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Determination of Thickness of Rotary Building Shells

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Abstract. This paper presents a non-invasive method for determining the thickness of rotary building shells. The method is based on the geometric locus of centres of circles which pass through a given point and intersect a given circle at angles of given measure.

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1. Introduction

After a building has been designed and constructed a geodetic test should figure out the dislocation and deformation of the whole building or only that of critical parts. The proper thickness of a shell plays an essential role for the safety and durability of an object. Shells of industrial objects are often submitted to aggressive influence of both water and atmospheric environment and thus undergo degradation. The knowledge of the real geometry of an object allows to determine necessary local strengthening. In this sense the possibility of determining the thickness of a shell is important.

The suggested method refers to rotary buildings since it is based on theoretically circular planar sections of buildings. However after introducing slight modifications it can be used for other objects, too.

Hyperboloid cooling stacks, digestive chambers in sewage plants of shapes consisting of conical and cylindrical surfaces, cylindrical tanks with portable water, or oval tanks are examples of buildings where thickness plays an essential role. Often the thickness varies for different heights. The distribution of the shell thickness in relation to the theoretical shape, which e.g. for cooling stacks is a one-sheet hyperboloid of revolution, can even be nonsymmetrical.

A lack of planning and low precision at the construction result in the fact that neither the inner nor the outside surface of the shell are surfaces which can be described mathematically in a closed form. In extreme cases big execution mistakes can be the reason for local deviations from the designed shape which exceed 1.0 m. The knowledge of the final shape of an object is thus often the result of measurements and therefore depending on their exactness.

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It is a common practice that due to a failure a part of the shell needs the determination of thickness. As a result of the necessary strength analysis a local support is suggested, which in turn requires a repair of the shell under permanent measurement control. Then only routine geometric measurements allow to evaluate the effectiveness of the restoration work.

Due to technology reasons there is a tendency not to use prefabricated products but monolith, reinforced concrete constructions that enforce the necessity of measuring the shell thickness at various stages of construction and at the usage of a building.

2. Principles of shell thickness measurement

For our considerations a cooling stack with the theoretical shape of a one-sheet hyperboloid of revolution has been chosen as an example.

The study is based on geometric properties of coplanar or noncoplanar circles passing through a given point and intersecting a given circle. The centers of circles passing through a given point in space and making a given angle α of intersection with a fixed circle belong to a curve of 4^{th} degree. In the special cases $\alpha = 0°$ or $90°$ or at a specific reciprocal location of the given elements this curve can undergo a degradation and constitute a curve of second degree or a line only.

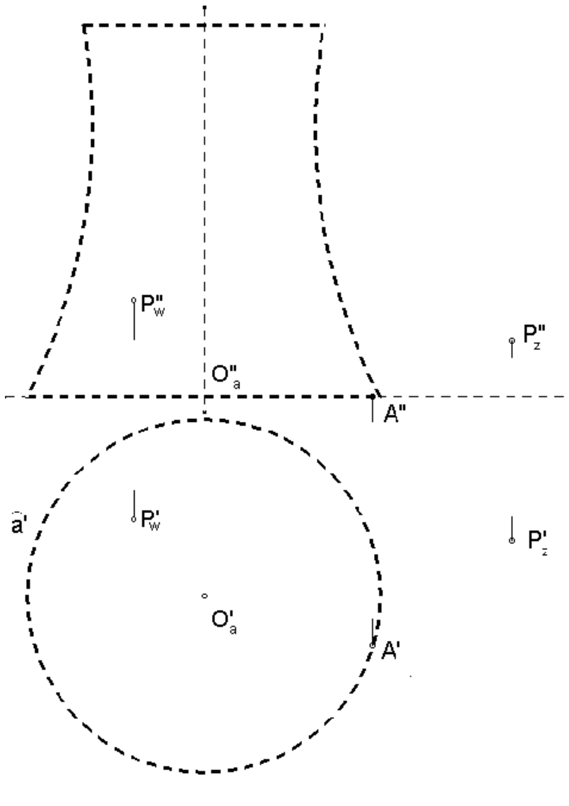

Figure 1: Points where the measurement apparatus is located

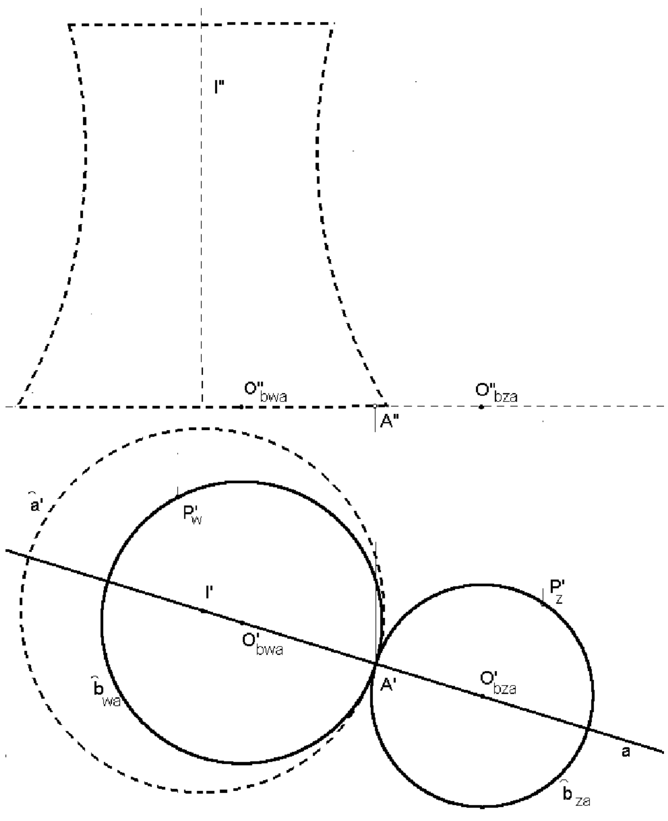

Figure 2: The base circle $\widehat{a}$ and the diameter line a through A

For determining the thickness of a given shell we specify the given circle as theoretical section of a shell with a plane perpendicular to the axis of revolution. And we set $\alpha = 0°$ as we look for circles tangent to the given section. In order to achieve the desired result it is necessary to follow consecutive steps. The geometrical background of the suggested method

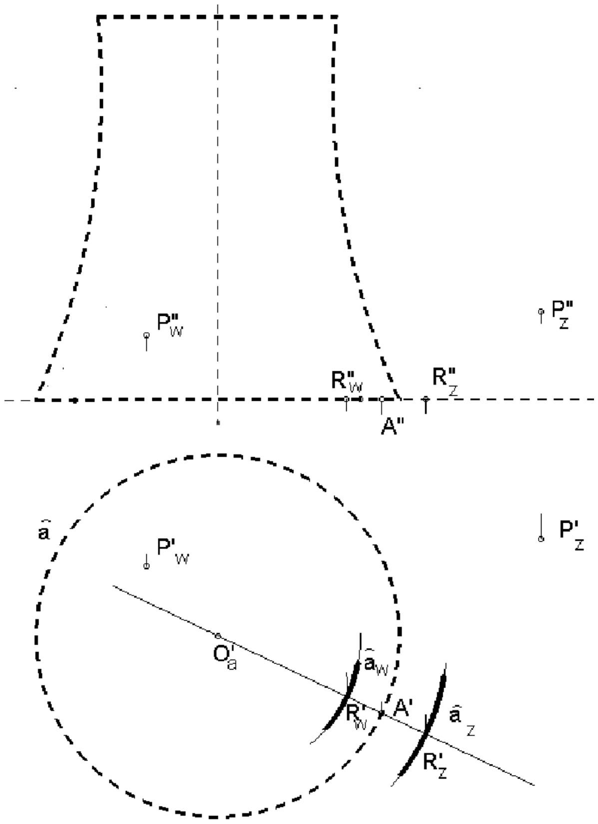

Figure 3: Circular arcs $\widehat{a}_z$ and $\widehat{a}_w$ representing the exterior and inner surface of the shell

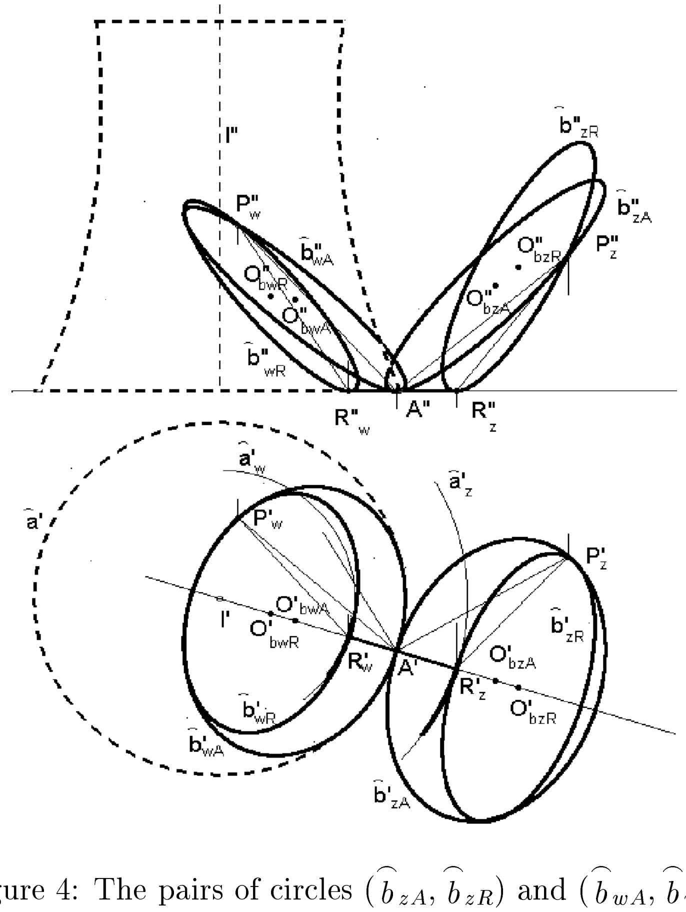

Figure 4: The pairs of circles $(\widehat{b}_{zA}, \widehat{b}_{zR})$ and $(\widehat{b}_{wA}, \widehat{b}_{wR})$

is presented below with accompanying drawings. For better understanding we combine this with the explanation of graphical constructions.

Let us specify a base circle $\widehat{a}$ from the technical documentation. It is a section of the theoretical model of the building by a plane perpendicular to its axis l. Let A be a point of this circle. Suppose that at this theoretical point it is necessary to determine the thickness of the shell. For the case of simplicity, we propose to measure the thickness between the inner and the outside surface of the shell along a line a which intersects the axis l perpendicularily.

We use two positions for our measurement apparatus: Point P_z is supposed to be an exterior point of the building such that its orthogonal projection P_z' into the plane of the base circle $\widehat{a}$ is outside the circle. Point P_w is situated in the interior such that its projection P_w' into the plane spanned by $\widehat{a}$ is inside the circle. Fig. 1 shows these positions in Monge's method of orthogonal projection.

The coordinates of the point A and the radius of the circle $\widehat{a}$ can be extracted from the plans whereas the coordinates of the points P_z and P_w are determined from nature.

The line a containing the segment whose length gives the thickness of the shell at point A, is the diameter of the circle $\widehat{a}$ which passes through A. For its determination the circle $\widehat{b}_{za}$ coplanar with $\widehat{a}$ was taken into consideration which passes through P_z' and contacts $\widehat{a}$ at

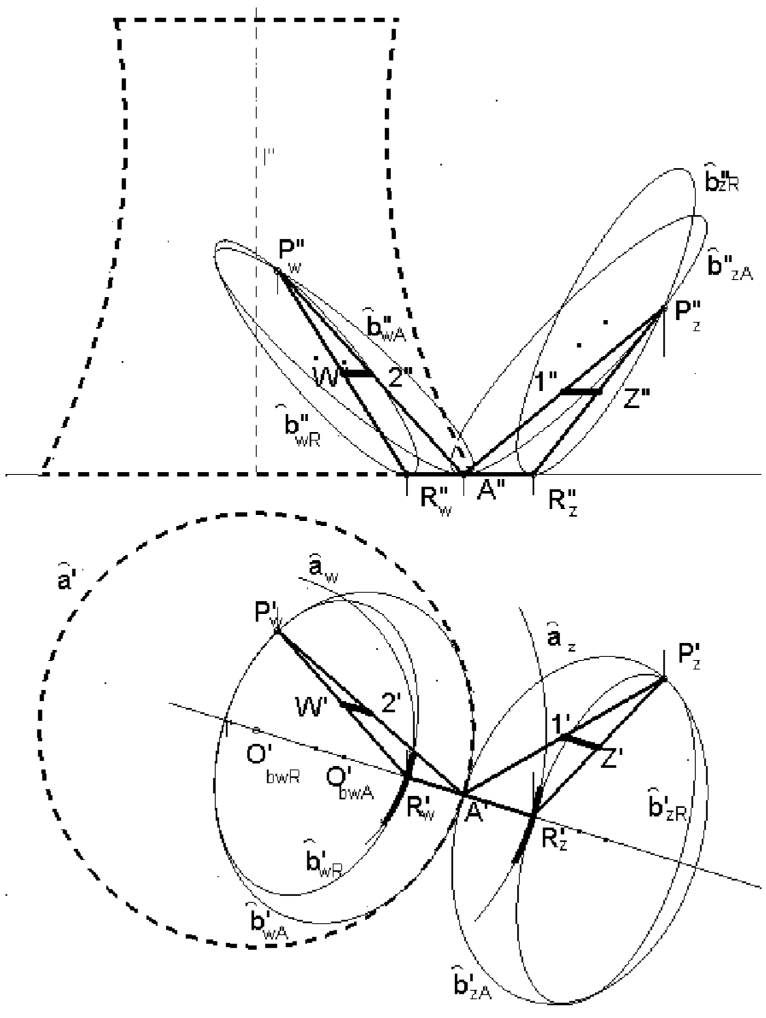

Figure 5: Relation beween the required segments and chord midpoints

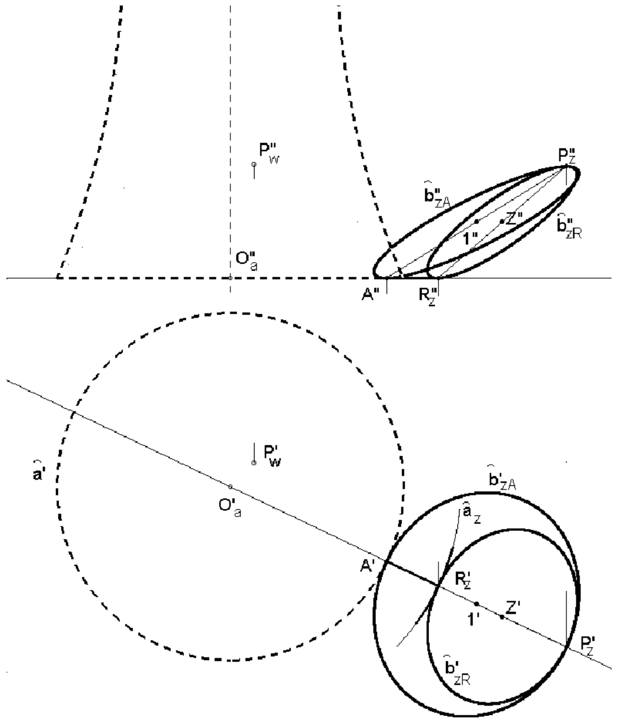

Figure 6: Particular case with point P_z in the meridian plane α of A

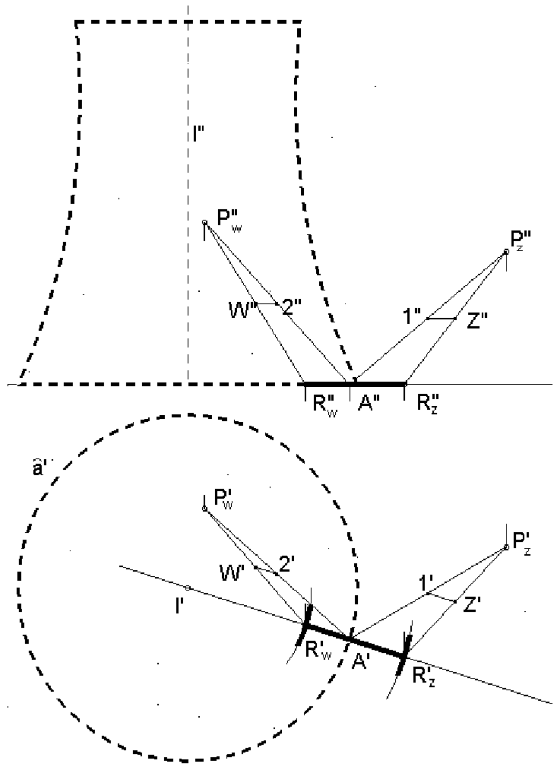

Figure 7: The required thickness equals the sum of two distances

the point A. The line a connects the centre O_{bza} of this circle $\overset{\frown}{b}_{za}$ with point A (see Fig. 2).

Similarly, for the inside surface of the building the direction of the same line a was determined by constructing a circle $\overset{\frown}{b}_{wa}$ through P'_w and tangent to the circle $\overset{\frown}{a}$ at point A. Again, the centre O_{bwa} of $\overset{\frown}{b}_{wa}$ is located on the line a.

For known coordinates of P_w, P_z, the center O_a of $\overset{\frown}{a}$ and A and for given location of the section plane it is easy to determine the parameters of the line a. Using the graphical software CABRI, the points O_{bza} and O_{bwa} can be drawn by means of macroconstructions. The situation described above is presented in Fig. 2.

For determining the thickness, we use in a neighbourhood of point A hypothetical arcs of circles $\overset{\frown}{a}_z$ and $\overset{\frown}{a}_w$ coplanar and concentric with the base circle $\overset{\frown}{a}$. These arcs symbolise the exterior and inner surface of the building shell and they are used only in theoretical considerations and illustrations (compare Fig. 3).

Now the required thickness is the difference of the radii of the circles $\overset{\frown}{a}_z$ and $\overset{\frown}{a}_w$, respectively, in a case where it is impossible to take measurements directly.

In practice, instead of theoretical points R_z and R_w on the circles $\overset{\frown}{a}_z$ and $\overset{\frown}{a}_w$, these points are marked as physical points on the building shell during measurements. E.g., point R_z can be determined as intersection of a ray (e.g. laser type) pointing from O_{bza} to the point A.

For the points R_z and R_w marked on the shell as well as for points P_z and P_w it is possible to determine pairs of circles $(\overset{\frown}{b}_{zA}, \overset{\frown}{b}_{zR})$ and $(\overset{\frown}{b}_{wA}, \overset{\frown}{b}_{wR})$ passing through the points P_z and P_w, respectively, and tangent to the circles $(\overset{\frown}{a}, \overset{\frown}{a}_z)$ and $(\overset{\frown}{a}, \overset{\frown}{a}_w)$ at the points (A, R_z) and

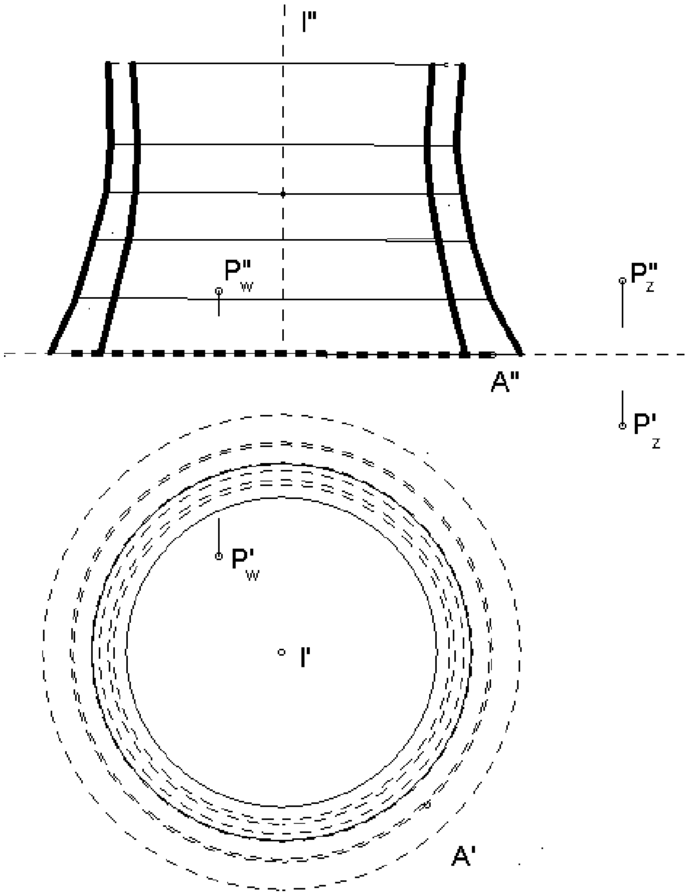

Figure 8: Different horizontal sections of the shell

(A, R_w), respectively. These circles can be drawn using macro constructions – in accordance with [2]. The circles are displayed in Fig. 4.

The segments $1Z$ and $2W$ connecting the midpoints of chords AP_z, R_zP_z and AP_w, R_wP_w of the circles $\overset{\frown}{b}_{zA}$, $\overset{\frown}{b}_{zR}$ and $\overset{\frown}{b}_{wA}$, $\overset{\frown}{b}_{wR}$ are parallel to the segments AR_w and AR_z. The length of the segments $1Z$ and $2W$ are half of the length of segments AR_w and AR_z (see Fig. 5). The sum of length of the segments AR_w and AR_z is the measure of shell thickness at the considered place, i.e.,

$$|AR_z| + |AR_w| = 2(|1Z| + |2W|).$$

When in particular the point P_z (or P_w) is located in the meridian plane α through A – as shown in Fig. 6 – then instead of chord midpoints we have the centres O_{bzA}, O_{bzR} (or O_{bwA}, O_{bwR}) of circles $\overset{\frown}{b}_{zA}$, $\overset{\frown}{b}_{zR}$ (or $\overset{\frown}{b}_{wA}$, $\overset{\frown}{b}_{wR}$).

If the coordinates of the point A, the radius of the circle $\overset{\frown}{a}$ and the length of the segments R_zP_z and R_wP_w are known, we obtain the thickness of the shell at point A by addition of the distances AR_w and AR_z (Fig. 7).

For consecutive levels we use points $A_{1,2,\ldots}$ (see Fig. 8 with a very exceeded scale) and repeat the procedure of intersecting the line a with the inner and exterior surface of the shell.

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