TEACHING ACTIVITIES OF JAN MAŘÍK

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Abstract. This paper contains a biography of the late Jan Mařík and a description of his teaching activities and methods.

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Any attempt to evaluate the influence of Jan Mařík on the development of Czech mathematics would be incomplete without a description of him as a teacher. To understand the considerable worth of his contribution one must describe his pedagogical efforts, emphasizing his pleasant and admirable personality.

Jan Mařík was born on November 12, 1920, in Užhorod (now in the Ukraine, but in Czechoslovakia at the time). He had Czech parents; his father was a civil servant who brought his family to Prague in 1923. Jan Mařík spent more than half of his life in Prague, where he attended school and began to study engineering at the Czech Technical University of Prague. It was here that he first confronted the misuse of strength and power. Actually World War II changed his life dramatically: Czech Universities were closed and he had to take a job with an insurance company. He later spent a year as a farm hand on his uncle's farm. This did not save him from having to go to the "Reich". He worked as an accountant in Steyr, Austria, from late 1942 until the end of the war. He returned home illegally, and soon after the liberation of Czechoslovakia he resumed his studies, switching, however, to mathematics at Charles University. Unfortunately, post-war Czechoslovakia developed in a direction completely different from his expectations.

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It was already after the communist coup in February 1948 when he completed his studies and became an assistant professor in the Mathematics Department of the Czech Technical University. He later worked in the Mathematical Institute of the Czechoslovak Academy of Sciences, where he completed his postgraduate studies. After he graduated from the University in 1948, he married his wife, Jiřina (now living in the USA). In 1953 when he joined Charles University as an assistant professor, his daughter Eva was born. She is now married with two children and also lives in the USA. Jan Mařík's career was brilliant. Within 11 years he received many degrees in mathematics, including a Doctorate of Sciences (DrSc.), the highest degree awarded in former Czechoslovakia. In 1955 he was promoted to Associate Professor and in 1960, to Full Professor of Mathematics at Charles University. From 1957 to 1970 he also acted as Editor in Chief of the Czechoslovak Mathematical Journal (CMJ).

With the exception of the fall of 1966, when he was a visiting professor at Århus University (Denmark), he spent almost all of the time prior to the events of 1968 in Czechoslovakia. After the Prague Spring in 1968, which along with many others he considered to be crucial to a freer life without the heavy burden of political influence, he accepted the offer of a visiting position from Michigan State University in East Lansing, Michigan. In September 1969 he left to spend a year in the United States.

After the Soviet occupation of Czechoslovakia in August, 1968, the new, pro-Russian communist government wanted to stop the extensive emigration taking place in those days and ordered the Czech Embassy in USA to cancel Jan Mařík's so-called exit visa. Jan Mařík wanted to keep his promise to spend a year in East Lansing and when he was offered tenure at Michigan State University, his decision to stay with his family in the United States was quite natural. By 1977, after he received US citizenship, his life got stabilized and the United States had become his second country.

In 1982 Jan Mařík became one of the two Managing Editors of the Real Analysis Exchange. At the time he used the pseudonym "John Marshall" to make it possible for Czech mathematicians to publish in the Journal. He also deserves credit for the close and productive collaboration that the Czechoslovak specialists in the field of real analysis had with their colleagues in the USA. Jan Mařík never broke his relatively numerous contacts with his former colleagues, and he was always interested in events in Czechoslovakia. He visited Czechoslovakia in 1987, not long before the changes of 1989. At that time he was already an American citizen. However he did not officially visit Charles University.

His last lecture at Charles University was held on September 12, 1991. This lecture on derivatives—a subject he studied intensively during the last years of his life—was very well received. The lecture took place during his last visit to Prague, after a meeting at Smolenice (now in the Slovak Republic). Everyone in the audience admired his clear exposition and his perfect use of the Czech language. He seemed then to be very happy because he felt how much his lecture and his return to Prague was appreciated. During his visit he met many colleagues and friends. He was probably already ill at the time, but nobody could have imagined that it was his last lecture at his Alma Mater. When he left for his home in East Lansing, his friends in Prague hoped to see him again soon. This did not happen. He died on January 6, 1994 at the age of 73. His wife brought his ashes to the Czech Republic for burial.

Jan Mařík was a very kind man. It is worth quoting from the obituary written by his colleagues, friends, and pupils and published in 1994 in Mathematica Bohemica (see [3]): "We knew Professor Jan Mařík as a man of integrity, an honest and just man, with a deep sense of truth. He was able to defend it even in situations when he was risking disgrace by those in power. We loved him for his friendly ways, fine humor and willingness to offer advice or help, for his exemplary correctness, his sense of the order of things and harmony of his words and deeds. He had a deep and admirable affection for Mathematics and he loved people in whom he found the same quality. Mathematical and human qualities do not always go so completely hand in hand. Jan Mařík was in this respect an exceptional personality. We perceived him as such while he was with us, as well as during the long time that we missed him in Prague. Our teacher, colleague, and friend died far from his home. With deep regret we pay tribute to the work and to the human message left to us by Jan Mařík." This passage captures something which was really very exceptional and too subtle to be described better in a few words.

Jan Mařík was a careful and thoughtful observer. His occasional ironic comments revealed his deep interest and closeness to the things which seemed not to be part of his life. He never joined any political party. In some way it was not always easy for his students to approach him; they were afraid to risk disclosing the gaps in their understanding of mathematics. But Jan Mařík always tried to do his best to stimulate them to ask questions no matter how trivial they might be.

It is essential to make one more comment. Jan Mařík was by no means limited to his deep love of mathematics. He was deeply devoted to his family. He loved his wife Jiřina, his daughter Eva, and his two grandchildren. He also enjoyed the simple things of life. He was a very good dancer, he liked to play tennis. He also played bridge and developed a bidding system for it. In particular, he liked to solve bridge problems. Finally, I think that he loved Prague and found great joy in being able to return at last. The question of whether Jan Mařík would have returned after the political changes if he were younger remains unanswered. Life does not allow such "ifs". His daughter Eva, however, was one of the first experts in economics to come to the Czech Republic to help it return to prosperity and democracy.

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As was mentioned above, Jan Mařík started his teaching career at the Technical University of Prague, which helped him to understand the role of mathematics in society. He was fully aware not only of its necessity for other sciences but also of its numerous applications. Despite the fact that those who like such characterizations would surely classify him as a pure mathematician and by no means an applied mathematician, his attitude was different. He was always very precise and he distinguished between perfect, less perfect and simply incorrect mathematical statements. When he felt the importance of something which lacked perfection, he tried to improve things. When others were reluctant to try things which were rather remote from their interests, he willingly started to give lectures on numerical methods for ordinary differential equations. He carefully struggled with theorems in which phrases like "sufficiently smooth function" were used, rewriting them into a precise and sometimes rather sophisticated form.

He was the first to lecture on potential theory. These lectures contained a relatively deep treatment of its indispensable basics such as surface integral, Gauss-Green theorem, etc. At that time he actively worked in the field. (See the contribution written by J. Král.) It is appropriate to mention here one admirable feature. Jan Mařík interpreted his pedagogical duties as a challenging inspiration for his scientific work. He also encouraged dissemination of his ideas concerning different subjects, either in the written form (see [4]), or in lectures for the wider mathematical community (see, e.g., [5, 6]). In [4] he intended to explain the basic properties of harmonic functions. He described them in the form of 30 exercises accompanied with detailed hints based on the popular Czech four volume course of analysis written by Vojtěch Jarník [1]. Jan Mařík did not expect background knowledge of surface integrals since he was aware that that subject was not sufficiently covered in lectures or in Czech books available at that time. In this connection it is interesting to quote from the Introduction to [1] (Integral Calculus II, published in 1955) where Vojtěch Jarník describes the parts of analysis which he has not sufficiently covered in his books.

"(...) But the real weakness of my book is the lack of a theory of k-dimensional integration in n-dimensional space for $k < n$, e.g. the theory of the surface integral so important particularly for physics. I do not know any text on the subject in world literature satisfactory both from the scientific and the pedagogical point of view. I do not feel to be the one called on to explain it. I know that some of our prominent mathematicians have deeply reflected on this problem and I hope that they will fill in this substantial gap in Czech literature." I am quite sure that Jan Mařík was one of those to whom those lines were addressed. In fact he wrote lecture notes for students on the subject but the text was never used on a wide scale.

During the third year of their studies of mathematics, Prague students had to attend special exercises; the title on the list of lectures was deceptively simple: "Practice in Mathematics". Jan Mařík taught the subject for about ten years. Students were asked to prepare short presentations of some more or less standard things, which for some reasons were omitted from regular courses or which slightly exceeded their contents. So far there was nothing to worry the students except that part of the assignment was to record carefully in written form the contents of the presentation. It was necessary to formulate some assertions and to prove them. The content differed but the scheme was the same every year. To many students it seemed like an endless procedure. Jan Mařík carefully corrected every version of such "minipapers" and asked students to correct their mistakes (including grammar), to improve formulations, to find missing assumptions and to rewrite the paper again and again. Rarely did a student pass after the first round. Jan Mařík as a very well-trained referee did not overlook any inaccuracy as subtle as it might be. Later he even wrote some notes for students (see [7]) from which we can learn about popular themes for presentations, e.g. infinite products, generalized series, convergence criteria and properties of the Γ-function although no part of analysis was à priori excluded. Sometimes the subjects were based on previously discussed problems raised by the students themselves.

Those students who successfully passed, gained immensely and mostly had no problem with writing their own dissertations. When Jan Mařík left for the USA and his colleagues tried to continue with "Practice", they understood what a great amount of work lay behind it. His great experience of the Editor in Chief of CMJ (see above) was valuable, but even so, he devoted much time and effort to the "Practice". His level of commitment far exceeded the limits of his successors in that activity (even though most of them had gone through his "Practice").

Before taking a closer look at the nature of topics discussed in the "Practice" I would like to emphasize the fact that without Jan Mařík's great human qualities no one was able to imitate his unique style of work with students. He loved mathematics and, for the benefit of future mathematicians, put in a tremendous amount of highly individual work with students. He liked difficult mathematics and liked students who cooperated with him in his efforts and who wanted to go deeper and extend their knowledge also in relation to other parts of the analysis. A student's approach was respected and there was no imposing of "the only solution". Jan Mařík was a demanding teacher and he asked for really hard work according to individual abilities.

It might seem quite common now to achieve what Jan Mařík did, but one must consider where and when it was done. Any comparison with the present is very unfair. Today conditions for individual work with students are in many respects much more favorable. A certain rather conservative atmosphere has vanished. Access to books and journals has greatly improved and e-mail has brought quite a new dimension to the exchange of information not limited to scientific work. Of course, the new situation is ideal only for *ideal* teachers and *ideal* students.

Jan Mařík quite often asked students during the "Practice" to explain certain statements, formulas or solutions of some exercises. He worked hard to create an atmosphere in which he could make a diagnosis in discussion. For him it was very important to analyse the roots of a student's mistakes or erroneous ideas. Sometimes he was not quite successful and years after he wrote in [9]: "I would like to say beforehand that I do not pretend to have found practical answers to the corresponding problems. But I have the feeling that we did not yet begin to ask the right questions". Let me try to illustrate his approach when he was teaching in "Practice".

To choose from the numerous themes would be difficult for everybody who was present at such a performance several times. Since I feel the duty to explain this point for those who have no such experience in detail, I decided to quote from two works of Jan Mařík which are not exactly available for everybody. In [9] he writes, "It is a popular theme of conversation among math teachers to complain how weak the students are and/or to speak about some ridiculous mistakes that occasionally show up. I do not think that there is anything wrong with such conversations; I do think, however that we should go a little deeper and examine the situation more closely. (...) We sometimes consider whether we should prove a certain theorem. But the real problem is how to make the students understand our theorem. Many of them do not realize that what they write should make sense. (...) A student who is unable to make statements about what we teach usually does not understand our statements. If a teacher persists, then some students—trying to oblige him—write something like *the series* $\sum\limits_{n=1}^{\infty} 2^{-n}$ *converges for each* n or *the radius of convergence of the series* $\sum\limits_{x=0}^{\infty} x^n/n!$ *is* $x/(n+1)$, etc. This, I hope, would not happen so often, if more teachers were persistent."

How did all of this work? Jan Mařík came (always on time) to the seminar and wrote a formula on the blackboard, e.g.

$$\int \frac{1}{x^2}\,\mathrm{d}x = -\frac{1}{x} + c\,, \quad x \neq 0\,. \tag{1}$$

Then he asked: "Have you ever seen such a picture?" After a short conversation about it (if it ever took place) he started to explain (I quote again from [9]).

"What should, e.g., $\int \mathrm{d}x/x^2$ mean ? A formula like (1) indicates something which is not true; it indicates, in particular, that if f is a function such that $f'(x) = 1/x^2$ for each $x \neq 0$, then there is a c such that $f'(x) = -1/x + c$ for each $x \neq 0$. (Analogous "theorems" connected with the formula $\int \mathrm{d}x/x = \log|x| + c$ are often applied in chapters about differential equations.) I think each student learning about derivatives should know that the function f, defined by $f(x) = -1/x$ for $x < 0$, $f(x) = -1/x + 2$ for $x > 0$, fulfils the relation $f'(x) = 1/x^2$ for each $x \neq 0$."

Then the conclusion in the end was as follows (according to [8]): 'Worst of all is the mechanical use of formulas. Every time it is necessary to know what we are dealing with. It is impossible to work with, say, Euler differential equation without knowing what a derivative is.' (Czech readers should see also [10].)

Jan Mařík's style of comments was sharp even when dealing with matters which were very much mixed with politics. Once in a speech for a rather broad audience he said, "An instruction from the Central Committee of the Communist Party (CC) on scientific work at Universities was received. What should we do? A meeting was organized, some plans launched, and everything remained as before. What comes to my mind is an act from the Emperor's baker (a well-known Czech comic movie). People are bemoaning the situation. 'Virgin Mary, you cannot leave it as it is!'. And the baker says, 'Of course she will leave it as it is when we do so as well.' I am afraid that it is tailored to our situation. The CC wants to help us. From what I have heard I have the impression that the CC does not know where our (at least the mathematicians') 'shoe pinches'." And after such an introduction he continued to explain his ideas on how to possibly improve conditions. It was a rather delicate situation, but he got away with it.

In the next paragraph I present several opinions and ideas expressed by Jan Mařík on various occasions. Some of them might be known to Czech readers, others probably not.

Abstract math is not so difficult but it is nonsense for young students to start with it.—Mathematics badly needs communication and any attempt to cultivate the whole of mathematics in our conditions is again nonsense.—It is much more difficult to ask questions than to answer them.—The more we try to cover in our lectures, the more students will be completely untouched by our effort.—It is important to teach students how to work with mathematical literature. I would like to express a heretic opinion; sometimes a very imperfect textbook could be better than an excellent one.—Exercises with students (extra hours devoted to practice in computation only) are important. To prepare for those is much more difficult than to have a lecture.—Examples must be very simple to illustrate principles.—In case of the student's wrong answer we have to distinguish between what he knows and what he does not understand. Students have to know what they are speaking about.—The style of writing (including the grammar) is an important part of a mathematician's work.

The above opinions form a small (positive) epsilon which some of my friends, pupils of Jan Mařík, and I myself learned from him. I hope that some of those will also inspire the reader and that I have succeeded to explain in more detail the above quoted part of [3]: (...) We perceived him as such while he was with us as well as during the long time that *we missed him in Prague.* This is true even now and more so: unfortunately, it is for ever.

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