systems (IFS) provide a mathematical connection between the classic fractals like the KOCH curve and natural shapes like coastlines (PEITGEN et al. [8]). PEITGEN, JURGENS, and SAUP describe how IFS work through the analogy of a copy machine with multiple reducing lenses. The resultant copies are fed back into the machine in an iterative process. An interesting thing happens. The configuration of the lenses is what determines the shape of the final image. What the original image was does not matter. For example, if there are three reducing lenses arranged in an equilateral triangle, it does not matter whether the original image was squares, circles, or ducks the final image will be the SIERPINSKI gasket.

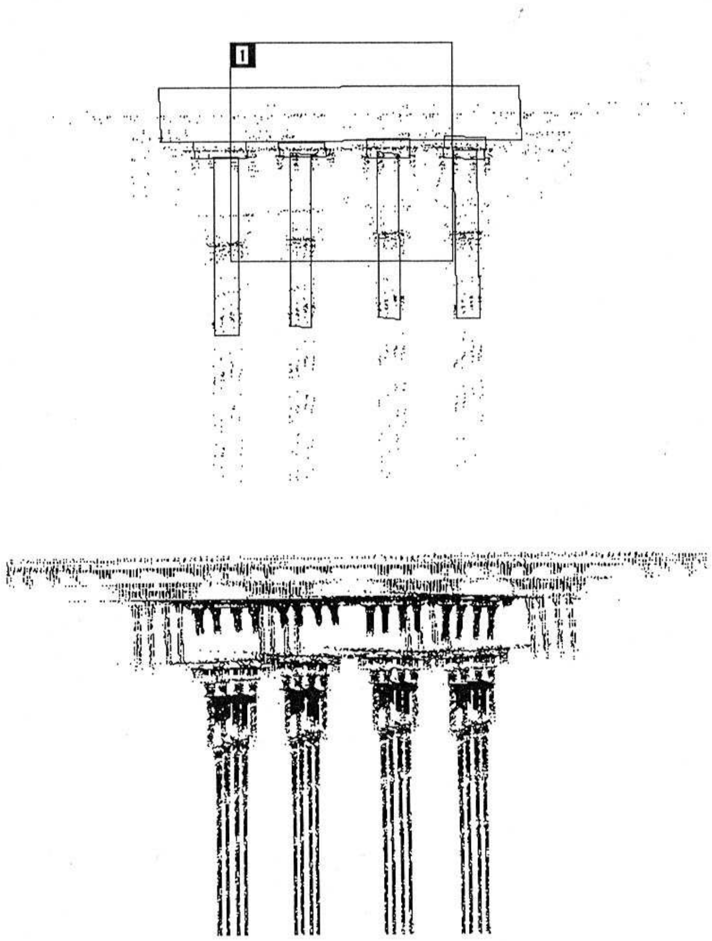

Figure 5: An iterated function system configuration in the form of columns holding up a lintel produces an image of fluted columns.

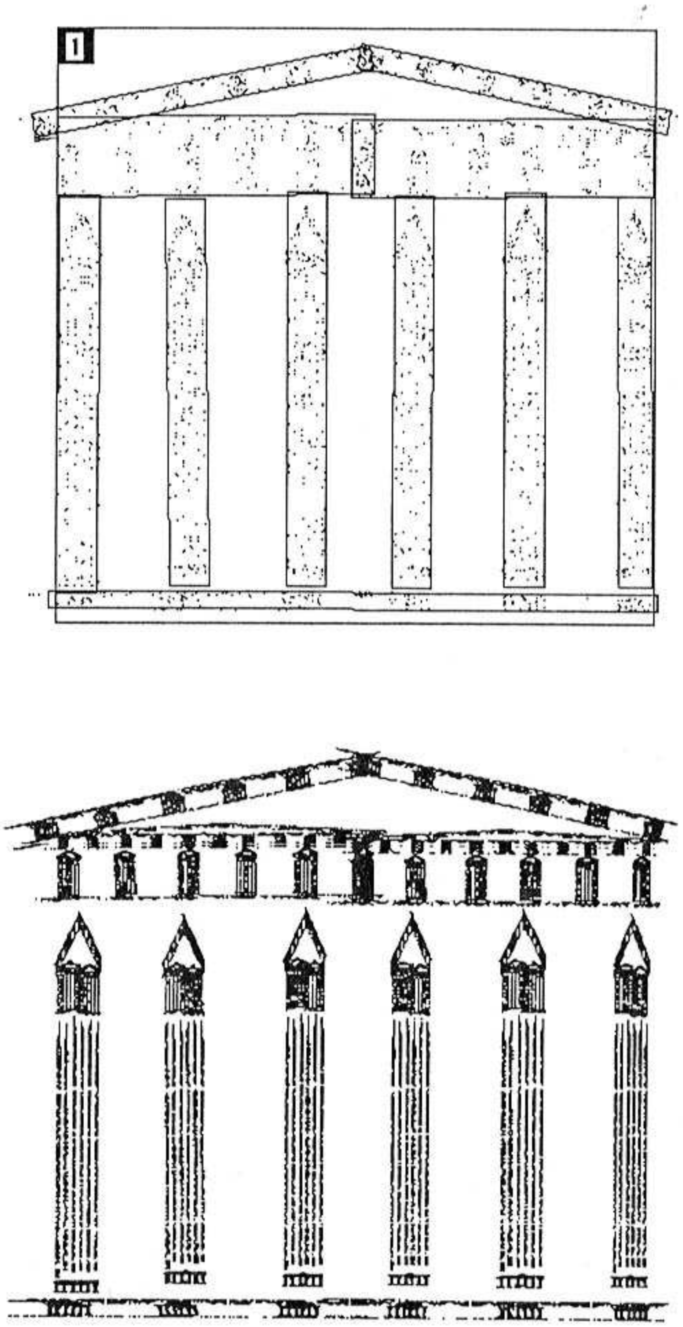

Figure 6: The four level deep fractal character of a Doric temple — the entire temple, the fluted columns, the triglyphs, and the mutules created with an IFS.

9. Conclusion, Fractal Geometry as Design Aid

Fractal geometry provides many avenues of exploration for the architect or artist who is interested in the new complex view of the world that science and mathematics are developing. However it does take a little effort to conquer some mathematics and a little faith to let the mathematics enter and influence the art.

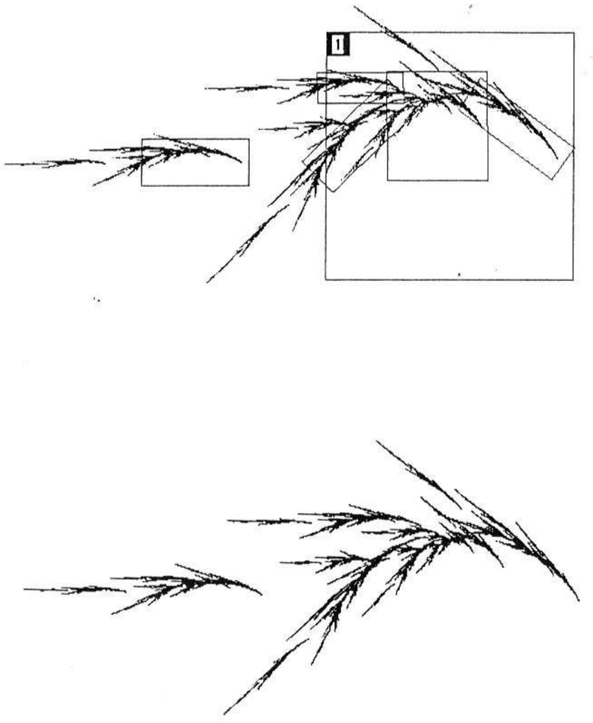

Figure 7: An iterated function system configuration used to explore the creation of organic ornament.

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Received August 14, 1998; final form December 13, 1999