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Large automorphism groups of 16-dimensional planes are Lie groups, II

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Abstract. Let $\mathcal{P}$ be a compact, 16-dimensional projective plane. If the group Σ of all continuous collineations of $\mathcal{P}$ is taken with the compact-open topology, then Σ is a locally compact group with a countable basis. The following theorem is proved: If the topological dimension $\dim \Sigma$ is at least 29, then Σ is a Lie group.

The automorphism group Σ of a projective plane $\mathcal{P}$ with compact, 16-dimensional point space P is a locally compact transformation group of P, and Σ has a countable basis [9, 44.3]. It is an open problem whether or not Σ is always a Lie group. If the topological dimension $\dim \Sigma$ is sufficiently large and if Σ is a Lie group, then the structure theory for Lie groups can be exploited to determine all possible planes. This has successfully been done in several cases, cp. [9, Chap. 8] and [8]. Therefore, the following criterion is useful:

Theorem. *If* $\dim \Sigma \geq 29$, *then* Σ *is a Lie group.*

In order to conclude that the connected component Σ^1 of Σ is a Lie group, a weaker hypothesis suffices [7]:

If $\dim \Sigma \geq 27$, *then* Σ^1 *is a Lie group.*

A theorem of Bödi [1], Proposition G in [7], and [9, 53.2] imply

($\square$) *If* Σ *is not a Lie group, and if the subgroup* Λ *of* Σ *fixes a quadrangle, then* $\dim \Lambda \leq 11$. *Moreover,* $\dim x^\Sigma = \dim \Sigma/\Sigma_x < 16$ *for each point* x.

The next result has been stated in [7, (a)] for connected subgroups of Σ, but the proof does not use connectedness:

Proposition. *If* Δ *leaves some proper closed subplane invariant, then* $\dim \Delta \leq 25$ *or* Δ *is a Lie group.*

All large semi-simple groups on a 16-dimensional plane $\mathcal{P}$ are known [5], [6]:

If $\dim \Delta > 28$ *and if* Δ^1 *is semi-simple, then either* $\mathcal{P}$ *is a Hughes plane (including the classical Moufang plane), or* $\Delta^1 \cong \mathrm{Spin}_9(\mathbb{R}, r)$ *with* $r \leq 1$.

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The full group of a Hughes plane is a Lie group [9, 86.12 and 53.2] or [9, 86.35]. The groups $\mathrm{Spin}_9(\mathbb{R}, r)$ contain the 28-dimensional compact group $\mathrm{Spin}_8\mathbb{R}$ (which fixes a triangle), and Σ is a Lie group by ($\Box$):

Corollary. *If* $\dim \Delta > 28$ *and if* Δ^1 *is semi-simple, then* Σ *is a Lie group.*

The *proof* of the theorem uses the approximation theorem [9, 93.8] for locally compact groups: there is an open subgroup Δ of Σ and an arbitrarily small compact, 0-dimensional normal subgroup $\Theta \lhd \Delta$ such that Δ/Θ is a Lie group. According to [9, 93.18], the connected component $\Delta^1 = \Sigma^1$ and the group Θ centralize each other, and $\Delta^1_{\ a}$ acts trivially on the orbit a^Θ. A group Ξ is called *straight* if each point orbit x^Ξ is contained in a line, and a well-known theorem of Baer implies that either Ξ is planar (i.e. the fixed elements of Ξ form an 8-dimensional subplane $\mathcal{F}_\Xi$), or Ξ is contained in a group $\Sigma_{[z]}$ of collineations with common center z, see [7, Th.B].

Assume now that $\dim \Sigma \geq 29$ and that Σ is not a Lie group, and choose Δ and Θ as above. Then Θ is not a Lie group, and the Proposition shows that Θ cannot be planar. By Baer's Theorem, there remain two possibilities: either Θ is not straight and some orbit a^Θ contains a triangle, or Θ consists of axial collineations with a common center. Note that Δ^1 is not semi-simple by the above Corollary.

(i) If a^Θ consists of more than 3 non-collinear points, then a^Θ generates a subplane, and ($\Box$) implies $\dim \Delta^1_{\ a} \leq 11$, $\dim \Delta \leq 26$. If a^Θ is just a triangle, however, and if the same is true for all orbits b^Θ with b near a, then $a^\Theta \cup b^\Theta = C$ generates a subplane, Θ induces on C a finite group Θ/Λ, the kernel Λ is not a Lie group, $\Lambda \neq \mathbb{1}$, and $\mathcal{F}_\Lambda$ would be a $\Delta^1\Lambda$-invariant proper closed subplane. This contradicts the Proposition. Hence Θ must be straight.

(ii) Because all arguments can be dualized, the elements of Θ also have a common axis W, and Θ is contained either in a group $\Sigma_{[a,W]}$ of homologies ($a \notin W$), or in a group $\Sigma_{[v,W]}$ of elations with center $v \in W$. The case that Θ consists of homologies and that Δ is connected has been treated in [7]. A contradiction is obtained by studying the possible actions of the Lie group Δ/Θ on the axis W. The reasoning remains valid, if instead of the center Z of Δ the centralizer of Δ^1 in Δ is used throughout. In the remaining case $\Theta \leq \Sigma_{[v,W]}$, the situation is different; it is the only one, in which the stronger hypothesis $\dim \Sigma \geq 29$ is needed. If Δ is connected, a theorem of Löwen [3] implies that Δ is a Lie group regardless of its dimension, cp. [4, (2.7)]. There seems to be no way, however, to extend Löwen's proof to non-connected groups. In the general case, a proof can be based on a careful analysis of a point stabilizer.

(1) Suppose again that $\Theta \leq \Sigma_{[v,W]}$ with $v \in W$. Choose any point $a \notin W$, and consider the connected component Γ of Δ_a. Because $\Gamma \cap \Theta = \mathbb{1}$, there is an embedding of Γ into the Lie group Δ/Θ. Hence Γ is itself a Lie group, and Γ has a minimal commutative, connected normal subgroup Ξ, or Γ is semi-simple. As has been noted before, Γ fixes the (infinite) orbit a^Θ pointwise. The dimension formula [9, 96.10] and ($\Box$) imply $14 \leq g = \dim \Gamma \leq 26$. Moreover, Γ acts effectively on W, and there is at most one point $u \in W \setminus \{v\}$ such that $u^\Gamma = u$, compare [7, Prop. G]

(2) Let $\mathcal{E} = \langle a^\Theta, z^\Gamma \rangle$ denote the smallest closed subplane containing the orbits a^Θ and z^Γ. If $z \in W \setminus \{v\}$ and $z \neq u$, then z^Γ is a non-trivial connected set, and $\mathcal{E}$ has dimension $d \in \{2,4,8,16\}$, see [9, 54.11]. Remember that $z^\Theta = z$ and that Γ and Θ commute. Consequently, $\mathcal{E}^\Theta = \mathcal{E}$. As a group of elations, Θ acts effectively on $\mathcal{E}$. Since each automorphism group of a plane of dimension $d \leq 4$ is a Lie group [9, 32.21 and 71.2], it follows that $\mathcal{E}$ is a Baer subplane or the plane $\mathcal{P}$ itself, for short, $\mathcal{E} \leq\!\bullet\, \mathcal{P}$.

(3) Similarly, if Π is a one-parameter subgroup of Γ and if $z^\Pi \neq z$, then $\langle a^\Theta, z^\Pi \rangle \leq\!\bullet\, \mathcal{P}$. Let Ψ denote the connected component of the centralizer of Π in Γ. The Lie group Ψ_z acts trivially on $\langle a^\Theta, z^\Pi \rangle$, and $\Psi_z^{\,1}$ is isomorphic to a subgroup of $\mathrm{SU}_2\mathbb{C}$ by [9, 83.22]. In particular, $\dim \Psi_z \leq 3$, $\dim \Psi \leq 11$. Note that $\mathrm{Cs}\Pi = \mathrm{Cs}\varrho = \Gamma_\varrho$ for any $\varrho \in \Pi \setminus \{\mathbb{1}\}$. The dimension formula [9, 96.10] gives $g - 8 \leq \dim \Gamma_z \leq \dim \varrho^{\Gamma_z} + 3$.

(4) Because a compact, commutative normal subgroup of Γ is contained in the center, it follows from (1) and (3) that either Γ is semi-simple, or Γ has a minimal normal subgroup $\Xi \cong \mathbb{R}^t$ with $t \geq g - 11$, compare [9, 94.26]. The semi-simple case will be discussed later.

(5) Assume that $\mathbb{R}^t \cong \Xi \lhd \Gamma$, and let $z^\Gamma \neq z \in W \setminus \{v\}$. If $z^\Xi = z$, then Ξ induces the identity on $\mathcal{E} \leq\!\bullet\, \mathcal{P}$, and Ξ would be compact by [9, 83.6]. Consequently, $z^\Xi \neq z$, and $\langle a^\Theta, z^\Xi \rangle \leq\!\bullet\, \mathcal{P}$ by the arguments of (2). Since Ξ_z fixes each point of $\langle a^\Theta, z^\Xi \rangle$, it follows that Ξ_z is compact, and then $\Xi_z = \mathbb{1}$. Therefore, Ξ acts freely on $W \setminus \{u, v\}$ or on $W \setminus \{v\}$, and $t \leq 8$, $g \leq 19$. In particular, $\dim a^\Delta \geq 10$, and the line av is not fixed by Δ.

(6) If $g = 19$, and if $\mathbb{1} \neq \varrho \in \Xi$, then (3) implies $\dim \Gamma_z = 11$, $\dim \varrho^{\Gamma_z} = 8$, and ϱ^{Γ_z} is open in Ξ by [9, 92.14 or 96.11(a)]. Hence Γ_z is transitive on $\Xi \setminus \{\mathbb{1}\}$, and a maximal compact, connected subgroup is transitive on the 7-sphere of the rays in $\Xi \cong \mathbb{R}^8$, see [9, 96.19]. With [9, 96.20–22] it follows that $\Gamma_z{}' \cong \mathrm{U}_2\mathbb{H}$. The central involution $\sigma \in \Gamma_z$ inverts each element of Ξ, and z is an isolated fixed point of σ on W. Therefore, σ is a reflection with center z and axis av. This contradicts the following Lemma on involutions, which will be needed repeatedly:

($*$) *Let α, β, and $\alpha\beta$ be pairwise commuting involutions in Γ. If Θ is not a Lie group, then exactly one of the 3 involutions is a reflection, and the torus rank $\mathrm{rk}\,\Gamma \leq 2$. Each reflection in Γ has axis av and some center $z \in W$. Moreover, Γ has no subgroup $\Phi \cong \mathrm{SO}_3\mathbb{R}$, and $\dim z^\Gamma \leq 6$.*

Proof. Any involution is either a reflection, or it is planar [9, 55.29]. If all 3 involutions α, β, and $\alpha\beta$ are planar, then the common fixed elements of α and β form a 4-dimensional subplane $\mathcal{F}$, see [9, 55.39(a)]. By definition, Γ is connected, Γ and Θ centralize each other, and $\mathcal{F}^\Theta = \mathcal{F}$. Because Θ consists of elations, Θ acts effectively on $\mathcal{F}$, and Θ would be a Lie group by [9, 71.2]. Hence we may assume that α is a reflection. Because Γ fixes the orbit a^Θ pointwise, each reflection in Γ has axis av, its center z lies on the fixed line W. Since the center of one of two commuting reflections is on the axis of the other [9, 55.35], the involutions β and $\alpha\beta$ are planar. If $\mathrm{SO}_3\mathbb{R} \cong \Phi \leq \Gamma$, and if α and β are chosen in Φ, then α and β are conjugate in Φ and therefore would be of the same kind, a contradiction. If $\dim z^\Gamma = k > 0$, then $\alpha^\Gamma\alpha$ is a k-dimensional set in the connected component E of the elation group $\Delta_{[v,av]}$, compare [9,61.19(b)]. The last statement in (5) implies that E is commutative, in fact, $\mathsf{E} \cong \mathbb{R}^k$. The connected group Γ induces linear maps of positive determinant on E. In particular, $\det \alpha = 1$. On the other hand, the reflection α inverts each element in E, and $\alpha|_{\mathsf{E}} = -\mathbb{1}$. Consequently, k is even. If $k = 8$, then $\Delta_{[v,v]}$ is transitive, and Θ would be contained in the Lie group $\Delta_{[v,W]} \cong \mathbb{R}^8$. This completes the proof of Lemma $(*)$.

(7) Now let $14 \le g \le 18$, and put $\Omega = \Gamma_z^{\ 1}$. Then $\dim \Omega \ge 6$, and for each $\varrho \in \Xi \setminus \{\mathbb{1}\}$ the last assertion in (3) gives $\dim \varrho^\Omega \ge 3$. Hence any minimal Ω-invariant subspace $\Upsilon \le \Xi$ has dimension $s \ge 3$. If $s = 3$, then Ω is transitive on $\Upsilon \setminus \{\mathbb{1}\}$, and $\Omega_\varrho \cong \mathrm{SU}_2\mathbb{C}$ by (3). Because Ω_ϱ fixes a subspace of Υ, the representation of Ω_ϱ on Υ is trivial. Consequently, Ω/Ω_ϱ would act sharply transitive on $\Upsilon \setminus \{\mathbb{1}\} \cong \mathbb{R}^3 \setminus \{0\}$, but such a group does not exist.

(8) Similarly, the case $s = 4$ leads to a contradiction: because $\mathrm{SU}_2\mathbb{C}$ has no 2-dimensional subgroup, one has again $\Omega_\varrho \cong \mathrm{SU}_2\mathbb{C}$ for each $\varrho \in \Upsilon \setminus \{\mathbb{1}\}$. Being compact, Ω_ϱ acts on Υ as an orthogonal group, in fact as a subgroup of $\mathrm{SO}_3\mathbb{R}$. Hence the central involution ω of Ω_ϱ is planar, the Baer subplane of its fixed elements is $\mathcal{F}_\omega = \langle a^\Theta, z^\Upsilon \rangle$. Either Ω_ϱ acts trivially on $\mathcal{F}_\omega$, or Ω_ϱ induces on $\mathcal{F}_\omega$ a group $\Phi = \Omega_\varrho/\langle\omega\rangle \cong \mathrm{SO}_3\mathbb{R}$. In the latter case, Φ fixes a quadrangle in $\mathcal{F}_\omega$ by its very definition. It follows that the fixed elements of Φ in $\mathcal{F}_\omega$ form a 2-dimensional subplane (use [9, 96.34]). Acting faithfully on this subplane, Θ would be a Lie group by [9, 32.21]. Therefore, Ω_ϱ is the kernel of the irreducible action of Ω on Υ, and $(\Omega/\Omega_\varrho)'$ is a non-trivial semi-simple linear group. Consequently, Ω' contains a 2-torus. Lemma $(*)$ implies $\dim z^\Gamma \le 6$, but then $14 \le g \le \dim z^\Gamma + \dim \Omega \le 6 + s + 3 = 13$. This contradiction shows that $s > 4$.

(9) By the last assertion, $\Omega = \Gamma_z^{\ 1}$ acts faithfully and irreducibly on $\Upsilon \cong \mathbb{R}^s$, and the semi-simple commutator subgroup satisfies $\dim \Omega' > 3$, hence $\dim \Omega' \ge 6$, see [9, 95.6]. If $s \in \{5, 7\}$, then $\Gamma_z{}'$ is almost simple and irreducible on Υ by Clifford's Lemma [9, 95.5]. Inspection of a list of irreducible representations [9, 95.10] shows that either $s = 5$ and $\dim \Gamma_z{}' \ge 10$, or $s = 7$ and $\dim \Gamma_z{}' \ge 14$, but $\dim \Gamma_z \le s + 3$. Hence $s \in \{6, 8\}$.

(10) Suppose that $s = 6 = \dim \Omega'$. Lemma $(*)$ implies $\mathrm{rk}\, \Omega = 1$, or $\mathrm{rk}\, \Omega = 2$ and $\dim z^\Gamma \le 6$. In the second case, $\dim \Omega = 8$, and the center of Ω is isomorphic to $\mathbb{C}^\times$. Consequently, $\mathrm{rk}\, \Omega' = 1$ and Ω' is almost simple and locally isomorphic to $\mathrm{SL}_2\mathbb{C}$. From [9, 95.6(b) and 95.10] it follows that Ω' acts irreducibly on Υ and $\Omega' \cong \mathrm{SO}_3\mathbb{C} > \mathrm{SO}_3\mathbb{R}$. This contradicts $(*)$.

(11) If $s = 6$ and $\dim \Omega' = 8$, then Ω' is isomorphic to a group $\mathrm{SU}_3(\mathbb{C}, r)$ or to $\mathrm{SL}_3\mathbb{R}$. None of these groups contains a central involution. Consequently, each involution in Ω' has a positive eigenspace in Υ and hence is planar. Moreover, there are 3 pairwise commuting involutions in Ω'. This is excluded by $(*)$.

(12) The case $s = 6$ and $\dim \Omega' = 9$ leads to a contradiction as follows: a 9-dimensional semi-simple group is not almost simple and has at least one 3-dimensional factor. On the other hand, the arguments of (6) show that Ω' acts

transitively on $\Upsilon \setminus \{\mathbb{1}\}$ and hence on the 5-sphere consisting of the rays in $\Upsilon \cong \mathbb{R}^6$. Therefore, Ω' contains an 8-dimensional almost simple factor $\mathrm{SU}_3\mathbb{C}$.

(13) From (7–12), it follows that $\Upsilon = \Xi \cong \mathbb{R}^8$. If $z \in W \setminus \{v\}$, $z \neq u$, then $z^\Xi \approx \mathbb{R}^8$ by step (5), and z^Ξ is open in W by [9, 53.1(a)]. Hence W is a manifold, and $W \approx \mathbb{S}_8$ according to [9, 52.3]. Since $W \setminus \{u,v\} \not\approx \mathbb{R}^8$, the group Ξ is sharply transitive on $W \setminus \{v\}$. Remember that Γ_z acts effectively on Ξ.

(14) Combination of (13) and $(*)$ shows that the group $\Omega = \Gamma_z$ does not contain any reflection. The semi-simple commutator subgroup Ω' has dimension at least 6. Because of $(*)$, its torus rank is 1, and Ω' is even almost simple. The only groups satisfying these conditions and having a faithful linear representation are $\mathrm{SL}_2\mathbb{C}$, $\mathrm{SO}_3\mathbb{C}$, and $\mathrm{SL}_3\mathbb{R}$, see [9, 95.10]. In the first case, the central involution would be a reflection. The latter two groups have a subgroup $\mathrm{SO}_3\mathbb{R}$ and hence are excluded by $(*)$. Together, steps (4–14) imply that Γ is semi-simple.

(15) If Γ has two or more factors, choose an almost simple factor B of maximal dimension and let A denote the product of the other factors, so that A and B commute elementwise. Consider $z \in W$ with $z^\Gamma \neq z$ and $\langle a^\Theta, z^\Gamma\rangle = \mathcal{E} \leq\!\bullet\, \mathcal{P}$ as in (2). Assume first that $z^\mathsf{A} = z$. Then A acts trivially on $\mathcal{E}$ and $\mathcal{E}$ is a Baer subplane, moreover, $\mathsf{A} \cong \mathrm{SU}_2\mathbb{C}$ by [9, 83.22]. Therefore, $\dim \mathsf{B} \geq 11$. Since B is almost simple, $\dim \mathsf{B} \geq 14$ and B acts almost effectively (i.e. with discrete kernel) on $\mathcal{E}$. But B fixes a^Θ, and the stiffness theorem [9, 83.17] gives $\dim \mathsf{B} \leq 7+4$, a contradiction. Similarly, $z^\mathsf{B} = z$ implies $\dim \mathsf{B} = 3$, and A is a product of 3-dimensional groups by the maximality of B. Hence $\dim \mathsf{A} \geq 12$. The kernel K of the action of Γ on $\mathcal{E}$ contains B, and $\dim \mathsf{K} = 3$ by [9, 83.22]. Consequently, A acts almost effectively on $\mathcal{E}$. Again, the stiffness theorem shows $\dim \mathsf{A} \leq 11$. Thus, $\langle a^\Theta, z^\mathsf{A}\rangle = \mathcal{A} \leq\!\bullet\, \mathcal{P}$ and $\langle a^\Theta, z^\mathsf{B}\rangle = \mathcal{B} \leq\!\bullet\, \mathcal{P}$.

(16) As in step (3), the last part of (15) implies $\dim \mathsf{A}_z \leq 3$ and $\dim \mathsf{A} \leq 11$. If $\dim \mathsf{B} \leq 6$, then $\dim \mathsf{A} \equiv 0 \bmod 3$ and $\dim \mathsf{A} = 9$. Therefore, $\dim z^\mathsf{A} \geq 6$ and $\mathcal{A} = \mathcal{P}$. Consequently, $\mathsf{B}_z = \mathbb{1}$, $\dim \mathsf{B} = 6$, and $\mathcal{B} = \mathcal{P}$. Now $\mathsf{A}_z = \mathbb{1}$ and $\dim \mathsf{A} \leq 8$, a contradiction. Since also $\dim \mathsf{B} \leq 11$ and B is almost simple, it follows that $\dim \mathsf{B} \in \{8, 10\}$ and $\dim z^\mathsf{B} > 4$. Hence $\mathcal{B} = \mathcal{P}$ and again $\mathsf{A}_z = \mathbb{1}$. Because $\dim \Gamma \geq 14$, the semi-simple group A has dimension at least 6, and $\mathcal{A} = \mathcal{P}$, so that $\mathsf{B}_z = \mathbb{1}$ and $\dim \mathsf{B} = 8$.

(17) By [9, 53.1(a)], the orbit z^B is open in W whenever $z^\Gamma \neq z$, and this is true for each point $z \in W \setminus \{v\}$ with at most one exception u, see step (1). Hence B is sharply transitive on $W \setminus \{v\} \approx \mathbb{R}^8$ or on $W \setminus \{u,v\} \approx e^\mathbb{R} \times \mathbb{S}_7$. In both cases, the homotopy group $\pi_3\mathsf{B}$ vanishes, but every almost simple Lie group X satisfies $\pi_3\mathsf{X} \cong \mathbb{Z}$, see [2] or [9, 94.36]. Therefore, Γ is almost simple.

(18) If the center Z of Γ is not trivial, and if $z^\mathsf{Z} \neq z \in W$, then Γ_z fixes each point of $\langle a^\Theta, z^\mathsf{Z}\rangle$, and $(\square)$ implies $\dim \Gamma_z \leq 11$, $\dim \Gamma < 20$. Therefore, Γ is of type G_2, or Γ is locally isomorphic to one of the groups $\mathrm{SU}_4(\mathbb{C}, r)$, $\mathrm{SL}_2\mathbb{H}$, $\mathrm{SL}_4\mathbb{R}$, $\mathrm{SL}_3\mathbb{C}$, or $\dim \Gamma \geq 20$ and Γ is even simple in the strict sence, cp. [9, 94.21]. In any case, Γ has a compact subgroup Φ which is locally isomorphic to $\mathrm{SU}_3\mathbb{C}$ or to $(\mathrm{SU}_2\mathbb{C})^2$. Note that $\mathrm{SO}_3\mathbb{R} < \mathrm{SU}_3\mathbb{C}$. Hence Φ contains a subgroup $\mathrm{SO}_3\mathbb{R}$ or $\Phi = \mathsf{A} \times \mathsf{B}$ with $\mathsf{A} \cong \mathsf{B} \cong \mathrm{SU}_2\mathbb{C}$. The first possibility is excluded by Lemma $(*)$.

(19) Finally, consider the alternative $\mathsf{\Gamma} > \mathsf{\Phi} = \mathsf{A} \times \mathsf{B}$ of the last step, and let $\alpha \in \mathsf{A}$ and $\beta \in \mathsf{B}$ be the central involutions of the two factors. Assume that β is not a reflection $(*)$. Then the fixed elements of β form a $\mathsf{\Phi\Theta}$-invariant Baer subplane $\mathcal{B}$, and $a^{\mathsf{\Theta}} \subseteq K = av \cap \mathcal{B}$. Lemma $(*)$ implies that α acts on $\mathcal{B}$ as a reflection with axis K and some center $z \in W$. Because a compact group of (z, K)-homologies of $\mathcal{B}$ has dimension at most 3, the group $\mathsf{\Phi}$ acts non-trivially on K. Since $\mathsf{\Phi}$ fixes each point of $a^{\mathsf{\Theta}}$, it follows from Richardson's theorem [9, 96.34] that $\mathsf{\Phi}$ induces on K a group $\mathrm{SO}_3\mathbb{R}$, and that the fixed points of $\mathsf{\Phi}$ on K form a circle S. The group $\mathsf{\Theta}$ acts effectively on S and hence would be a Lie group. This contradiction completes the proof of the theorem.

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