Journal for Geometry and Graphics
Volume 4 (2000), No. 2, 197–207.

Chaos and Geometric Order in Architecture and Design

Paweł Rubinowicz

Institute of Architecture and Spatial Planning, Technical University of Szczecin
Zołnierska 50, PL 71-210 Szczecin, Poland
email: pawel@rubinowicz.com.pl

Abstract. Since the beginning of human history, the geometric order and chaos exists in the architectural and urban structures together. In context of future dissertation, this paper presents an opinion, that for a good quality of architectural space the balance between order and chaos is necessary. The architectonic space is created by design and other self-organising processes as well. In the long term it is unforeseeable and unstable. The development of the chaos theory creates a new perspective for better understanding of chaos and complex processes in architecture. Some aspects of this theory can by applied in design.

Key Words: architectural design, urban planning, theory of chaos

MSC 2000: 51N05

1. Chaos and geometric order

It is relatively easy to distinguish between geometric order and chaos in architectural compositions, but the definition of these concepts is difficult. The following definitions can be assumed: The geometric order is represented by ideal mathematical forms (in 2D: e.g. line, circle, quarter, or 3D: e.g. plane, sphere, cube) and ideal relationships (e.g. perpendicularly, parallelism, symmetry, rhythm/regularity). Chaos is the opposite of geometric order; it is represented by forms and relationships that are complex and difficult to describe with the language of classic mathematics.

From the point of view of spatial perception, other definitions can be assumed. In Fig. 1 two graphic compositions are presented, which consists of about 1600 points each. The average density of points is constant in the whole area of both compositions. In the first composition the circular area of regular points is visible on the background of random points. The other composition is inverse: the circular area of random points is visible on the background of regular points. Based on this example, we can indirectly define chaos as an interference of geometric order and geometric order – as an interference of chaos.

ISSN 1433-8157/\$ 2.50

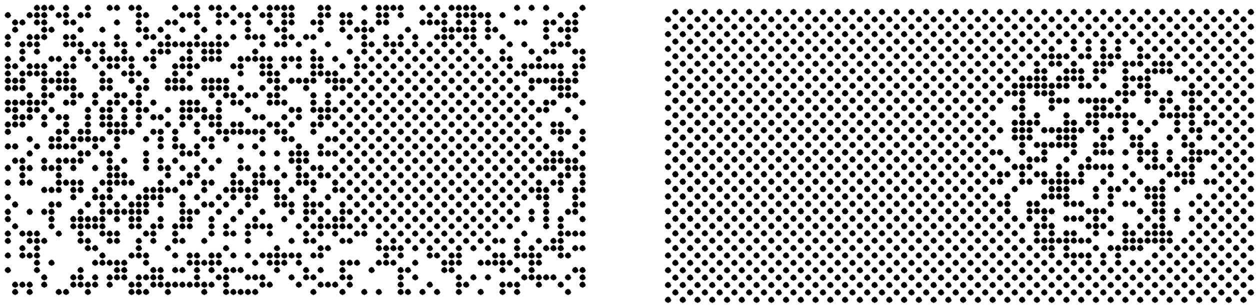


Figure 1: Two graphic compositions: a regular area on a chaotic background (left part) and chaotic area on a regular background (right part)

A new aspect in defining chaos and geometric order is the mathematical theory of chaos that has developed since the 60's. According to this theory the order is a special coincidence of a wider chaotic arrangement and chaos is a deterministic and not scholastic phenomenon [1]. Very complex phenomenon (e.g. atmospheric phenomenon, turbulence, the number of natural population, exchange fluctuations) can be generated through simple formulas. An example of such a formula is one mathematical sequence:

$$x_0, x_1, x_2, \ldots x_n, \text{ where } x_{n+1} = k\, x_n^2 - 1.$$

For $k = 1,35$ and $x_0 = 0,4$ the generated sequence has a periodical recurrence. But for $k = 2,0$ the generated sequence is chaotic [7].

Based on the presented considerations, especially in the context of mathematical chaos theory, we can conclude that the geometric order and chaos are strongly connected together. Is this connection also visible in architecture and does it have an application in design?

2. Ideal and "non-ideal" structures

Since the beginning of human history, geometric order has been applied in architectural structures. This order emphasises their unusualness and confers the high importance, monumentality and even the sacral dimension. The natural world is constructed according to more complex rules. The ideal forms and relationships distinguish architecture against the background of nature. Platon's (427–347 B.C.) philosophy observed the world differently from the real world. His model of the world was idealised. Such philosophy emphasises human supremacy over the natural world and ideal forms over more complex forms. This is typical for many phases of the history of architecture. But, throughout history, parallel to the ideal structures, the "non-ideal" structures have arisen. Departure from the ideal order was often a compromise with real financial possibility and different functional requirements. The creation of the architectural space is not an isolated event, but the continuous process of technological modernisations, destroying, adapting and property changes. In the scale of the city, the accumulation of this process causes spatial diversity and complexity. This is not a consequence of conscious design planning, but free transformation. In this case architecture starts to be chaotic.

Geometric order and chaos exist in architecture together. How do people observe the two opposite aesthetics? The square on the front of the basilica of St. Peter's in Rome is one of the best urban works in baroque (left part of Fig. 2). The architect Gialnorenz Bernini

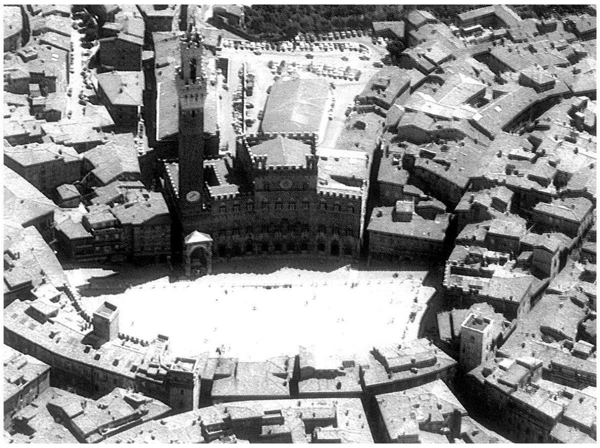


Figure 2: Geometric order and chaos in urban structures: the square on the front of St. Peter's basilica in Rome from 1656–1667 (left part) and Piazza del Campo with surroundings in Siena, Italy, from 1289–1355 (right part)

Figure 3: The urban complexity: panorama of the medieval San Gimignano, Italy (left part) and contemporary New York City, Manhattan (right part)

designed this square on a plan of oval. This square is surrounded by the rhythm of columns. The composition is based on an ideal geometry. This place becomes very important. The city centre of medieval Siena with the central located Piazza del Campo represents another aesthetics (right part of Fig. 2). It took 200 years to form this place. On an irregular street plan many types of tenements have arisen. The final structure of the city centre is very complex and diverse chaotic [13]. Similar spatial phenomenon is visible in the characteristic panorama of another Italian city from this time – San Gimignano. In the Middle Ages this city had its best period. At this time about 75 fortified towers (up to 50 meters high) were created. Only 15 towers have survived up to this date (left part of Fig. 3; [13]). The complexity, typical for medieval Italian cities, is not isolated to space and time. The panorama of New York City can be a good contemporary example (right part of Fig. 3).

The analysis of urban examples presented here, lead to the following conclusions: The geometric order, typical for St. Peter's square, causes in an observer a feeling of classic beauty and harmony. But as well, the complex and chaotic structures of medieval Siena, San Gimignano or contemporary New York creates an individual atmosphere and peculiar beauty. Are these conclusions also correct in a scale of single architectonic structures?

There exist a lot of obvious examples of architectural structures based on the geometric order – e.g. the pyramids in Egypt, Doric temple, and Gothic cathedrals. Aspiration for the ideal geometry is visible in the theoretic design of Isaac Newton Cenotaph. Etienne-

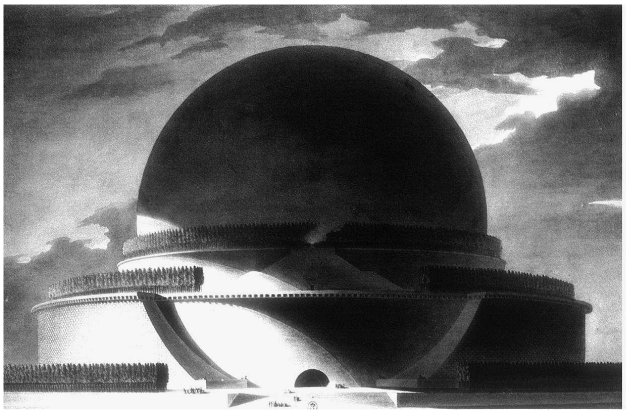


Figure 4: The ideal architectural structures: theoretic design by Etienne-Louis BOULLEE from 1784 (left part) and the Louvre Pyramid in Paris from 1983 (right part).

Louis BOULLEE designed it at the end of 18^{th} century (left part of Fig. 4, [2]). A similar aspiration is visible by the contemporary architect of Chinese descent – Ieoh Ming PEI. His glass pyramid is the modern entrance to the Louvre museum in Paris (right part of Fig. 4, [11]). In the wider context, the geometric order is typical for the architecture of all modernism – the main aesthetics and philosophical trend in the first half of the 20^{th} century [2]. The modernist MIES VAN DER ROHE formulated an artistic manifest “less is more”. In this manifest he favours the simple geometric forms over the more complex forms. In 1957 another architect Robert VENTURI published an opposite idea “less is boring”. He prefers complexity in place of monotonous and “boring” spatial simplicity [4]. This idea was widely accepted in architecture and it was the basis for a new trend – post-modernism. The glorification of complexity and regularity exists also in architecture of present time. Many well-known architects (Zaha HADID, Daniel LIEBESKIND, Frank GEHRY and others) presently take inspiration from chaos. Such inspiration is visible in the UFA Cinema Centre in Dresden designed by COOP HIMMELBLAU — the group of architects: Wolf PRIX and Helmut SWICZINSKY (Fig. 5). Eight cinema theatres are cantilevered in one block. A crystal, glass shell wraps up a wandering public space [3]. This structure is ”non-geometric”. It makes the impression of random composition of different forms. The final effect may shock and delight as well. Paradoxically – the absence of geometric order emphasises the movie’s structure in the context of the city.

Concluding: in the architectural composition the geometric order, as well as chaos are the basic components. The geometric order evokes the feeling of harmony, seriousness and monumentality. Chaos revives the architectural space and gives it an individual dimension. Elimination of chaos from the architectural composition causes “spatial boredom”. Elimination of geometric order causes the illegibility of compositions. Therefore, for a good quality of architectural space, the balance between order and chaos is necessary. The presence of geometry in designs is obvious. But, are there, in an architect’s workshop, the tools for simulating, analysing and understanding chaos?

3. Design or self-organisation

There exists in nature a lot of phenomena that are possible to foresee and describe in the language of mathematics e.g. eclipse of the sun. But, there are also phenomena that have a

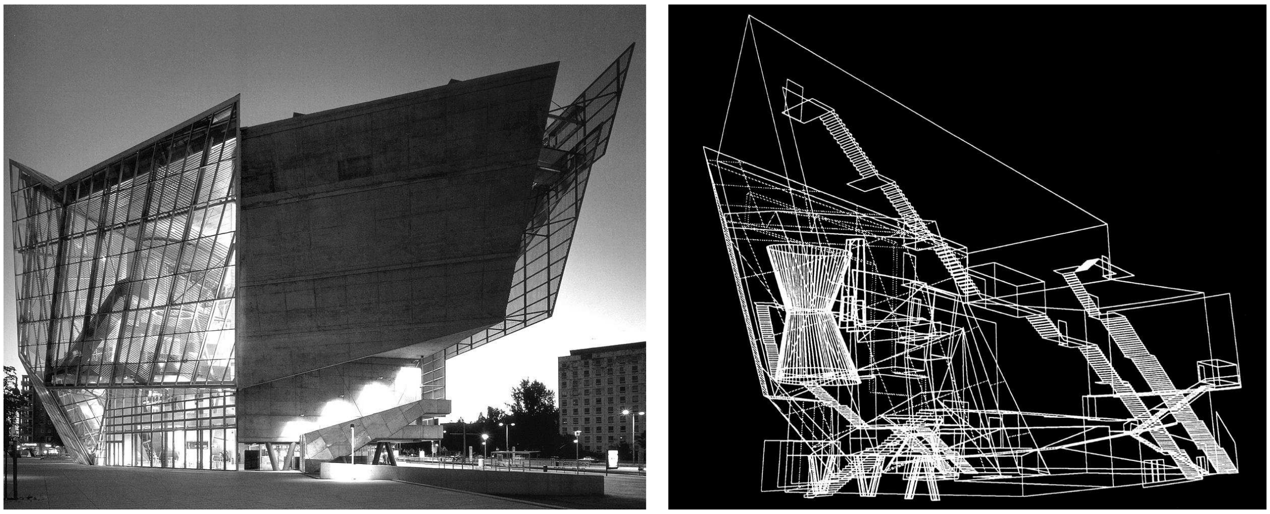


Figure 5: Irregular and "non-geometric" structure of the UFA Cinema Centre in Dresden, Germany, designed by Coop Himmelblau and finally realized in 1998

complex course, and in the longer term are not possible to foresee e.g. atmospheric circulation [12]. Are the processes, which occur in the architectural space foreseeable? Is the architectural space created – by the design process or grown – by other external or internal factors (self-organisation)?

In the urban structure of Barcelona the subtle combination of design and self-organisation is visible today. In the 19th century, after the city's walls were pulled down, a large extension of the city was realised – according to the plans, made by Ildefons Cerda [14]. The new part of the city was organised in grids of identical urban blocks – 110 meter $\times$ 110 meter with angled corners. Using this method, over 600 blocks have been created up to today. Originally, the block was developed only partly. But, after years, the whole areas of most blocks were completely developed, and repeatedly built. Finally, all the blocks were developed in their individual way. The analyse of the developing process for a small region of the city is presented in Fig. 6. On the left part of the figure is an aerial-view photograph made in the 90-s, over a hundred years after the initialisation of the urban plan. The geometric synthesis of the aerial-view shows an original urban plan: a few blocks typical for Creda's plan (middle part of figure). But, the special computer analysis of the aerial-view photograph shows another property of the urban structure – their complexity. This analysis is based on the algorithm of separating the bright and dark regions of photograph. The generated picture consists of borderlines. For Barcelona's aerial-view these lines are not reconstructions of the original urban plan, but the chaotic "local whirls of the structure" (right part of the figure). Concluding, the urban structure of this part of the city has simultaneous characteristics of geometric order (the original plan) and chaos ("local whirls of the structure").

In the architectonic scale, similar phenomena are visible. Fig. 7 shows the elevations of a five-floor tenement house in the historical part of Barcelona. The photograph of this house (left part of the figure) after geometric synthesis (middle part of the figure) shows the original order of the elevation which is compatible to the architectonic design. But, the computer analysis of the elevation's photograph (analogical like in the previous example) shows the complexity of the composition (right part of the figure). The original order is not visible. The process of exploitation has changed the original, simple, monotonous elevation (rhythms of two types of windows, as shown in the middle part of the figure). Modifications of the same

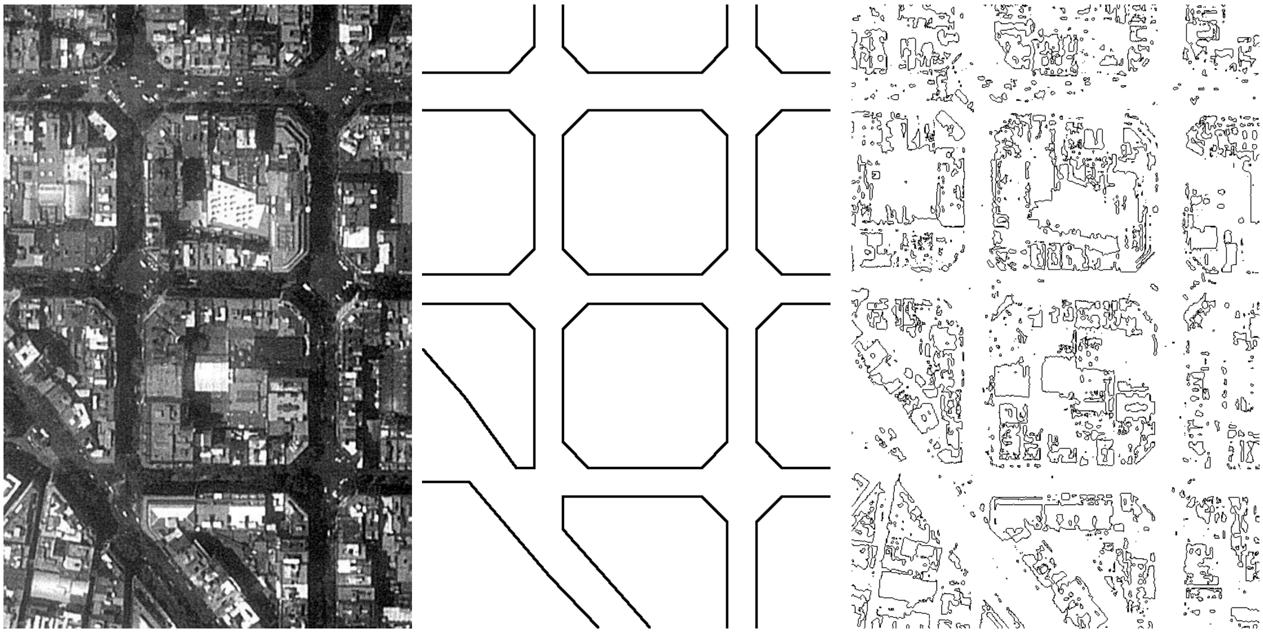


Figure 6: The analysis of the urban structure of a region of Barcelona, Spain: the aerial-view photograph from the 90-s (left part); the geometric synthesis, which shows the original urban plan (middle part); the special computer analysis (described in text) of the aerial-view photograph, which shows the complexity and chaos of the actual urban structure (right part)

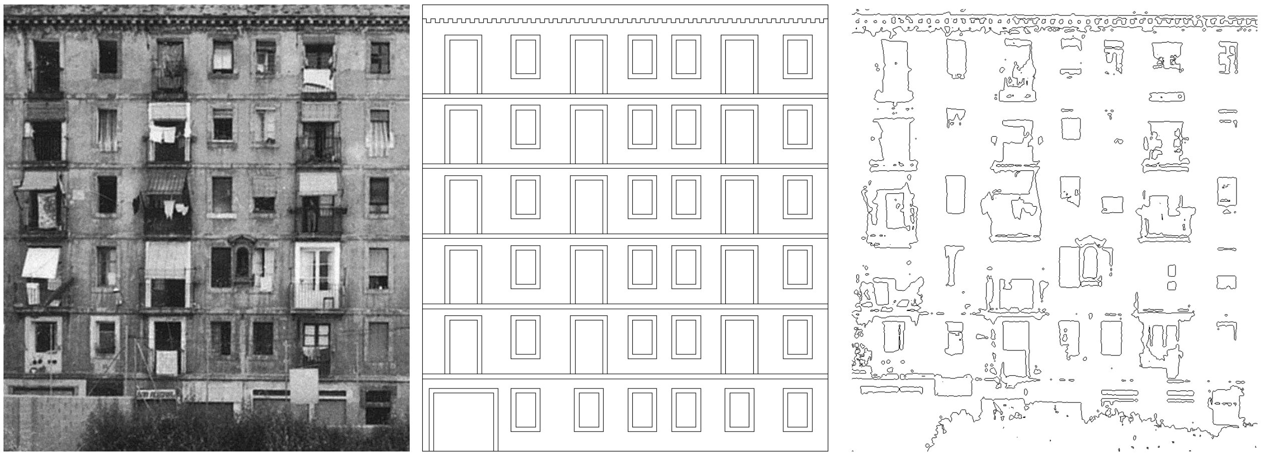


Figure 7: A tenement house in the historical part of Barcelona, Spain: the elevation's photograph from the 90-s (left part of figure); the geometric synthesis shows the original architectural design (middle part); the special computer analysis (described in text) of the photographs, which shows the real complexity and chaos of the actual form of elevations (right part)

windows made by occupants, different forms of installed sun protectors and others cause the increase of complexity in the elevation's composition. Finally, each window has an individual form. In presented urban and architectonic examples, the combination of geometric order and chaos is visible. The geometric order is a result of controlled planning or design process. And chaos is created by the natural transformations — self-organisation.

Finally, we can formulate two conclusions: Firstly, the influence of the self-organisation process, means that the architectonic space in the long term is unforeseeable. Secondly, because of the continuity of this process, the architectonic space is unstable – still changing. How should the architect's workshop be organised then?

4. The cellular automata in architecture

In typical architectural designs the unforeseeably and instability of architectonic space is not respected. Firstly, the architectural design describes space with three-dimensions and doesn't respect the fourth dimension — time. Secondly, tools for the creation of complex and chaotic compositions don't exist in the workshop of most architects. These limitations concern the classic design and the Computer Aided Design (CAD) as well. The computer technique for architects was dynamically developed in the 90-s. This technique increased the effectiveness of the architect's work, but fundamentally – hasn't changed design methods. The Parametric Modelling presented by the author in other papers ([6], [8]) is an example of alternative, more complex methods of computer supported architecture design. Among other things, this method adds some elements of chaotic compositions into the computer model and design [9]. But the simulation of complex processes, which occurs in the architectonic space using this method is not realistic. Such simulation may be possible with the computer application of the theory of cellular automata [15].

The mathematical theory of cellular automata was formulated in the 70-s. The working rule of the two-dimensional cellular automaton [7] is shown in Fig. 8. The automaton consists of cells located in a grid. Each cell has a specific condition – it may be alive or dead. In the figure, the living cells are presented in black and the dead cells are presented in white. The automaton works in successive steps. In individual steps, some cells are alive and some have to die and special rules decide about this. There are a lot of different variations for such rules. In the case presented in the figure, there is the following rule: the cells are living, when in their surroundings (the location place of the selected cell and its 8-th neighboured cells) a minimum of 5 cells are alive, otherwise they are dead because of "loneliness". The original arrangement of cells (left part of figure) is changed in the first step of automaton (right part of figure). Some cells were alive others died and the rest save their condition. For example the cell E2 died, because in it's surroundings only 3 cells were alive (D3, F1, F3). But the cell E3 is alive, because in it's surrounding, six cells were alive.

This simple play causes interesting results. There is, in Fig. 9, results of the cellular automaton consisting of 10,000 cells (a grid of 100 cells $\times$ 100 cells). Originally the cells were in a chaotic arrangement except for 9 empty (dead) square areas. The following rule was assumed: cells are living, when in their surroundings, a minimum of 3 and not more than 8 cells are alive, otherwise they are dead. In successive steps of the automaton, these square areas change their location and shapes. After 45 steps the original order was completely invisible. The geometric order was transformed into a more chaotic composition, similar to the organic structure. In Fig. 10, another automaton is presented — based on the same rules as in Fig. 8. Originally, the cells were arranged chaotically. In the steps, the process of self-organisation is visible. After 4 steps the arrangement of cells is consequently stabilised (middle part of figure).

Between cellular automata and architectonic space some analogies are visible. An effect of automaton work, which is presented in the middle part of Fig. 10, looks similar to the structure of a big city – e.g. London from 1939 (right part of Fig. 10). In a sense, the city is a variation of a cellular automaton. Individual buildings may be the cells of such automaton.

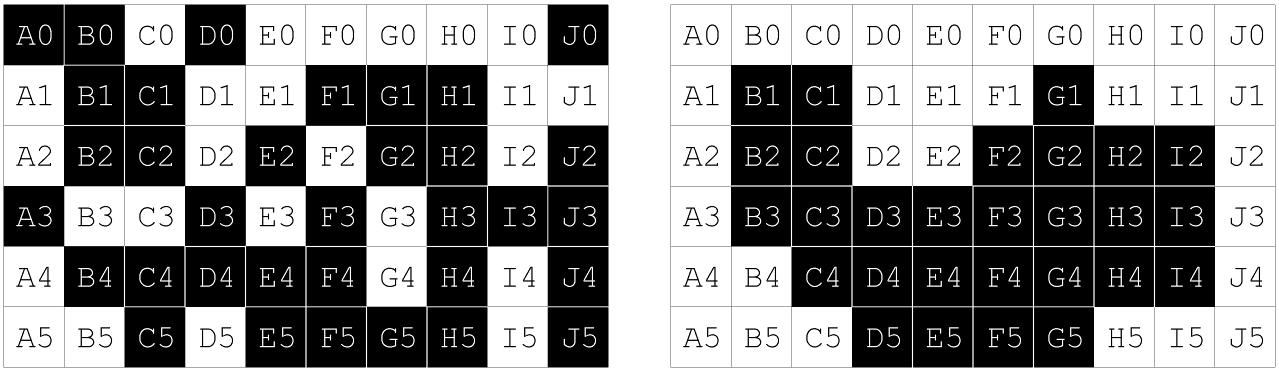
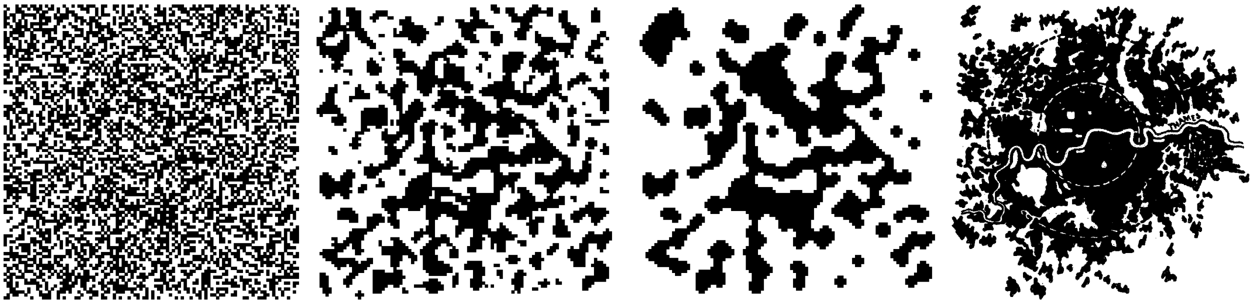


Figure 8: The working principle of the cellular automaton: the original set of cells (left part) and the set of cells after the first step of automaton's work (right part). This automaton is determined by the rule: cells are living (black), when in their surroundings, a minimum of 5 cells are alive, otherwise they are dead (white)

ORIGINAL SET AFTER 15 STEPS AFTER 30 STEPS AFTER 45 STEPS

Figure 9: The steps of cellular automaton are determined by the rule: cells are living when in their surrounding a minimum of 3 and not more than 8 cells are alive, otherwise they are dead. This automaton contains 10,000 cells (grid 100 cells x 100 cells). The order of 9 empty (dead) squares is successively devastated during the process of automaton work



ORIGINAL SET AFTER 2 STEPS AFTER 4 STEPS LONDON IN 1939

Figure 10: The steps of cellular automaton which contains 10,000 cells (grid 100 cells x 100 cells) and is determined by the rule: cells are living, when in their surrounding a minimum of 5 cells are alive, otherwise they are dead (left and middle part); and the urban structure of London in 1939 (right part)

Figure 11: Cellular automata and their analogies in architectonic space: a tenement house from the 19^{th} century and future buildings – similar to a cell with living neighbours (left part); and a tenement house from the 19^{th} century without architectonic context – similary to a dying cell without living neighbours (right part). Examples from Szczecin, Poland

The interaction of individual cells is a base of automaton work. In the architecture, the interaction of buildings is easy to observe. The existing urban and architectural context has a large impact on the shape of new architectural structures. The single building may be similar to the form generated by an automaton. A good example for this may be a residential building designed by an architect of Israeli descent – Moshe Safdie for an Expo world-wide exhibition in Montreal (left part of Fig. 12). The main aim made by the architect was the creation of privacy for occupants and conditions like in detached houses. Therefore every flat has a terrace on the roof, although the building has few floors. Such principles may be translated into a language of cellular automata: cell (a dwelling module) dies when over it another cell exists (in that way, the creation of, a terrace on the roof is not possible). The shape of the building looks like a structure generated by a three-dimensional cellular automaton (right part of Fig. 12).

Concluding: Between cellular automata and the architectonic space some analogies are visible. There exists a potential possibility of using this mathematical theory in design. The limitations of a classic architects workshop concerns the absence of tools for fourth-dimensional modelling (including time) and for the creation of complex and chaotic compositions. In the case of cellular automata, these limitations don't exist any more. Successive steps of automaton may simulate architectonic processes. The structures generated by cellular automata can be absolutely complex and chaotic.

5. Conclusions

Geometric order and chaos are the basic components of the composition of architectural and urban structures. Coexistence of these components in architectonic space is very natural. In general, geometric order is a result of design and planning, and chaos – is created by self-organising processes. Finally the architectonic space in the long term is unforeseeable and unstable. The development of the chaos theory creates a new perspective for better understanding complex processes in architecture. Some aspects of this theory can be applied

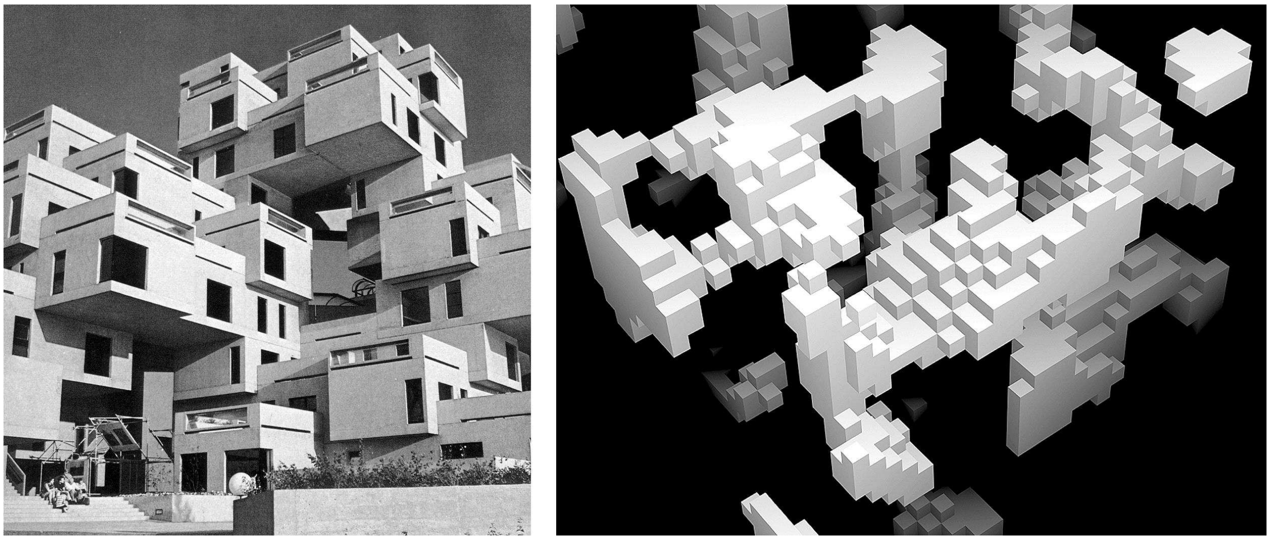


Figure 12: Cellular automata and their analogies in architectonic space: "Habitat 67", the residential building designed by Moshe SAFDIE on the Expo world-wide exhibition in Montreal in 1967 (left part); and the cellular automaton in 3D made using the computer program MODEL [10] (right part)

in design, especially with use of the special computer techniques. The mathematical model of cellular automata may be a potential way for realising this. Examples presented in this paper show analogy between cellular automata and real architectonic and urban structures.

References

[1] P. COVNEY, R. HIGHFIELD: *Frontiers of Complexity – The Search for Order In a Chaotic World.* .

[2] P. GOESSEL, G. LEUTHAEUSER: *Architektur des 20. Jahrhunderts.* Benedikt Taschen Verlag, Cologne, Germany, 1990.

[3] C. JENCKS: *Ecstatic Architecture.* Academy Editions, London 1999.

[4] C. JENCKS, K. KROPF: *Theories and Manifestoes of Contemporary Architecture.* Academy Editions, London 1999.

[5] B. MANDELBROT: *The Fractal Geometry of Nature.* W.H. Freeman and Company, New York 1983.

[6] Z. PASZKOWSKI, P. RUBINOWICZ: *Toward the Parametric Modeling in Architectural Design.* Proc. 7th ICECGDG Cracow 1996, vol. 1, pp. 33–36.

[7] H. PEITGEN, H. JUERGENS, D. SAUPE: *Chaos and Fractals.* Springer Verlag, New York 1992.

[8] P. RUBINOWICZ: *Computer Parametric Modeling As a New Design Strategy.* Proc. 4th Conference on Computer in Architectural Design. Białystok, Poland, 1996, pp. 205–214

[9] P. RUBINOWICZ: *Parametric Modeling – Random Factors in Architecture.* Proc. 8th ICECGDDG, Austin 1998, vol. 1, pp. 81–85.

[10] P. RUBINOWICZ: *MODEL – Computer application for parametric modeling A and B.* instructions available on web site: `www.rubinowicz.com.pl`.

[11] J. Steele: *Architecture Today*. Phaidon Press Limited, London 1997.

[12] I. Steward: *Does God Play Dice? The New Mathematics of Chaos*. Basil Blackwell, Oxford 1990.

[13] R. Toman: *Die Kunst der italienischen Renaissance*. Koenemann Verlag, Cologne, Germany, 1994.

[14] A. Whittick: *Encyclopedia of Urban Planning*. McGraw-Hill, USA, 1974, pp. 931–932.

[15] S. Wolfram: *Theory and Application of Cellular Automata*. World Scientific, Singapore 1986.

Received August 1, 2000; final form December 13, 2000