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NEWS AND NOTICES

MIROSLAV KATĚTOV 1918–1995

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Shortly before the end of 1995, Czech mathematical community lost one of its most prominent members, a distinguished mathematician, a worldwide known specialist in general topology, the founder of the interdisciplinary seminar now called the Katětov seminar.

Miroslav Katětov was born on March 17, 1918 in Bělinskij (originally Čembar) near Penza as a son of POW Czech legionnaire and Russian mother. Since 1923 he lived with his mother in Czechoslovakia. After finishing his studies at secondary school 1927–35, to which he was admitted at the early age of nine years, he studied at the Faculty of Science, Charles University in Prague (1935–39). Here he made friends with L. S. Rieger, later a prominent Czech logician, this friendship lasted till Rieger's death in 1963. It was after he submitted his thesis but before he passed the final examinations that the Nazis closed the Czech universities, so that he graduated only after the World War II in June 1945. The official oponent of his thesis was Prof. V. Jarník who unofficially asked Prof. E. Čech, then in Brno, for his opinion since the regulations did not allow oponents or examiners from another university.

During World War II M. Katětov worked in the Institute of Human Labour. His task was to help in mathematical-statistical part of work in standardization of psychological tests and in the analysis of the data obtained. Among other, he intensively used the factor analysis which was then a new method in psychology. Here Katětov got acquainted with applications of mathematical methods in psychology, and he began to be interested in psychology proper. He returned to this field in the seventies when he founded the Seminar on mathematical methods in psychology.

Since June 1945 Katětov was employed as Assistant at Faculty of Science of Charles University, later at the newly established Faculty of Mathematics and Physics. In 1961 he joined the Mathematical Institute of Czechoslovak Academy of Sciences where he worked as a Principal Scientific Officer till his retirement. However, even then he remained a fellow of the Institute, as well as Professor Emeritus of Charles University.

From the long list of offices and degrees of M. Katětov let us mention only the most important ones. He defended his habilitation thesis in 1947 and was appointed Associated Professor (Dozent) on February 4, 1948. On October 1, 1953 he was appointed Full Professor. In 1952–53 he was the first Dean of Faculty of Mathematics and Physics, while in 1953–57 he was Rector of Charles University, he resigned from the office on his own request. In 1962 he was elected Ordinary Member of the Czechoslovak Academy of Sciences (having been its Corresponding Member since the foundation of the Academy in 1953). In 1960–70 he was Director of the Mathematical Institute of Charles University after its founder Prof. Eduard Čech. Let us note that the generation gap between Čech and Katětov never affected their friendship or Čech's respect of Katětov as a mathematician.

When the Scientific Board for Mathematics of the Academy was established, Katětov held its chair 1962–64. During 1965–69 he was member of Presidium of the National Committee for Scientific Degrees. He was awarded State Prize in 1953 for his results in Mathematics.

An integral part of Katětov's personality was his active interest in political life. He joined the Czechoslovak Communist Party in 1945. In 1970, his membership was cancelled. In spring 1989 he was one of the founders of the Circle of Independent Intelectuals and after the revolution he took part in the transformation of the Czechoslovak Academy of Science.

Beyond the field of Mathematics, Katětov was well known as a chess player. He actively cultivated chess since his young years, represented Czechoslovakia during 1946–51 and gained the title of International Master. Later he quitted playing chess for lack of time, but always had a chess journal on his desk.

Katětov's scientific activity falls within general topology, functional analysis and general theory of entropy. A "common denominator" of almost all works of M. Katětov is the notion of the "covering property", even if this fact is not always immediately apparent from the formulation of the result. As an example let us mention the now famous Katětov-Morita theorem claiming that $\dim X = \operatorname{Ind} X$ for any metric space X. Indeed, the crucial point of the proof is the construction of a special countable sequence of coverings, which allows to prove that $\dim X \geqslant \operatorname{Ind} X$.

Papers belonging to general topology concern extremal properties of spaces, which are conceptually linked to papers on filters and ultrafilters, works on dimension theory and on properties similar to that of paracompactness, and are topped by a study of general structures of continuity. Many of Katětov's results got ahead of their time so much that only after their later re-discovery the mathematical community realized and acknowledged Katětov's priority.

The first decade of Katětov's publication activities can be characterized by a slogan "each paper—a fundamental contribution to general topology".

In paper [1], which is the German translation of Katětov's RNDr. thesis, he constructed the maximal H-closed extension of the Hausdorff space, now termed the Katětov extension $\varkappa X$.

Let us recall: A Hausdorff space is said to be H-closed if it is a closed set after being embedded into any Hausdorff space Y. It is well known that every compact Hausdorff space is H-closed, which had been proved already by Alexandrov and Uryson in [AU]. Katětov knew both fundamental papers on the β-envelope [Č], [S] and used maximal centered systems of open sets in a context beyond the framework of Boolean algebras. Katětov's extension consists of the original points of the space X and of the ideal points, which are the maximal centered families of open sets nonconvergent in the space X. The subspace of all ideal points in $\varkappa X$ is discrete. This guarantees the most important property that makes the extension X similar to the Čech-Stone extension βX: every continuous mapping from X onto a dense subset of the Hausdorff space Y can be continuously extended to a subspace $Z \subseteq \varkappa X$ sufficiently large as to satisfy $Y = f[Z]$.

Stone's assertion that a Hausdorff space X is compact if and only if each of its closed subspaces is H-closed is proved in [1], Katětov's proof in the context of the whole theory requires practically null effort and at present represents a standard proof in monographs.

Other types of H-closed extensions (Katětov's, Fomin's and Wallman's, or the Čech-Stone compactification) are dealt with in [5]. The main result is a full innner characterization of these hulls by the combinatorial properties of the embedding of the given space into the corresponding extension. Always we have a mutual relation between the intersection of closed sets in a subspace and the intersection of their closures in the extension.

The existence and properties of Hausdorff spaces without isolated points in which there exist no two disjoint dense subsets, now called irresolvable spaces, represented a problem since the times of Čech's seminar (1936–39). The answer was given by E. Hewitt in 1943 and independently by M. Katětov [4] in 1947. Not only such spaces do exist, but moreover they have further remarkable properties many of which characterize them:

— every bounded real function defined on an irresolvable space has a limit at every point, this property is characteristic for irresolvable spaces,

— every bounded continuous real function defined on a dense subset can be continuously extended to the whole space,

— there exist H-closed irresolvable spaces,

— every regular irresolvable space is totally disconnected and hence completely regular,

— for any infinite κ, there exists a regular irresolvable space of cardinality κ.

In his paper Katětov also formulated the famous problem whether any real function defined on an irresolvable space has at least one point of continuity. This problem was solved only in 1986 by K. Kunen, A. Szymański and F. D. Tall: it is undecidable in ZFC (see [KST]).

This problem was attached from the other side in a paper [27] from 1962 where sufficient conditions are given for at least κ disjoint dense sets to exist in a given space. Similarly, also κ-resolvable spaces are an object of interest even nowadays.

In [15] M. Katětov was the first to publish the solution of Birkhoff's problem whether there exists an infinite rigid Boolean algebra, i.e. a Boolean algebra whose only automorphism is the identity. Katětov solved the problem very elegantly using the Stone representation theorem. His rigid Boolean algebra is dual to the Čech-Stone compactification of a certain rigid countable normal space whose each point is the limit of a nontrivial sequence. Rigid algebras proved important fifteen years later after the discovery of the forcing, especially in the context of complete algebras.

During the World War II, after the Nazis closed the Czech universities, M. Katětov attended meetings of mathematicians which took place in the house of Prof. V. Jarník. He got interested in functional analysis and at these seminars gave a report on Banach's book "Théorie des opérations linéaires". These lectures gave rise to the papers [2], [3] published in Czech parallelly with a shorter German version. Their character is not that of a modern original research paper. It is rather a systematical exposition of fundamental notions for topological vector spaces, intended for the wider nonspecialized public. Nevertheless, the exposition smoothly passes into original author's results. It is remarkable that Katětov in these papers developed all tools needed for the proof of Mackey-Arens theorem [A], [M]. The topology of uniform convergence on w-compact convex symmetric sets in the dual agrees with the duality, the convexity being essential. Katětov proved this assertion independently in [10] in 1948. Papers [2], [3] formed the basis for the lecture notes [77].

A compact space X is metrizable if and only if X^3 is hereditarily normal. This theorem is denoted as Corollary 2 in [9]. Although the properties of the space X as related to the properties of the space X^2 were studied by tens of authors, until now not a single further proposition has been found which would deduce some property of a space from an information of the properties of its cube. Many topologists are still amazed by the fact that it is at all possible to think of such a theorem.

Since 1947 Katětov has systematically studied covering properties of topological spaces. This is easily understandable if we take into account the fact that paracompactness was at the time a new and evidently perspective notion. His results led both to the dimension theory and to the normal, paracompact, uniform and proximity spaces.

A comprehensive survey of all facts connected with the covering was published in Czech in a supplement to the monograph of E. Čech "Topological Spaces" [23], except for the assertion concerning dimension. The supplement "Fully normal spaces" contains all which is essential in the papers [9, 16, 17, 21].

Let us present some of the results:

A topological space X is normal if and only if for every pair of real functions $f \leqslant g$ where f is upper- and g lower semicontinuous there exists a continuous function h such that $f \leqslant h \leqslant g$, if we require for $f < g$ the existence of a function h such that $f < h < g$ then we obtain a characterization of normal countably paracompact spaces.

The contents of the next result (Katětov theorem) reminds the Tietze theorem for normal topological spaces: every uniformly continuous bounded real function defined on an arbitrary subspace of a uniform space can be uniformly continuously extended to the whole space.

A paracompact space is realcompact if and only if the same is true for each of its closed discrete subspace, that is, if the cardinality of any closed discrete subspace is less than the first measurable cardinal.

Countably paracompact normal spaces were investigated by Katětov independently and simultaneously with C. H. Dowker [D], he found their characterization and posed the problem whether each normal space is countably paracompact. (The problem was solved negatively by M. E. Rudin in 1971.) The term "countable paracompactness" itself belongs to Dowker as well as the assertion that a normal space X is countably paracompact if and only if $X \times [0,1]$ is normal.

Theory of dimension was Katětov's lifelong affection. His result from the fifties deeply influenced the progress of this discipline. From various definitions of the notion of dimension, three are the most important, namely the small inductive dimension $\operatorname{ind} X$ (Menger-Uryson), large inductive dimension $\operatorname{Ind} X$ (Brouwer-Čech), and the covering dimension (Čech-Lebesgue).

The paper [12] contains a suprising characterization of the dimension of a compact topological space X in terms of the ring $\mathcal{C}(X)$ of continuous real-valued functions on X. The key role in the proof of the Stone-Weierstrass theorem is played by a system of functions separating points. It was Katětov who posed himself the question what this system can tell about the properties of the space X, and answered it in the following way: For a compact metric space X we have $\dim X \leqslant n$ if and only if there exist n functions $f_1, f_2, \ldots, f_n \in \mathcal{C}(X)$ such that the least subalgebra containing all f_n's, all constant functions, and closed with respect to square roots (i.e. $f^2 \in \mathcal{A} \Longrightarrow f \in \mathcal{A}$) is dense in the algebra $\mathcal{C}(X)$.—Chapter 16 in the monograph of Gillman and Jerison [GJ] is fully devoted to a detailed account of Katětov's result.

Already Uryson knew that a compact metric space X satisfies $\dim X = \operatorname{ind} X = \operatorname{Ind} X$. The same identity was known to hold for separable metric spaces since late twenties (L. A. Tumarkin, W. Hurewicz). In the paper [19] the definitive general theorem is established: $\dim X = \operatorname{Ind} X$ for all metric spaces X. This result was proved independently by K. Morita, and it is now usually referred to as the Katětov-Morita theorem. It was for this research that Katětov received the State Prize in 1953. The situation for general metric spaces was fully described only after P. Roy in 1962 constructed a complete metric space X with $\operatorname{ind} X < \dim X$.

It is not generally known that the change in the original definition of the notion of dimension dim, namely the replacement of open coverings with coverings which are functionally open (cozero), which makes it possible to extend the whole theory from normal to fully regular spaces, is also Katětov's contribution [14]. The identity $\dim X = \dim \beta X$ is an easy consequence of Katětov's definition.

The paper [22] studies a less known type of dimension of a metric space X, the metric dimension $\mu \dim X$. It is proved that $\mu \dim X \leqslant \dim X \leqslant 2\mu \dim X$. This paper marks further, much later papers devoted to metric spaces from the viewpoint of invariants similar to dimension (Bolzano dimension, Dushnik-Miller dimension, entropy).

From the ideas contained in the lecture [30] delivered at the International Congress of Mathematicians 1962 in Stockholm let us mention two. Trying to define a notion of "continuous structure" as general as possible, Katětov explicitly gives the following method of forming a new structure from the given one by means of the covariant functor Φ from the category of sets into itself: The new structure is the set X equipped with the old structure on ΦX, a mapping of the new structured sets F: $\mathcal{X}$ is continuous if and only if $\Phi(F)$ is a continuous mapping of ΦX into ΦY equipped with the old structures. This idea was widely developed in the categorial theory of structures. Katětov himself studied as an example the free real module ΛX over the set X equipped with a compatible locally convex topology [28, 32]. Another example will be mentioned in the next paragraph. The other idea was the stressing of the importance of the projective and inductive generation of continuous structures. In Prague this led to the introduction of the so called amnestic functor and the S-functor ([Hu]) which is now used under the name of the topological functor, see [AHS].

Merotopic spaces are structures of continuity more general than the current topological, uniform and proximity spaces. Merotopy is determined by a filter Ξ consisting of a covering of the set X, that is, $\bigcup \mathcal{V} = X$ for all $\mathcal{V}$ Ξ, if $\mathcal{V}_1, \mathcal{V}_2 \in \Xi$, then also the cover $\{V_1 \cap V_2 \colon V_1 \in \mathcal{V}_1, V_2 \in \mathcal{V}_2, V_1 \cap V_2 \neq \emptyset\} \in \Xi$, if $\mathcal{V}$ Ξ, $\mathcal{W}$ is a covering of the set X and $\mathcal{V}$ refines $\mathcal{W}$, then $\mathcal{W} \in \Xi$. Unlike in the case of uniformity there is no requirement of openness of the elements of the covering here, and therefore merotopies can be equivalently described by families of "small" sets as well as by families consisting of mutually "near" sets. Let us note that these notions had been used by some authors before Katětov, but Katětov proved fundamental theorems on these spaces, two of which we will present here:

Merotopic spaces are exactly the quotients of uniform spaces.

A subcategory of the so-called filter merotopic spaces is Cartesian closed, i.e. the space of functions X^Y can be canonically equipped with a structure in such a way that $X^{Y\times Z}$ be isomorphic to $(X^Y)^Z$.

Katětov's term "merotopic spaces" later gave way to Herrlich's term "nearness spaces". The intensity of research of the successors can be documented by the survey paper [He].

It was already in 1960 that Katětov demonstrated the importance of the cardinal characteristic $\mathfrak{d}$, the dominating number, the least cardinality of a set F of sequences of positive integers such that each sequence is majorized by a sequence from F. In the papers [25, 26] he investigated the following cardinal numbers: the character of the set of integers in the space of reals, the least cardinality of the cofinal part of the family of compact subsets of rational numbers ordered by inclusion, the least cardinality of a covering of irrationals by compact sets, and proved that they are all equal to $\mathfrak{d}$. If rationals are replaced by irrationals in the above consideration, then the pseudocharacter will evidently be countable, but the character, as Katětov showed, is again equal to $\mathfrak{d}$.

Not a single one of the above presented results was omitted by E. K. van Douwen in his Handbook of Set Theoretic Topology article [vD].

In the sixties the interest in filters and ultrafilters increased rapidly. This trend did not skip Prague. It was then that Katětov became friends with the young Zdeněk Frolík, and filters and ultrafilters were a frequent topic of their discussions. Frolík then proved nonhomogeneity of the space $\beta\mathbb{N}\setminus\mathbb{N}$. Katětov was interested in the operations on filters and in convergence with respect to a filter. In the paper [37] Katětov investigated products of filters. Let us recall: If $\mathcal{F}$ is a filter on a set A, $\mathcal{G}$ a filter on a set B, then the filter $\mathcal{F}\cdot\mathcal{G}$ on the set $A\times B$ consists of all $X\subseteq A\times B$ such that $\{a\in A\colon \{b\in B\colon (a,b)\in X\}\in\mathcal{G}$, i.e. the product in the current Fubini sense. If $\mathcal{F}$ is a filter on A and $\mathcal{G}$ a filter on B, then $\mathcal{F}$, $\mathcal{G}$, have the same type ($\mathcal{F}\sim\mathcal{G}$) if there is a bijection $f\colon A\longrightarrow B$ such that $F\in\mathcal{F}\iff f[F]\in\mathcal{G}$. For filters Katětov studied the two transitive relations which are now called Rudin-Keisler and Rudin-Frolík orderings in the case of ultrafilters ([CN]—Comfort and Negrepontis were apparently not aware of [37]) and proved that for ultrafilters they really represent orderings of types. To this end he used a lemma on three sets, proved and published in [35]. It is true that the lemma is a special case of de Bruijn-Erdős theorem [BE] which Katětov did not know at the time, but he was the first who demonstrated the importance of this special case for the theory of ultrafilters. Katětov also proved that for an ultrafilter $\mathcal{F}$ and an arbitrary filter $\mathcal{G}$ with an empty intersection neither $\mathcal{F} \sim \mathcal{F} \cdot \mathcal{G}$ nor $\mathcal{F} \sim \mathcal{G}$ can hold, similarly as $\mathcal{N}$ does not hold for the Fréchet filter $\mathcal{N}$. Naturally a question arose whether there can at all exist a nontrivial filter $\mathcal{F}$ with the property $\mathcal{F} \sim \mathcal{F} \cdot \mathcal{F}$, in the paper [47] it is proved that such a filter does exist on any infinite set, and it is constructed there.

In [39], [40] the convergence with respect to a filter is studied. The main theorem in both the papers asserts that—if we assume CH—there exists a special filter $\mathcal{F}$ on a countable set such that for every topological space X all $\mathcal{F}$-limits of sequences of continuous functions are exactly all Baire functions on X. The theorem fails to hold, which is proved by the diagonalization method in [43]. The main result then is the description of the class of spaces for which the theorem is correct.

In the course of the activities of the Seminar of mathematical methods in Psychology a number of psychological experiments turned Katětov's attention to the notion of information. His effort to grasp this notion mathematically resulted in a numerous series of papers [45, 49, 52, 53, 57, 58, 59, 60] which was interrupted only by his death. Roughly speaking, if $\langle X, \varrho, \mu \rangle$ is a set equipped with a semimetric ϱ, i.e. $\varrho(x,x) = 0$ and $\varrho(x,y) = \varrho(y,x)$, and a finite measure μ for which ϱ is $\mu \times \mu$-measurable, he posed the question of existence of functionals defined on the class of such spaces which in two important special cases, namely those of X finite and the metric satisfying $\varrho(x,y) = 1$ for all $x \neq y$, and of X without measure but with a metric, result in the Shannon entropy or the Kolmogorov entropy, respectively. Functionals of this type with further reasonable properties Katětov called the extended Shannon entropy. The passage from the finite to the infinite situation Katětov solved by a tree sequence of binary decompositions first of the given space and then of the elements of the decomposition into sets of lesser and lesser diameter, the branches of the tree end at the moment when the diameter is less than ε.

Now the connection with various types of dimensions, which were dealt with by Katětov in [54], [61], is no more a surprise.

However, Katětov desired to achieve more than we have just described. Step by step he developed a unified theory covering as special cases various further kinds of known entropies (beyond already mentioned Kolmogorov entropy of totally bounded metric spaces, e.g., differential entropy, entropy in the sense of Posner, Rodemich and Humphrey, topological entropy, entropy of a mapping, Bowen's entropy). This research was extremely demanding from the technical point of view and differed essentially from procedures used in the information theory. The importance of this monumental activity can be justly evaluated only by the future.

In 1970, M. Katětov founded Seminar on mathematical methods in Psychology in the Faculty of Mathematics and Physics. He opened it as a specialized seminar from Applied Mathematics, having had prepared for it many years before.

At the beginning, the seminar was devoted to the mathematical problems in psychology (theory of information, mathematical linguistics, neurolinguistics, artificial inteligence, problem solving, theory of perceptrons, theory of complexity, probability, plans and the structure of behavior, genetic epistemology, theory of measurement, perception etc.) M. Katětov wrote then a series of huge texts devoted to various aspects of using mathematical methods and structures in psychology, but mostly copied for the seminar use only. From that period (seventies), also Katětov's papers on modelling of multiple sclerosis by means of the catastrophe theory came [50], [q], [r]. A remarkable paper [48] models a seemingly alogical behavior of a subject in a certain standard psychological test; the model has a surprising predictive power. As already said, the study of theory of information in psychology inspired a series of Katětov's mathematical papers on entropy theory for metric spaces.

Since the end of seventies, the contents of the seminar widened in further areas of applications of mathematics (biology, medicine) and, contrary to original Katětov's intentions, it opened also to the philosophical questions in eighties, becoming thus transdisciplinary.

It is possible that Katětov was not the best lecturer for introductory courses. On the other hand, by his lectures of advanced parts of Mathematics and by his seminars he aroused interest in scientific work in more than one generation of Czech mathematicians. He was a peerless paragon for all his students and students of his students. His unfailing memory, comprehensive knowledge, the ability to see and to precisely and pregnantly formulate the essentials, these were gifts given only to few.

Unlike most of his colleagues, Miroslav Katětov believed all his life that Mathematics is communicable and everybody is able to grasp it. This is probably the reason why he wrote so many articles and texts popularizing Mathematics or explaining some parts of it to laymen or specialists from remote professions. This concerns papers [64, 67, 72, 79], [b]–[p], [s]–[v], all written with anxious care for mathematical precision combined ideally with easy-to-grasp presentation. The last paper that Katětov finished immediately before his death was a contribution for the Handbook from the history of general topology devoted to his most favorite topic, dimension theory [76]. He verified with extraordinary care all historical facts, thus avoiding the frequently cited inaccuracies. This of course concerns all the other Katětov's papers devoted to the history and development of Mathematics [66, 66, 67, 68, 70, 71, 73, 74, 79, 80, 84].

Miroslav Katětov passed away on December 15, 1995. On his desk he left manuscripts of papers to be completed…

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