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SEVENTY YEARS OF PROFESSOR MIROSLAV FIEDLER

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In the paper [R1], J. Sedláček and A. Vrba described the life and work of Professor Miroslav Fiedler on the occasion of his sixtieth birthday. Also, the list of M. Fiedler's publications was included.

On April 7, 1996, Prof. Fiedler celebrated his seventieth birthday. We shall try here to continue the paper [R1] and show that though retired since 1992, his activities have by no means diminished during the past ten years.[1]

He still is Chief Editor of the Czechoslovak Mathematical Journal, editor of three other journals: Linear Algebra and Its Applications, Mathematica Slovaca, and Numerische Mathematik. Since 1994, he has been chairman of the Czech National Committee for Mathematics. In recognition of his merits he was awarded the Hans Schneider ILAS (International Linear Algebra Society) Prize in 1993.

We shall briefly describe Prof. Fiedler's scientific achievements. (The list of publications below includes a few papers which were listed in [R1] but were not published at that time. The numbering continues that in [R1].)

In the past ten years, M. Fiedler published nearly 50 papers. A vast majority of them concern matrix theory, in particular special classes of matrices.

Prof. Fiedler resumed studying Hankel matrices (his first paper on this topic was published in 1964) and related classes, such as Toeplitz, Bézout, and Loewner matrices, in mid-eighties [106, 107, 109, 110, 117, 118, 121, 122, 123, 135, 140, 143].

While the basis of their theory was given by famous mathematicians of the end of the last and the beginning of this century, these matrices have become very popular again since the seventies, especially due to their occurrance in linear systems theory. M. Fiedler (in some cases jointly with his colleague and for decades the closest collaborator V. Pták) studied mutual relations and connections with associated polynomials and rational functions.

[1] Last year a special issue of Linear Algebra and Its Applications was dedicated to M. Fiedler and V. Pták. It contains a survey of their scientific career up to now [R2].

It may be surprising that Loewner matrices played a role in the solution of the following problem [132]: Given a polynomial $\varphi(x)$ with all its roots real numbers, find its symmetric companion matrix (the characteristic polynomial of which equals $\varphi(x)$) [133], [134].

Another series of papers, in some cases jointly with T.L. Markham, concerned the classes related to M-matrices [119, 120, 125, 126, 129, 136, 159], completion problems [114, 116], Hadamard products of matrices [120, 124, 148, 154], and generalized inverses [129, 139, 141, 151]. Several of these papers [125, 126, 136] gave an exhausting answer to topics studied previously by other authors.

Let us also mention an interesting new notion introduced and studied in a recent joint paper of M. Fiedler and V. Pták [156], the notion of spectral geometric mean of two positive definite matrices A and B (of the same order). The spectral geometric mean of A, B is the matrix F (always existing and unique) which satisfies $F = CAC$ and $F = C^{-1}BC^{-1}$ for some positive definite matrix C.

In graph theory, one of Prof. Fiedler's pioneering ideas was his definition of algebraic connectivity [59] as the second smallest eigenvalue of the Laplacian matrix of the graph (i.e. the matrix of the quadratic form $\sum_{(i,k)\in E}(x_i - x_k)^2$ if $G = (V, E)$, $V = \{1, \ldots, n\}$ being the set of vertices and E the set of edges.) It is interesting to note that it found important applications in the numerical solution of large systems of linear equations as well as in the so called seriation problems. In fact, it served as a basis for spectral methods in both areas. It turned out that the eigenvector (now generally called *Fiedler vector*) of the Laplacian corresponding to the algebraic connectivity has good both the separation and ordering properties for the vertex set of the graph.

Another original Fiedler's idea was to study classes of minimax problems for graphs ([131], [137], [142]) based on minimizing (or, maximizing) various characteristics of a weighted graph when all weightings on edges with a constant sum are considered, thus obtaining *absolute characteristics*. In particular, an explicit formula for the absolute algebraic connectivity of a tree was obtained [130].

Quite recently, Prof. Fiedler returned [144, 151] to the topic which had interested him decades ago — geometry of simplexes and its connection with graphs, matrices and resistive electrical networks. In [151], he found a simple relationship between the Menger matrix and the Moore-Penrose inverse of the Gram matrix of outward normals to the simplex (normalized in such a way that the sum of the normals is zero).

In conclusion, we use the opportunity to extend to Professor Miroslav Fiedler our best wishes of good health, full success in his scientific work and much happiness in his personal life.

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[R2] *Z. Vavřín*: Miroslav Fiedler and Vlastimil Pták: Life and Work, Linear Algebra Appl. *223/224* (1995), 3–29.

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