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Descriptive Geometry Courses for Students of Architecture – On the Selection of Topics

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Abstract. Descriptive Geometry (DG) is an applied mathematical discipline dealing with the practical performance of the process of representation as well as with the analysis and generation of objects in three-dimensional space by methods of drawing. Due to the decreasing share of DG in the curriculum of architectural studies it is no longer possible to teach DG even roughly to its full extent. As contribution to the development of a curriculum "Descriptive Geometry for architects" the geometrical topics actually used by professional architects as well as those which assist the student directly or indirectly in developing skills fundamental in the daily work of professional architects are explored.

Key Words: Descriptive Geometry courses, architecture, selection of topics

MSC 2000: 51N05

1. Introduction

In order to determine the topics and methods of Descriptive Geometry (DG) which are in particular relevant for professional architects we will not only clarify the value of the various ways of projection for architectural drawings but also analyse the application of spatial objects in the architect's field of activity.

2. Selecting ways of projection

The following aspects are relevant for selecting the type of projection:

- The predefinition of architectural objects by orthogonal projection,
- the ways of projection with realistic effects and
- the enhancement of the realistic effects by other means.

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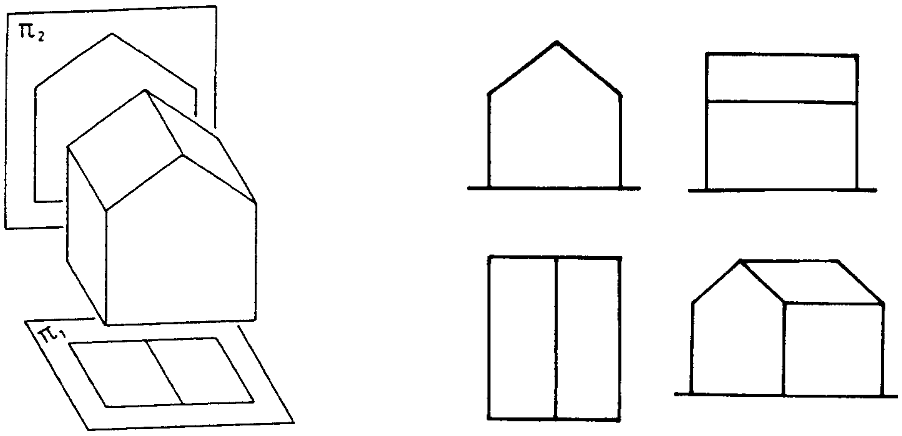
2.1. Orthogonal projection

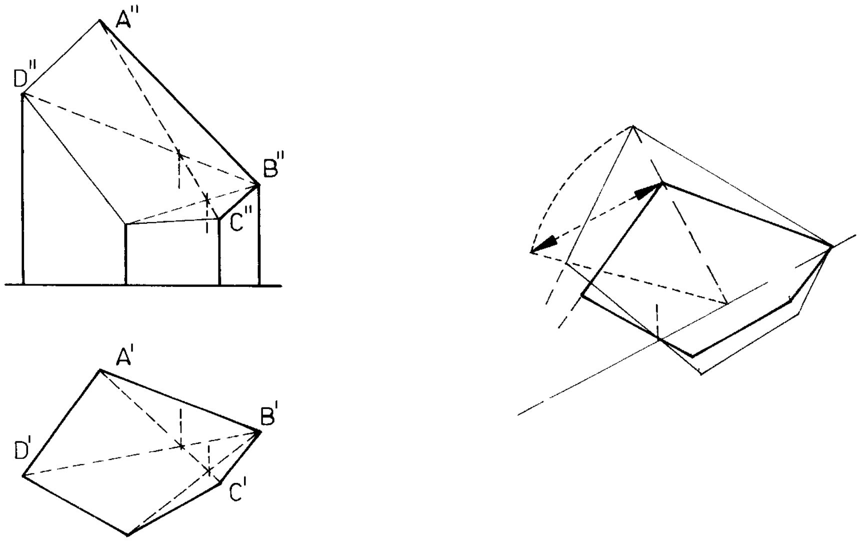
Architects depend on drawings to develop projects, to locate problematical areas and to find solutions regarding design and construction. These solutions have to be secured and they provide the basis for all following phases of architectural planing and realisation. The drawing facilitates the thinking-process and enables the architect to communicate with all the other specialists involved in the process of building. DG represents an essential contribution to the predefinition of architectural designs by means of orthographic projection onto special image planes:

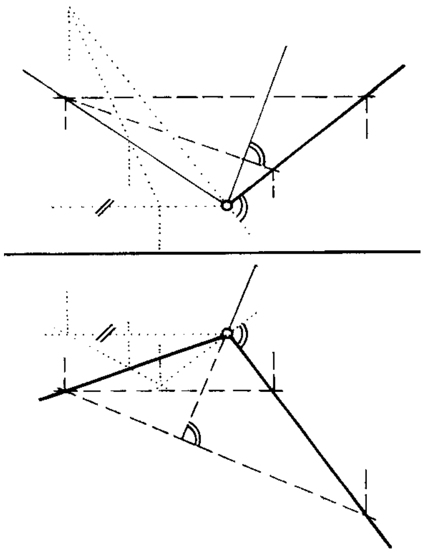
- *ground plan* (the image plane π_1 is horizontal),
- *elevation* (the image plane π_2 is vertical and parallel to a main plane of the object),
- *side view* and *section* (the image plane is vertical but not parallel to a main plane of the object).

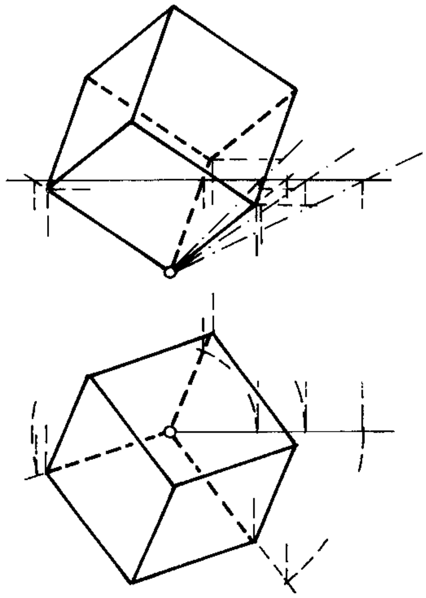


With these orthogonal projections all tasks of spatial geometry relevant to a professional architect can be solved by combination of only eleven basic tasks:

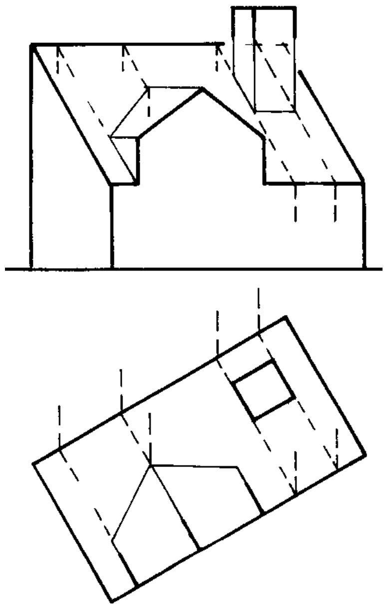
1. Determination of a fourth point of a plane,
2. intersection of a straight line and a plane,
3. intersection line of two planes,
4. determination of a line orthogonal to a plane,

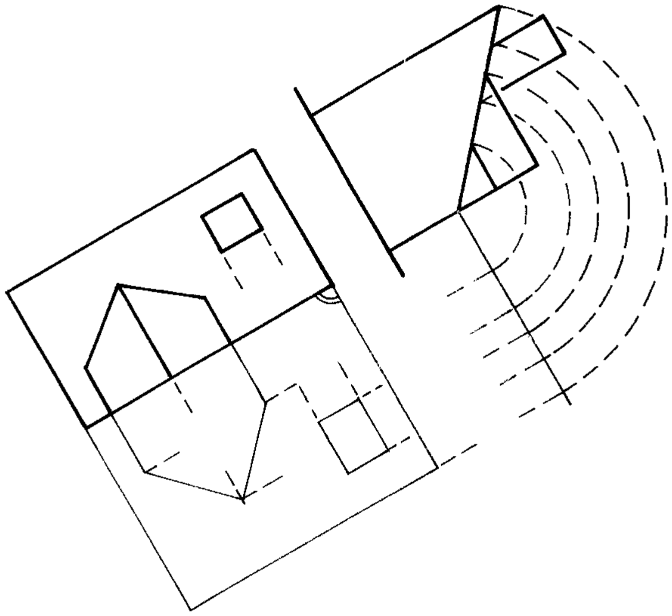






5. determination of a plane orthogonal to a line,
6. rotating a point about a straight line,
7. true length of a line segment,
8. true size of the angle between a line and the image plane,
9. true size of the angle between a plane and the image plane,
10. true size of the angle between two intersecting lines,
11. true size and shape of a plane figure.

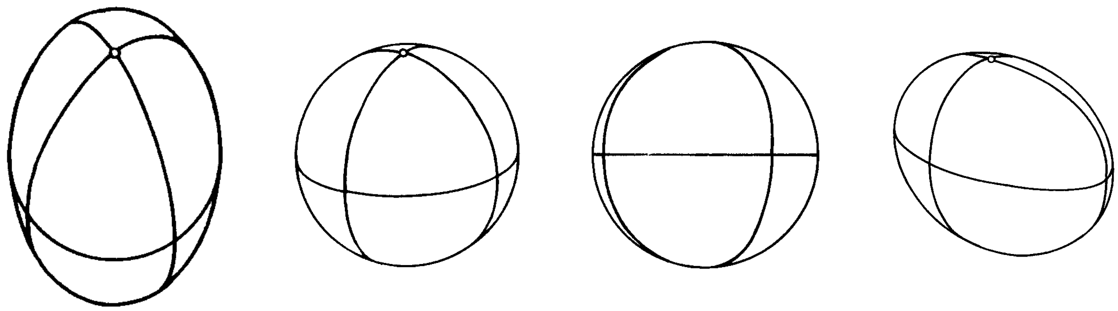




Due to the great importance of this topic the aspects of orthogonal projection mentioned above should build the main part of teaching DG for students of architecture. From experience we know that these topics are for students the most difficult ones to understand. Once understood, the practising architect works with these methods everyday. Therefore most of them forget that they had to learn it once and that this was a hard job.

2.2. Ways of projection with realistic effects

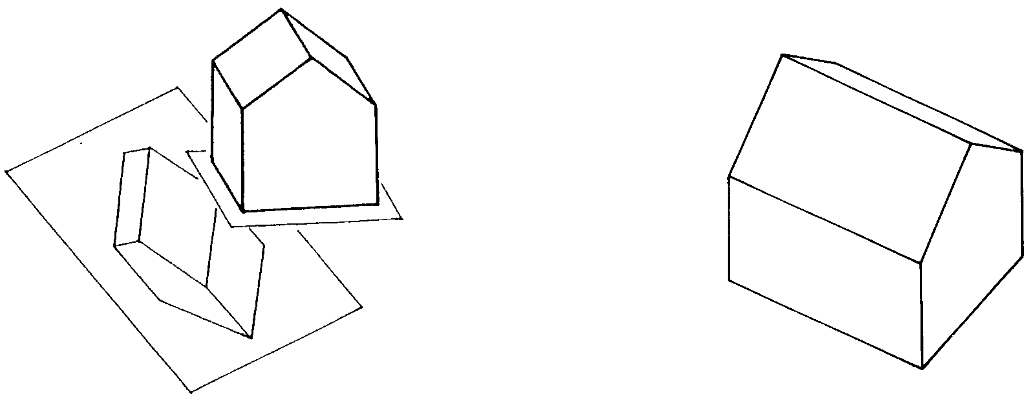
Architects utilise realistic views significantly less than orthographic views. Realistic views are mainly used to show laymen the effect of the architectural design. For the architect it is important that these drawings come as near as possible to reality. The more the projective beams and the visual rays (of the observer) correspond, the better the realistic effect of a drawing. In most cases the *oblique parallel projection* is unfavourable, e.g. because the outline of a sphere is an ellipse. The *orthographic parallel projection* is beneficial; the outline of a sphere is a circle. The *central projection* is ideal because the projective beams and the visual rays can actually correspond; the outline of a sphere is a circle. But the effect of a central projection will be unfavourable when the observer looks from the wrong position: if the centre of the sphere is not on the main visual ray, the outline of a sphere is an ellipse or even an other conic.



Regarding this, the four ways of projection with the most realistic results are determined:

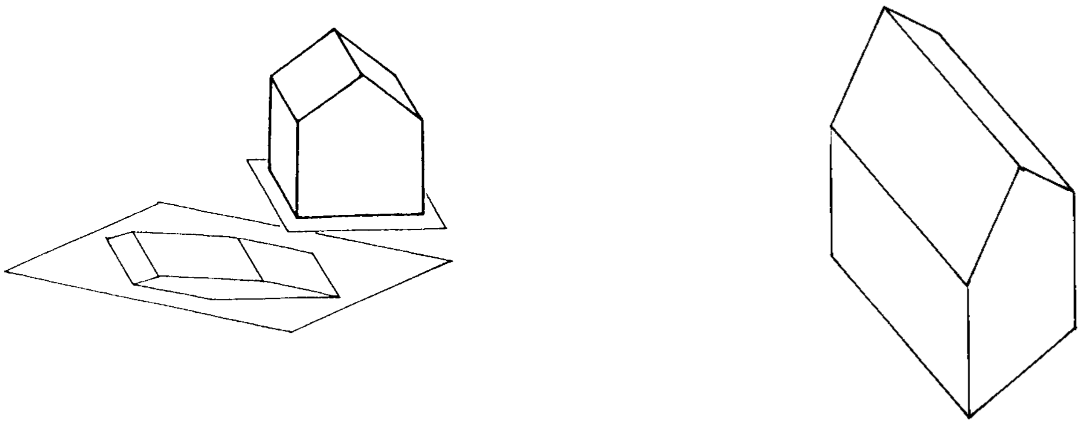
- *Axonometry*

  The orthographic parallel projection onto an inclined image plane has especially good effects and is easy to draw.



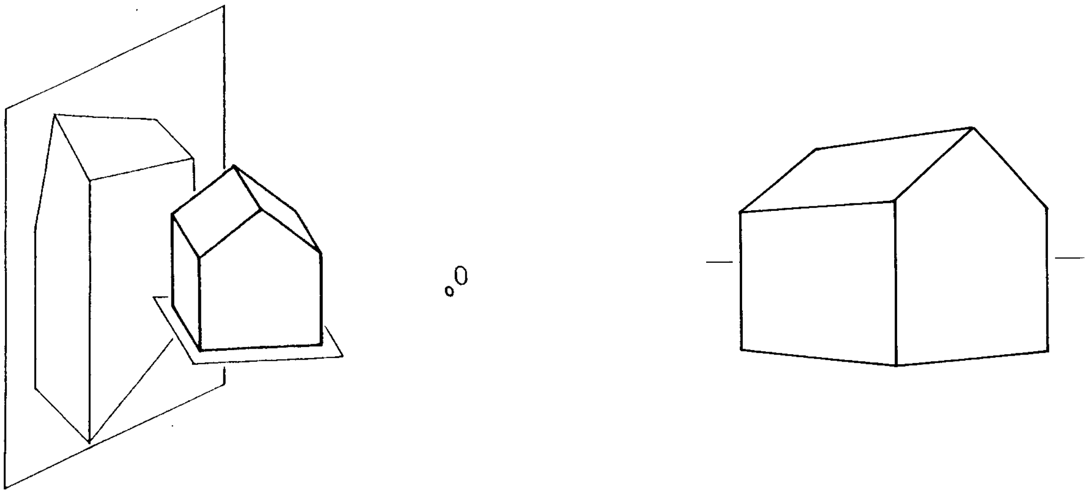
- *Birds-eye-view*

  Even though the oblique parallel projection onto a horizontal image plane has disadvantages in regard of its realistic effects it should be taught because it is the easiest realistic drawing.



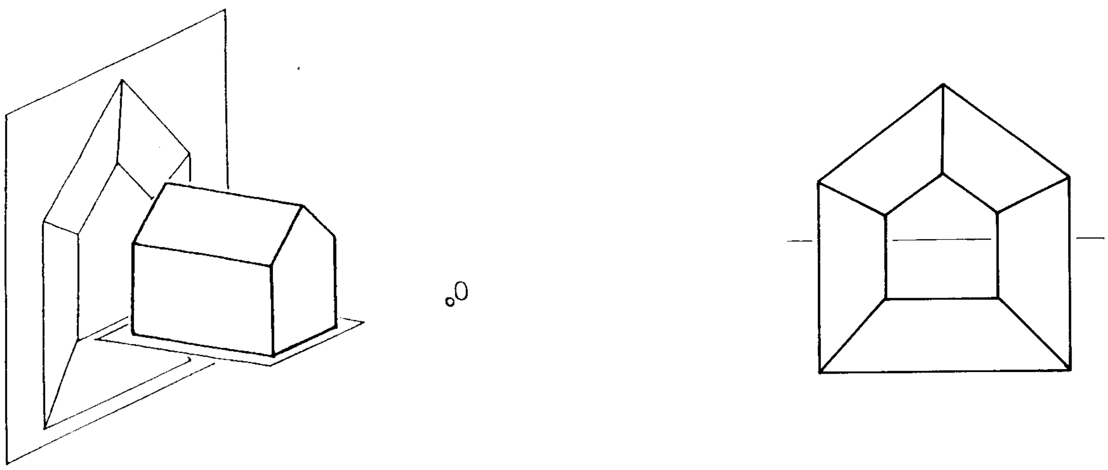
- *Perspective*

  The central projection onto a vertical image plane has the best realistic effect; with this perspective the architect can show his design in the most favourable way.



- *Frontal perspective*

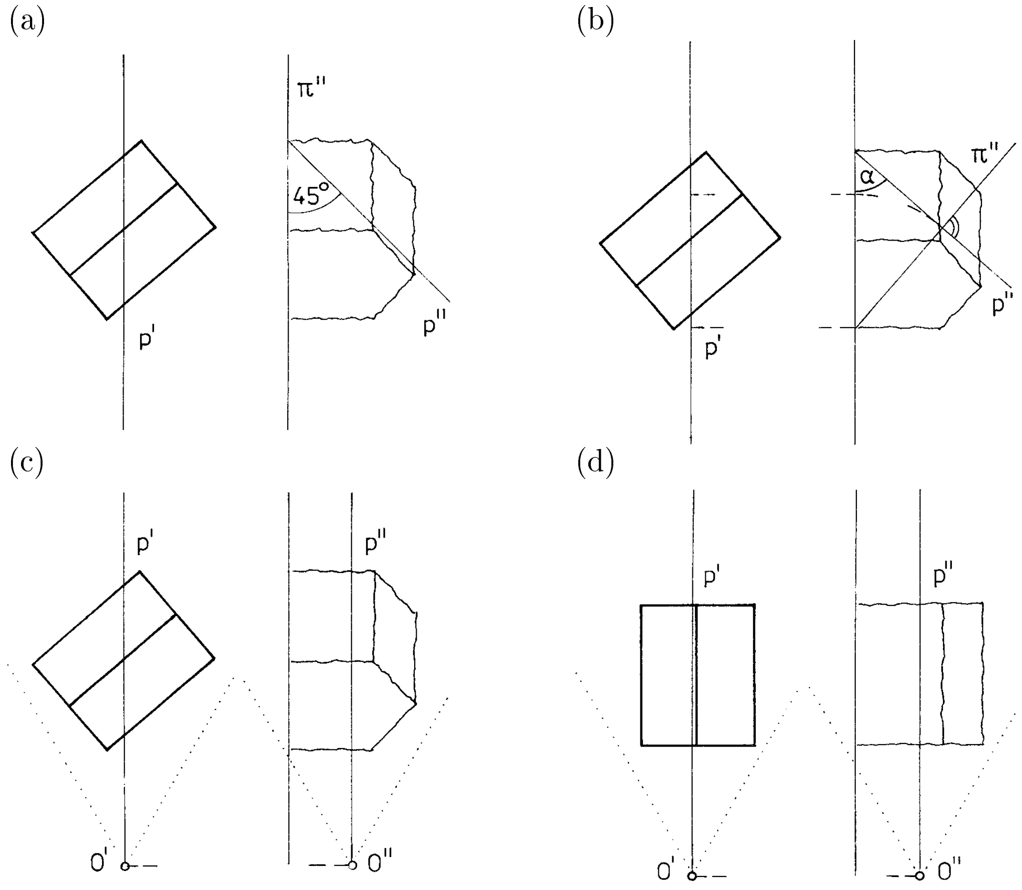
  The central projection onto an image plane parallel to the elevation plane is a special case of the perspective: The object cannot be shown from every angle but the method of construction is easier.



These reflections imply that the birds-eye-view and the perspective are particularly important to architects and therefore should be taught in full detail. The axonometry and the frontal perspective are also suggested to be included in the curriculum.

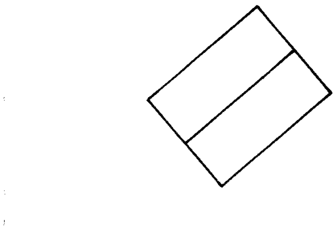
In order to present a concise and simple method of drawing views for architects, a composition procedure has been designed which works for all ways of projection, requires little effort and allows drawings of every object. The basis of the composition procedure consists of four steps: (a) birds-eye-view, (b) axonometry, (c) perspective, (d) frontal perspective.

I. Choice of view:



II. Determining the ground plan:

(a)



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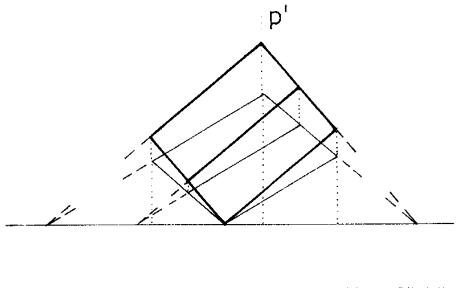

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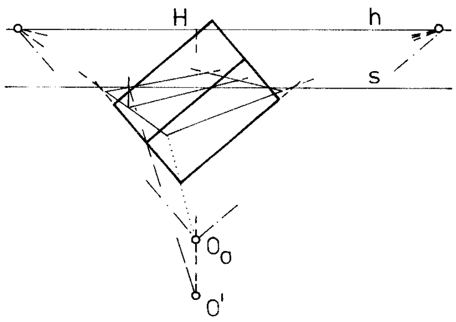

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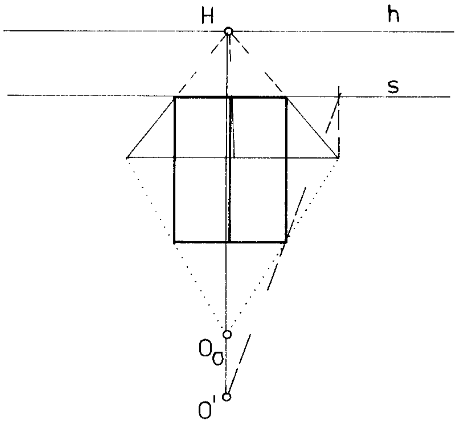
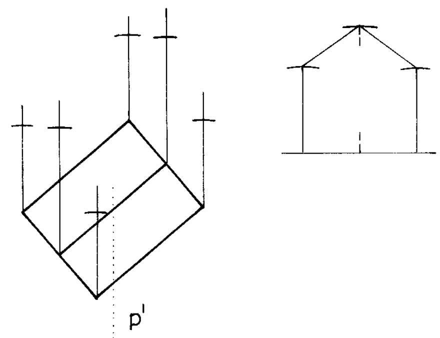


III. Protracting the heights:

(a)



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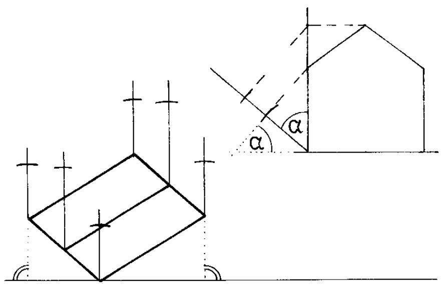

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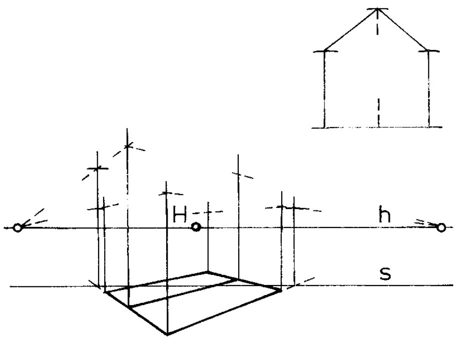

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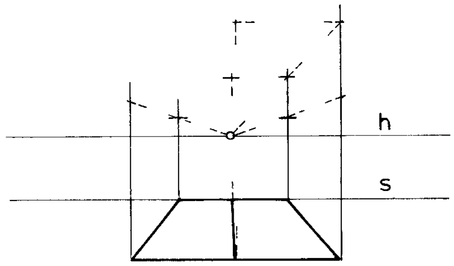
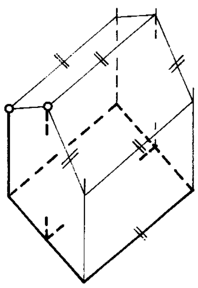


IV. Simplifications for the reproduction of parallel lines:

(a)



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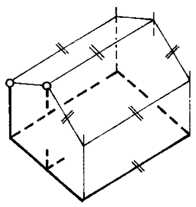

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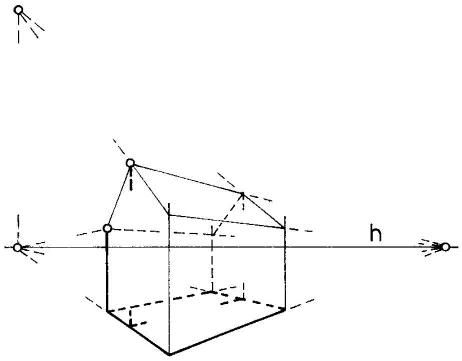

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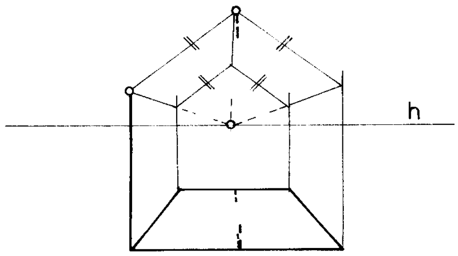


Supplementary elements are developed which are able to reduce the effort in construction even more:

V. Simplifications for the subdivision of lines,

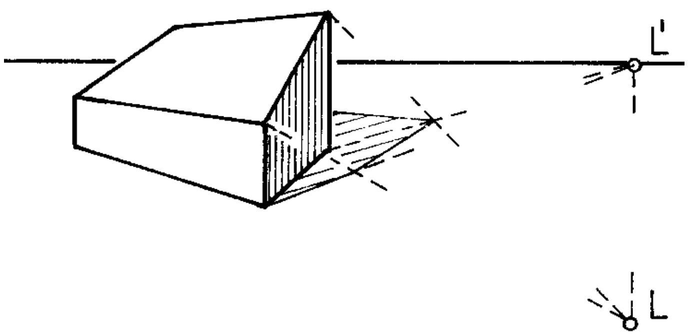
VI. simple reproduction of figures in vertical planes,

VII. simple reproduction of figures in inclined planes,

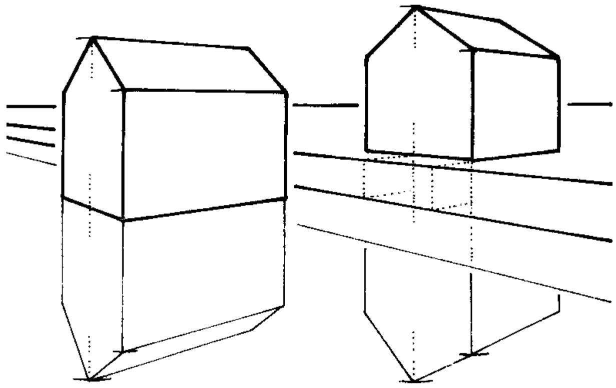
VIII. reconstruction of the parameters of a projection.

2.3. Enhancement of realistic effects

The realistic effect of ground plans can be improved by including shadows; this technique is often also useful for elevations, axonometry and perspectives. The corresponding geometrical ideas should be part of the curriculum. The construction of shadows is possible by means of the eleven above mentioned basic tasks.



Illustrations of reflections in architectural drawings are not really necessary. On the other hand there are some trivial special cases which are worth to mention in the mandatory course.

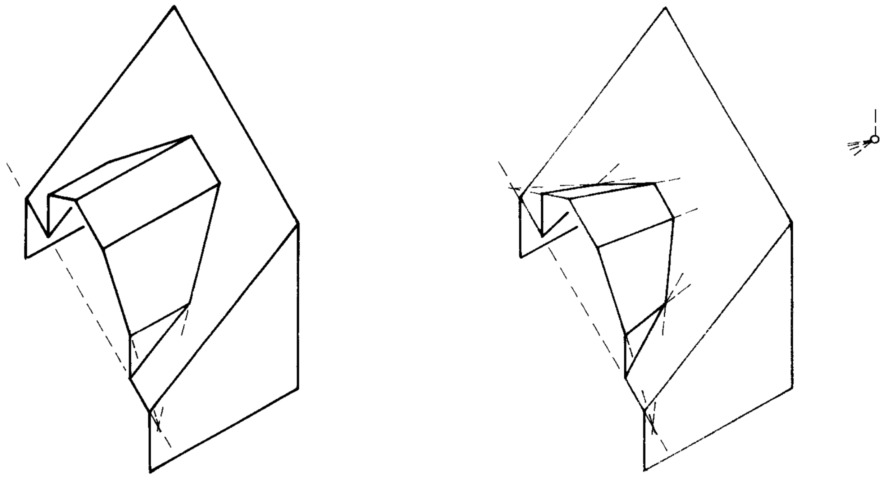


3. Selection of objects

The following objects are to distinguish: Polyhedra, curved surfaces and the intersections of curved surfaces.

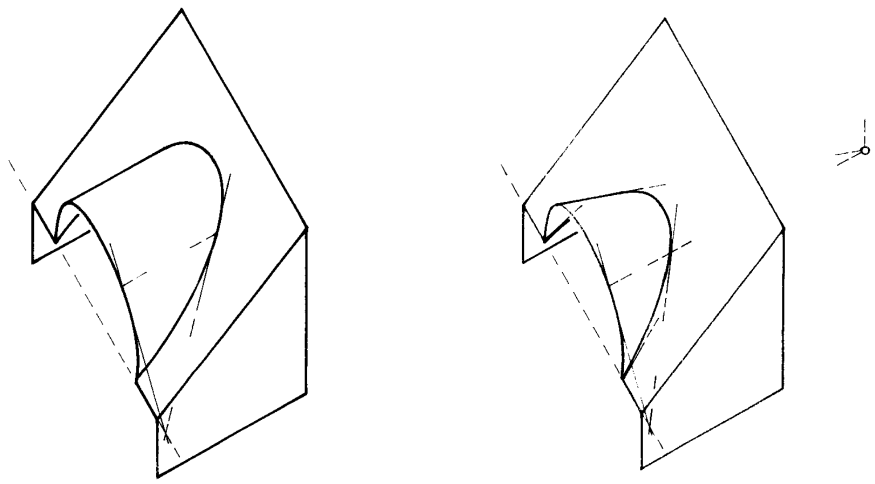
3.1. Polyhedra

Polyhedra represent the spatial objects being most important to architects. Everybody will agree that the preponderant part of architecture is formed by prisms and cuboids whereas pyramids are rarely used and regular polyhedra are nearly never found. On the other hand there are a lot of general polyhedra in architecture. To handle polyhedra in drawings the architect must be able to master only the eleven above mentioned basic tasks. Some easement of labour can be achieved by using affinity or perspectivity when a prism or a pyramid is cut by a plane.

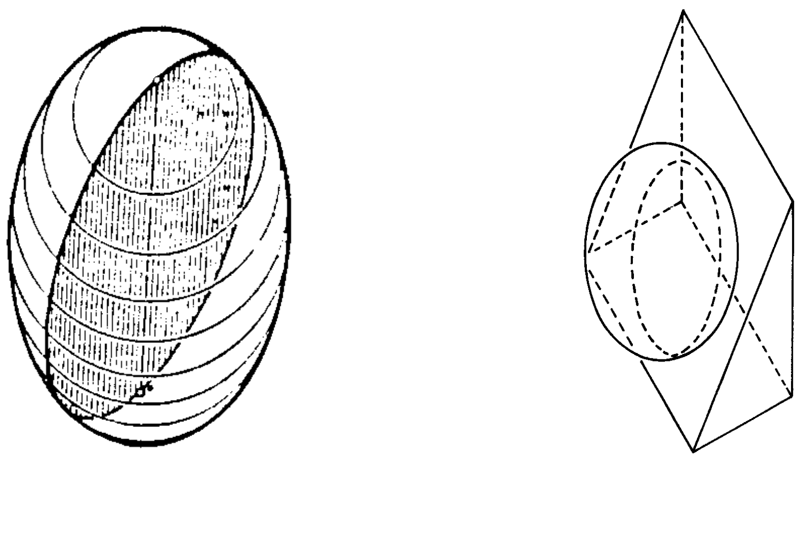


3.2. Curved surfaces

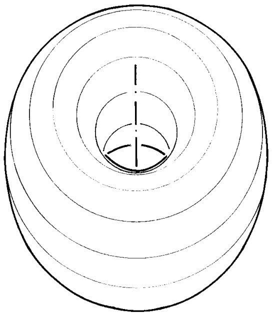
Straight cylinders are often used as architectural elements; the advantage of straight cylinders is that they are formed by straight lines (beams) and circles of the same size, therefore the prefabrication is easy to plan. The straight cylinder is mostly positioned horizontally or vertically. Elliptical, parabolic and general cylinders are seldom used in architecture: The straight lines and the repetition of the same curves on the surface are advantageous, but the bending differs in each point of the curves. There are only few *straight circular cones* in architecture (and nearly no general cones): Their disadvantage is that all straight lines intersect in one point and all the circles differ in their radius. To handle cylinders and cones in drawings the architect can fall back on the thorough elaborateness of prisms and pyramids. Furthermore ellipses, the construction of tangents and the development of the surfaces have to be dealt with.

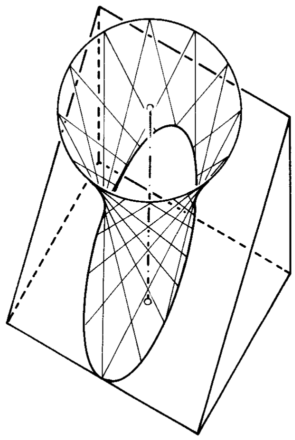


There is a limited number of *spheres* in architecture: Due to its double bending a sphere can be built as a shell and will carry a multiple of its own weight. But the fabrication of spheres is expensive.

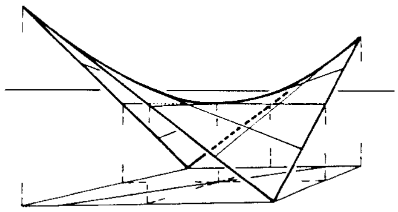


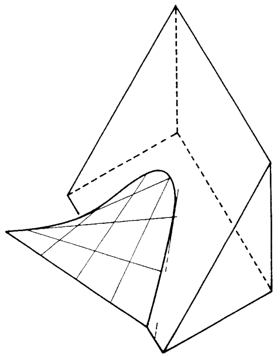
General *surfaces of revolution* are less suitable as parts of buildings: The meridians can be manufactured repeatedly in the same form, but the parallel circles on the surface differ in size. Most examples of surfaces of revolution in architecture are tori (rotated circles) or one-sheet hyperboloids of revolution (rotated straight lines).



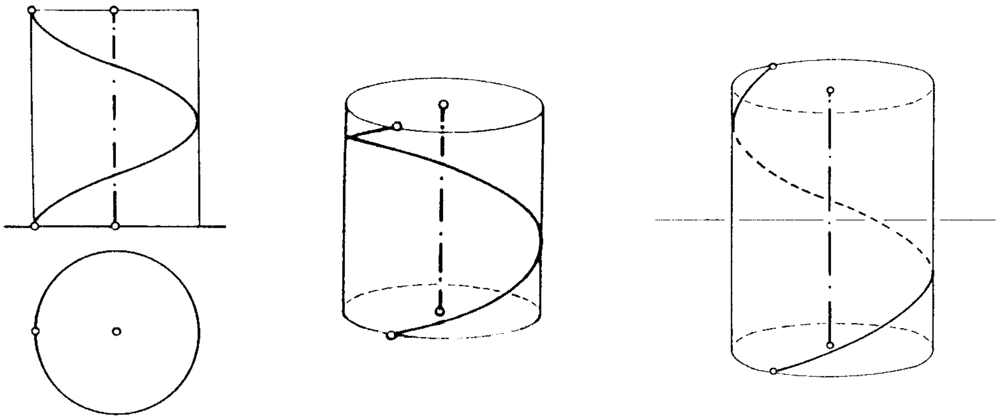


General surfaces generated by the translating a curve don't supply the architect with solutions for his tasks. The only exception is the *hyperbolic paraboloid*: On its surface there are two cohorts of straight lines, therefore form boards can be used for its fabrication. Furthermore two cohorts of parables are on the surface which opportunely divert the forces. So great spans with small construction heights are possible.



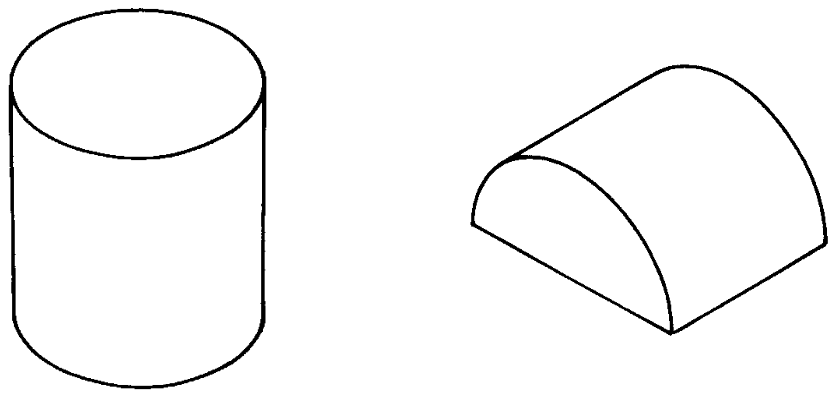


Building parts generated by a screwing motion can almost only found at spiral staircases and ramps.

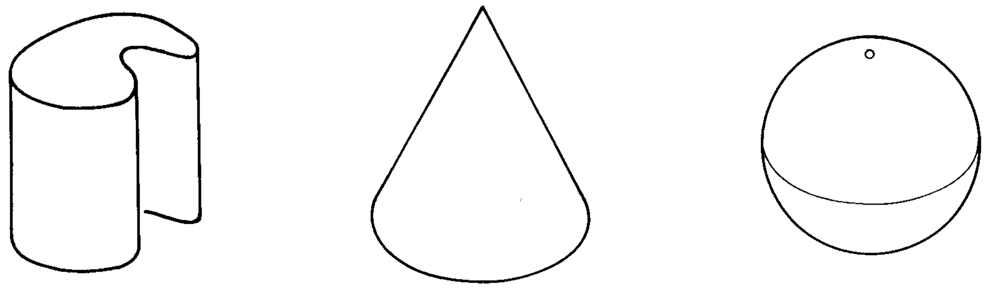


All the surfaces treated until now can relatively easily be generated by a movement of a curve. Of course there are other surfaces which can be relevant for solving architectural tasks. As they do not follow simple geometrical laws and their construction is very complicated to handle. These surfaces are not part of the curriculum of DG. On the other hand it should be noticed that only those parts of buildings are to produce easily and economically, which are easy to draw. If the architect still intends to use a general surface, he has to use a model or a computer specialist.

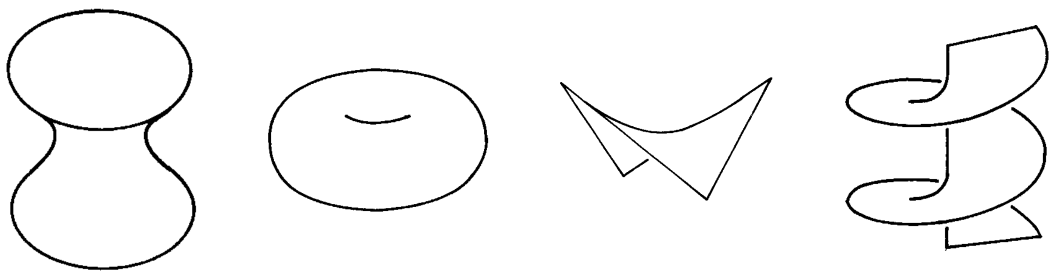
Recapitulating it can be argued, that straight cylinders are most frequently used as architectural elements among curved surfaces; therefore students of architecture should be taught the necessary knowledge of cylinders thoroughly.



General cylinders, straight circle cones and spheres can be dealt with in a straighter way.



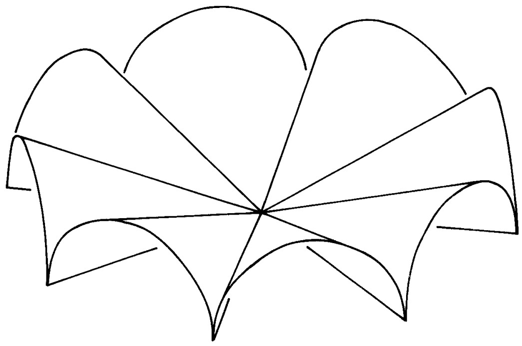
Some other surfaces (e.g. hyperbolic paraboloid and spiral surfaces) can only be touched briefly in a mandatory DG course and should be dealt with in an advanced and optional course.



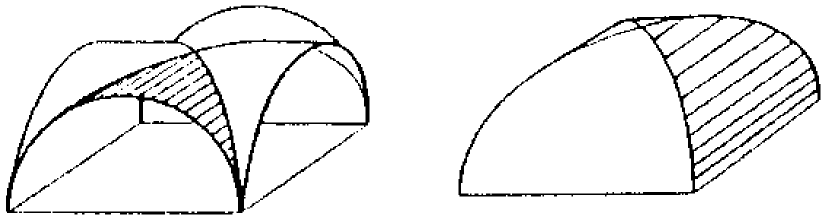
The remaining surfaces are of no significance for architecture.

3.3. Intersections of curved surfaces

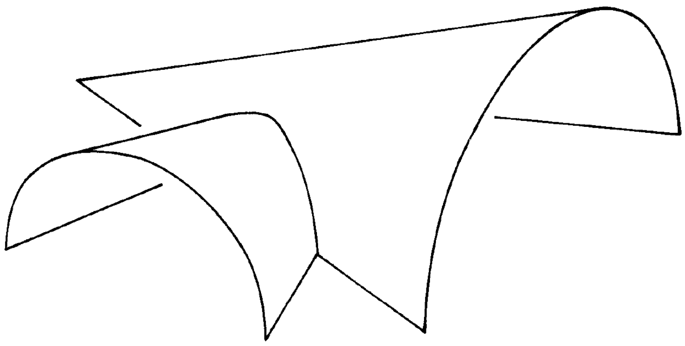
As mentioned above, curved surface are rarely used in architecture; even more infrequently these surface are positioned in a way, that they intersect.



Most of these rare cases are built by intersecting straight cylinders. Combining two cylinders in certain cases the line of intersection lays in a plane. These cases are easier to handle and cheaper to build than curved lines.



Intersections of curved surfaces are very rare in architecture and can therefore be neglected in the curriculum.



However, since even two straight circle cylinders can intersect in a curved line, students of architecture should be taught the basic principles dealing with intersections, e.g. the methods of auxiliary planes to find points of intersection, and the method of tangential planes to find the tangents of the curved line.

4. Conclusion

This selection of topics in a DG course enables students of architecture to solve geometrical tasks which occur in their future jobs. Furthermore they are able to efficiently acquaint themselves with the skills to solve special geometrical problems if they occur in practice. The results of this work can be used for developing a curriculum for teaching architecture at university and they represent the basis to tie the teaching of Descriptive Geometry to other subjects within the study of architecture.

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