# GRAPHS WITH THE SAME PERIPHERAL AND CENTER ECCENTRIC VERTICES

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*Abstract.* The eccentricity $e(v)$ of a vertex $v$ is the distance from $v$ to a vertex farthest from $v$, and $u$ is an eccentric vertex for $v$ if its distance from $v$ is $d(u,v) = e(v)$. A vertex of maximum eccentricity in a graph $G$ is called peripheral, and the set of all such vertices is the peripherian, denoted $\operatorname{Peri} G$). We use $\operatorname{Cep}(G)$ to denote the set of eccentric vertices of vertices in $C(G)$. A graph $G$ is called an S-graph if $\operatorname{Cep}(G) = \operatorname{Peri}(G)$. In this paper we characterize S-graphs with diameters less or equal to four, give some constructions of S-graphs and investigate S-graphs with one central vertex. We also correct and generalize some results of F. Gliviak.

*Keywords*: graph, radius, diameter, center

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## 1. Introduction

We consider nonempty and finite graphs without loops and multiple edges. All terminology as well as notation except that given here is taken from [1].

The set of vertices of a graph $G$ is denoted by $V(G)$, and the set of edges by $E(G)$. Let $G$ be a connected graph with vertices $u$ and $v$. The *distance* $d(u,v)$ between $u$ and $v$ is the length of a shortest $u-v$ path in $G$. The *eccentricity* $e(v)$ of a vertex $v$ is the distance from $v$ to a vertex farthest from $v$, and $u$ is an *eccentric vertex* for $v$ if $d(u,v) = e(v)$. The *radius* $r(G)$ of $G$ is $\min\{e(v);\ v \in V(G)\}$, while the *diameter* $d(G)$ of $G$ is $\max\{e(v);\ v \in V(G)\}$. A *diametral path* is a path of length $d(G)$ joining a pair of vertices of the graph $G$ that are at distance $d(G)$ from one another. A vertex with minimum eccentricity is called a *central vertex* and the set of all such vertices is the *center* of $G$ denoted by $C(G)$. A graph is *self-centered* if its every vertex is in the center. The neighborhood of a vertex $v \in V(G)$ is denoted by $N_G(v)$. For any nonempty subset $S$ of vertices in $G$, the *induced subgraph* $\langle S\rangle$ is the maximal subgraph of $G$ with the vertex set $S$. A vertex $u$ is a *center eccentric vertex* of $G$ if it is an eccentric vertex of some central vertex of $G$. A vertex $v$ is *peripheral* if $e(v) = d(G)$, and the set of such vertices is the *peripherian* of $G$. We use $\mathrm{Cep}(G)$ to denote the set of all center eccentric vertices and $\mathrm{Peri}(G)$ to denote the peripherian of $G$. A connected nontrivial graph $G$ is called an S-*graph* if $\mathrm{Cep}(G) = \mathrm{Peri}(G)$.

Buckley and Lewinter [2] proved the existence of an S-graph $G$ with $r(G) = a$ and $d(G) = b$ for every positive integers $a$, $b$, $a \leqslant b \leqslant 2a$, and showed how to embed a graph $G$ into an S-graph. They also proved that the cartesian product of two graphs is an S-graph if and only if both these graphs are S-graphs.

## 2. Main results

As follows from the definition of an S-graph, any self-centered graph as well as any tree are S-graphs. In particular, the complete graph $K_n$, $n \geqslant 1$ is an S-graph. Further we will investigate only S-graphs that are not self-centered.

Let $G_1$, $G_2$ be two disjoint connected graphs. Let $x \in V(G_1)$, $y \in V(G_2)$. We say that a graph $G$ arose from $G_1$ and $G_2$ by the identification of the vertices $x$, $y$ with a new vertex $t$ ($t \notin V(G_1)$, $t \notin V(G_2)$), if

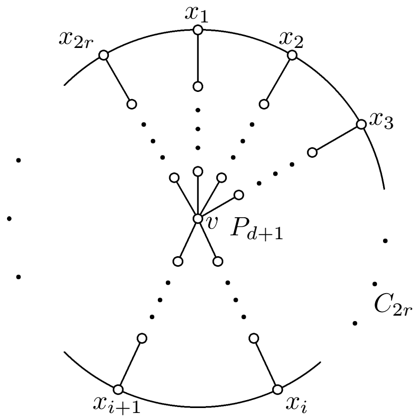
$$\begin{aligned} V(G) &= V(G_1) \cup V(G_2) - \{x, y\} \cup \{t\}, \\ E(G) &= E(G_1) \cup E(G_2) - \{xu, yu;\ xu \in E(G_1), yu \in E(G_2)\} \\ &\qquad \cup \{tu;\ xu \in E(G_1) \text{ or } yu \in E(G_2)\}. \end{aligned}$$

Gliviak [4] gave the following construction of S-graphs with one central vertex.

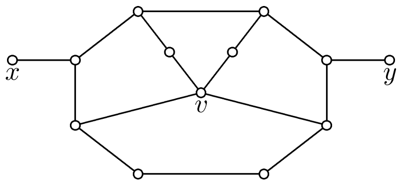
Construction. Let $r \geqslant 1$, $n \geqslant 2$ be natural numbers. Let $G_i$, $i = 1, 2, \ldots, n$ be vertex disjoint graphs having at least one vertex $v_i$ of eccentricity $r$. Let the graph $H$ arise from graphs $G_i$ by identification of all vertices $v_i$ with one common vertex $w$.

He claims (Theorem 2) that a graph $Q$ is an S-graph of radius one if and only if $Q$ is either a complete graph $K_n$, $n \geqslant 2$ or can be constructed according to the previous construction, as well as (Theorem 3) that for any $r \geqslant 2$ a graph $Q$ is an S-graph of radius $r$ with one central vertex if and only if $Q$ can be constructed according to this construction.

This construction gives S-graphs with one central vertex, but not all such S-graphs. In any S-graph $G$ which we get by the construction the central vertex is a cut vertex of $G$ and the diameter of such a graph is equal to $2r(G)$. As follows from Fig. 1 there exist S-graphs with one central vertex, which cannot be constructed according to the above construction. In Fig. 1a there is an S-graph with radius $r \geqslant 1$, diameter



a)



b)

Fig. 1

$2r$, and one central vertex $v$. The peripherian of this graph consists of all vertices belonging to $C_{2r}$. In Fig. 1b there is an S-graph with radius 3, diameter 5 and one central vertex $v$. The peripherian of this graph is $\{x, y\}$.

**Theorem 2.1.** *Let $G$ be a graph with $r(G) = 1$ and $d(G) = 2$. Then $G$ is an S-graph if and only if $|C(G)| = 1$.*

P r o o f. If $C(G) = \{c\}$, then the eccentricity of any vertex from $V(G) - \{c\}$ is equal to two. Thus $\mathrm{Peri}(G) = \mathrm{Cep}(G) = V(G) - \{c\}$, and $G$ is an S-graph.

Conversely, let $G$ be an S-graph. Let $|C(G)| \geqslant 2$ and $x, y \in C(G)$, $x \neq y$. Since $x, y \in \mathrm{Cep}(G)$ and no vertex from $C(G)$ can belong to $\mathrm{Peri}(G)$, we have a contradiction. This completes the proof. □

**Corollary 2.2.** *A graph $G$ with $r(G) = 1$ is an S-graph if and only if it is a complete graph $K_p$, $p \geqslant 2$, or has one central vertex.*

As mentioned above, for any two positive integers $a$ and $b$, $a \leqslant b \leqslant 2a$, there exists an S-graph with radius $a$ and diameter $b$. The following theorem shows that there exists no S-graph $G$ with $d(G) = r(G) + 1$ and $|C(G)| = 1$.

**Theorem 2.3.** *Let $G$ be an S-graph with $r(G) \neq d(G) \geqslant 3$ and $|C(G)| = 1$. Then $r(G) < d(G) - 1$.*

P r o o f. Let $G$ be an S-graph with $C(G) = \{c\}$ and $d(G) \geqslant 3$. Let $r(G) = d(G) - 1$. The eccentricity of any vertex $t \in N_G(c)$ is greater than $r(G)$, but $e_G(t) \leqslant d(G) = r(G) + 1$. Therefore $e_G(t) = d(G)$ and $t \in \mathrm{Peri}(G) = \mathrm{Cep}(G)$, which is a contradiction with $d_G(t, c) = 1 < r(G)$. This completes the proof. □

**Lemma 2.4.** *Let $G$ be an S-graph, $r(G) \geqslant 2$ and $r(G) \neq d(G)$. Then the distance of any two vertices from $C(G)$ in $G$ is less than $r(G)$ and $d(\langle V(G) - C(G)\rangle) \geqslant d(G)$.*

P r o o f. The distance of no two vertices from $C(G)$ can be equal to $r(G)$, since no vertex from $C(G)$ can belong to $\operatorname{Peri}(G)$.

If $e_{\langle V(G)-C(G)\rangle}(x) < d(G)$ for any vertex $x \in V(G) - C(G)$, then the eccentricity of any vertex from $V(G)$ in $G$ is less than $d(G)$, which is a contradiction. $\square$

**Theorem 2.5.** *Let $G$ be a graph with $d(G) = r(G) + 1$ and $r(G) \geqslant 2$. Then $G$ is an S-graph if and only if any vertex from $V(G) - C(G)$ is a center eccentric vertex of $G$.*

P r o o f. Let $G$ be an S-graph. Then $\operatorname{Peri}(G) = V(G) - C(G)$, because the eccentricity of any vertex from $V(G) - C(G)$ is equal to $d(G)$. Since $G$ is an S-graph, every vertex from $\operatorname{Peri}(G)$ is eccentric for some vertex from $C(G)$.

Conversely, let any vertex of $V(G) - C(G)$ be a center eccentric vertex of $G$. Since $\operatorname{Peri}(G) = V(G) - C(G)$, the graph $G$ is an S-graph. This completes the proof. $\square$

**Corollary 2.6.** *Let $G$ be a graph with $d(G) = 3$ and $r(G) = 2$. Then $G$ is an S-graph if and only if no vertex from $V(G) - C(G)$ is joined to all central vertices of $G$.*

**Theorem 2.7.** *Let $G$ be a graph with diameter four and radius two. Let $Q = \langle V(G) - C(G)\rangle$, where $C(G)$ is the center of $G$. Then $G$ is an S-graph if and only if*

1) *$\langle C(G)\rangle$ is a complete graph and*
2) *the cardinality of the set $T = \{x \in V(Q);\ N_G(x) \cap C(G) = \emptyset\}$ is at least two, and for every vertex $x \in T$ at least one vertex $y \in V(Q)$ such that $d_Q(x, y) \geqslant 4$ belongs to $T$.*

P r o o f. Let $G$ be an S-graph. As follows from Lemma 2.4, $\langle C(G)\rangle$ is a complete graph, and $d(Q) \geqslant 4$. Any vertex $t \in V(Q) - T$ has eccentricity $e_G(t) = 3$. Thus any vertex with eccentricity four belongs to $T$, i.e. $\operatorname{Peri}(G)$ is a subset of $T$. Since $G$ is an S-graph, $\operatorname{Peri}(G) = \operatorname{Cep}(G)$. But $T = \operatorname{Cep}(G)$, because $T$ consists of all center eccentric vertices of $G$. Obviously, $|\operatorname{Cep}(G)| \geqslant 2$. Let $x \in T$. If there is no vertex $y \in V(Q)$ such that $d_Q(x, y) \geqslant 4$ belongs to $T$, then $e_G(x) \leqslant 3$, which is a contradiction with $x \in \operatorname{Peri}(G)$.

Conversely, let 1) and 2) hold. As follows from 2), $d(Q) \geqslant 4$ and $T$ consists of all center eccentric vertices of $G$. Thus $\operatorname{Cep}(G) = T$. Let $x \in T$ and $y \in T$ be such that

$d_Q(x, y) \geqslant 4$. Then $e_G(x) = 4$, and $T$ consists of all diametral vertices of $G$. Thus $\mathrm{Peri}(G) = T$.

This completes the proof. □

There exists no self-centered S-graph with one central vertex. As follows from Theorem 2.3, there exists no S-graph $G$ with one central vertex and $r(G) = d(G) - 1$. Next we prove that for all other possible cases of radius and diameter there does exist an S-graph with one central vertex.

**Theorem 2.8.** *For each pair of positive integers $d$ and $r$, $r \geqslant 2$, $r + 2 \leqslant d \leqslant 2r$, there exists an S-graph $G$ with $r(G) = r$, $d(G) = d$ and $|C(G)| = 1$.*

Proof. If $d = 2r$, then we can use the graph formed by $P_{d+1}$.

Let $d = r + k$, $2 \leqslant k < r$. To construct an S-graph $G$ with the required properties we can proceed as follows.

Let $T_1, T_2, \ldots, T_m$, where $m = 4k - 2$, be disjoint copies of the trees from Fig. 2, such that $T_1$ and $T_{2k}$ are copies of the tree $H$, $T_k$ and $T_{3k-1}$ are copies of $S$ and all the other $T_i$, $1 \leqslant i \leqslant m$ are copies of $Q$.

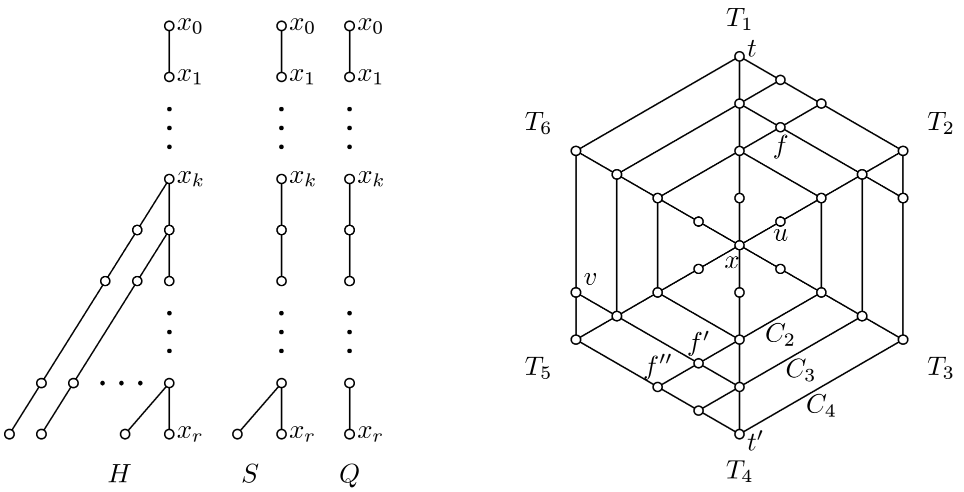



Fig. 2

Let the tree $T$ arise from $T_1, T_2, \ldots, T_m$ by the identification of all the corresponding copies of the vertex $x_0$ from $T_1, T_2, \ldots, T_m$ in this order, with one common vertex $x$.

Let $G$ be the graph which we get by forming a cycle $C_j$ on all vertices with the distance $j$ from $x$, for $j = k, k+1, \ldots, r$. (The corresponding graph $G$ with the cycles $C_2, C_3, C_4$ for $r = 4$ and $d = 6$ is in Fig. 2). There are $l = r - k + 1$ such cycles $C_j$

with lengths $2d-2(r-j+1)=2(k+j-1)$ for $j=k,k+1,\dots,r-1$, and $2d$ for $j=r$.

It is easy to check that the eccentricity of any vertex $t$ of $C_{2d}$ is equal to $d$ and this eccentricity is attained for the vertex $t'$ of $C_{2d}$ whose distance in $C_{2d}$ from $t$ is $d$ (Fig. 2).

Let $f$ be a vertex from $C_j$, $k \leqslant j \leqslant r-1$ and let $f'$ be the vertex from $C_j$ with distance $d-r+j-1=k+j-1$ from $f$ in $C_j$. Let $f''$ be a nearest vertex from $C_{2d}$ for the vertex $f'$ in $G$. Then $d_G(f,f'')=d-1=e_G(f)$.

Let $u$ be a vertex with a distance $i \leqslant k-1$ from $x$, and let $u$ belong to $T_n$, $1 \leqslant n \leqslant m$. Let $v$ be a vertex from $C_{2d}$ belonging to $T_{(n+2k-1) \bmod (4k-2)}$. Then $d_G(u,v)=r+i=e_G(u)$.

Thus $\mathrm{Peri}(G)$ consists of all vertices of $C_l$, $\mathrm{Cep}(G)=\mathrm{Peri}(G)$ and $C(G)=\{x\}$, which completes the proof. □

For any vertex $v$ of a connected graph $G$ there exists a spanning tree $T$ that is distance preserving from $v$, i.e. $d_T(v,u)=d_G(v,u)$ for any vertex $u \in V(G)$. If $G$ is an S-graph with one central vertex $x$, then its distance preserving spanning tree $T_x$ from $x$ is also an S-graph with $d(T_x)=2r(G)$, $C(T_x)=C(G)=\{x\}$ and $\mathrm{Peri}(T_x)=\mathrm{Cep}(T_x)=\mathrm{Peri}(G)=\mathrm{Cep}(G)$. Therefore any S-graph $G$ with one central vertex can be constructed by adding new edges to a proper tree of radius $r(G)$ and diameter $2r(G)$. On the other hand, not every tree $T$ with radius $r$ and one central vertex, is a distance preserving spanning tree of an S-graph with radius $r$, diameter $d(G)<d(T)$ and one central vertex. For example, $P_7$ cannot be supplemented (by adding new edges) to an S-graph $G$ with one central vertex, of diameter 5 and radius 3. As follows from the proof of Theorem 2.8 for the case $r(G)=3$ and $d(G)=5$ there exists such a distance preserving spanning tree given by the construction of $T$ as in Fig. 2.

Given an S-graph $G$ we will say that an edge $e \in E(G)$ is *superfluous* in $G$, if $G-e$ is also an S-graph with the same radius, diameter and peripherian as in $G$. An S-graph $G$ is said to be *critical*, if it has no superfluous edge. For example, $G$ is a critical S-graph with one central vertex, radius $r(G)$ and diameter $d(G)=2r(G)$ if and only if it is a tree with one central vertex, radius $r(G)$ and diameter $2r(G)$. We suggest to investigate critical S-graphs with one central vertex.

Buckley and Lewinter [2] studied two other interesting classes of graphs called F-graphs and L′-graphs. A connected graph $G$ is an F-graph if it has at least two central vertices, and for each pair of central vertices $u$ and $v$, $d(u,v)=r(G)$ holds. They showed that the only graph that is both an F-graph and an S-graph is $K_n$, $n \geqslant 2$.

If no diametral path of a connected graph $G$ contains a central vertex, $G$ is called an L$'$-graph.

Let $G$ be an S-graph with one central vertex $c$ and $d(G) < 2r$. Since $G$ is an S-graph, $\mathrm{Peri}(G) = \mathrm{Cep}(G)$ and the distance between $c$ and any diametral vertex is equal to $r(G)$. Since no diametral path in $G$ can contain the central vertex $c$, $G$ is an L$'$-graph. The opposite is not true as there are L$'$-graphs that are not S-graphs. One such graph $G$ with diameter 5 and radius 3 is shown in Fig. 3. The center of this graph is $C(G) = \{t\}$, $\mathrm{Cep}(G) = \{x, v, q, y\}$ and $\mathrm{Peri}(G) = \{x, y\}$.

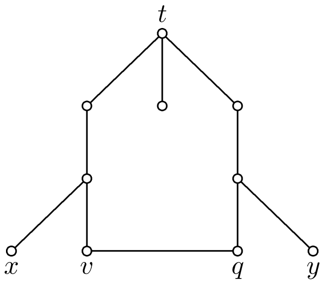



Fig. 3

Let $H$ and $Q$ be two graphs and let $s$ be an arbitrary vertex from $V(H)$. We say that the graph $G$ arose from $H$ by the substitution of $s$ by $Q$ if

$$V(G) = V(H) \cup V(Q) - \{s\}$$

and

$$E(G) = E(H) \cup E(Q) \cup \{xy;\ x \in V(Q) \text{ and } sy \in E(H)\} - \{sy;\ sy \in E(H)\}.$$

Gliviak [4] used this substitution to prove the existence of an S-graph $G$ with a prescribed radius $a$ and diameter $b$ containing a given graph $Q$ as an induced subgraph of $G$. To prove this assertion for the case $a \geqslant 2$ and $b = 2a - 1$ it is suggested to construct such an S-graph by the substitution of any central vertices of $P_{2a}$ by $G$ and for $a \geqslant 2$ and $b = 2a$ by the substitution of the central vertex of $P_{2a+1}$ by $G$. If $a = 2$ and $d(G) \geqslant 2$ then the resulting graph is not an S-graph. This can be corrected by substituting not a central but a diametral vertex of $P_{2a}$ and $P_{2a-1}$, respectively.

Next we formulate a more general theorem based on the substitution of a vertex of an S-graph.

**Theorem 2.9.** *Let $G$ be an S-graph with $r(G) \geqslant 3$ and $x \in V(G)$. Let $H$ be a graph disjoint with $G$. Let $G'$ be the graph, arising from $G$ by the substitution of $x$ by $H$. Then $G'$ is an S-graph, and*

1) $r(G) = r(G')$, $d(G) = d(G')$
2) $\mathrm{Cep}(G') = \mathrm{Peri}(G') = \mathrm{Cep}(G) = \mathrm{Peri}(G)$ *if* $x \notin \mathrm{Cep}(G)$
3) $\mathrm{Cep}(G') = \mathrm{Peri}(G') = \mathrm{Cep}(G) \cup V(H) - \{x\}$, *if* $x \in \mathrm{Cep}(G)$.

Proof. As follows from the substitution, $e_G(s) = e_{G'}(s)$ for any $s \in V(G)$ and $e_G(x) = e_{G'}(t)$ for any $t \in V(H)$. Thus $r(G) = r(G')$ and $d(G) = d(G')$.

Let $x \notin \mathrm{Cep}(G)$.Then $d_G(x,c) < r(G)$ for any central vertex $c \in C(G)$, and also $d_{G'}(t,c) < r(G)$ for any $t \in V(H)$. Any $s \in V(G)$ is a center eccentric vertex in $G$ if and only if it is a center eccentric vertex in $G'$. Thus $\mathrm{Cep}(G) = \mathrm{Cep}(G')$. Since $x \notin \mathrm{Cep}(G) = \mathrm{Peri}(G)$, the eccentricity of $x$ in $G$ is less than $d(G)$. Then also $e_{G'}(t) < d(G)$ for any $t \in V(H)$, and no vertex from $V(H)$ is a diametral vertex in $G'$. Any $s \in V(G)$ is a diametral vertex in $G$ if and only if it is a diametral vertex in $G'$. Thus $\mathrm{Peri}(G) = \mathrm{Peri}(G')$.

Let $x \in \mathrm{Cep}(G)$. Then $x$ is an eccentric center vertex in $G$, and any $t \in V(H)$ is an eccentric vertex in $G'$. Thus the set of the center eccentric vertices in $G'$ is $\mathrm{Cep}(G) \cup V(H) - \{x\} = \mathrm{Cep}(G')$. Since $G$ is an S-graph, $x \in \mathrm{Peri}(G)$. Then any vertex $t \in V(H)$ is a diametral vertex in $G'$. Therefore the set of diametral vertices in $G'$ is $\mathrm{Peri}(G') = \mathrm{Peri}(G) \cup V(H) - \{x\} = \mathrm{Cep}(G) \cup V(H) - \{x\}$. This completes the proof. □

**Theorem 2.10.** *Let $H$ be a graph. For any positive integers a,b such that $a + 2 \leqslant b \leqslant 2a$, $a \neq b$ and $a \geqslant 3$ there exists an S-graph $G$ with $r(G) = a$, $d(G) = b$ and $\langle C(G)\rangle = H$.*

Proof. Let $Q$ be an S-graph with radius $a$ and diameter $b$ with one center vertex $c$. Let $G$ be the graph which we get by the substitution of $c$ by $H$ in $G$. As follows from Theorem 2.9, $G$ is an S-graph with $r(G) = r(Q)$, $d(Q) = d(G)$, $\mathrm{Peri}(Q) = \mathrm{Cep}(Q) = \mathrm{Peri}(G) = \mathrm{Cep}(G)$ and $\langle C(G)\rangle = H$. □

**Theorem 2.11.** *Let $G_1, G_2, \ldots, G_n$, $n \geqslant 2$, be disjoint S-graphs with $r(G_i) = r \geqslant 2$ and $C(G_i) = \{x_i\}$ for $i = 1, 2, \ldots, n$. Let $H$ be a self-centered graph disjoint with $G_i$, $1 \leqslant i \leqslant n$ and $V(H) = \{t_1, t_2, \ldots, t_n\}$. Let $G$ be the graph that we construct by identifying the corresponding pairs of vertices $t_i$ and $x_i$ with a new vertex $u_i$, for $i = 1, 2, \ldots, n$. Then the graph $G$ is an S-graph with $d(G) = 2r + r(H)$, $r(G) = r + r(H)$, $C(G) = V(H)$, $\mathrm{Peri}(G) = \bigcup_{i=1}^{n} \mathrm{Peri}(G_i)$ and $\mathrm{Cep}(G) = \bigcup_{i=1}^{n} \mathrm{Cep}(G_i)$*

Proof. For every $i = 1, 2, \ldots, n$ the eccentricity $e_G(u_i) = r + r(H)$ and the eccentricity of any vertex $z \in V(G)$, $z \neq u_i$ for $i = 1, 2, \ldots, n$ is $e_G(z) > r + r(H)$. Thus $r(G) = r + r(H)$ and $C(G) = \{u_1, u_2, \ldots, u_n\}$.

The distance of any two vertices $x, y \in V(G)$ is $d_G(x,y) \leqslant 2r + r(H)$. The equality holds only if $x$ and $y$ are center eccentric vertices of the graphs $G_i$ and $G_j$, respectively, for which $d_G(u_i, u_j) = r(H)$. Thus $\operatorname{Peri}(G) = \bigcup_{i=1}^{n} \operatorname{Peri}(G_i)$.

Similarly, the distance of a vertex $x \in V(G)$ from $u_i$, $i \in \langle 1, n\rangle$ is $d_G(x, u_i) = r + r(H)$ only if $x$ is a center eccentric vertex of $G_j$ for which $d_G(u_i, u_j) = r + r(H)$. Therefore $\operatorname{Cep}(G) = \bigcup_{i=1}^{n} \operatorname{Cep}(G_i)$. □

In the conclusion we give an estimate of the number of edges of an S-graph.

**Theorem 2.12.** *Let $G$ be an* S-*graph with $p$ vertices and $q$ edges. Let $d(G) = d \geqslant 3$. Then $p - 1 \leqslant q \leqslant d + 1/2(p - d + 1)(p - d + 4)$.*

P r o o f. Ore [5] proved this upper bound for any connected graph with diameter $d \geqslant 3$. As follows from the example in Fig. 4, this upper bound is attained also for S-graphs. The lower bound is obvious and it is attained for $P_{d+1}$. □

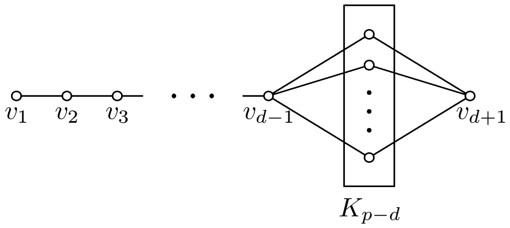



Fig. 4

*References*

[1] *Chartrand, G., Lesniak, L.*: Graphs and Digraphs. Wadsworth and Brooks, Monterey, California, 1986.
[2] *Buckley, F., Lewinter, M.*: Minimal graph embeddings, eccentric vertices and the peripherian. Proc. Fifth Carribean Conference on Combinatorics and Computing. University of the West Indies, 1988, pp. 72–84.
[3] *Buckley, F., Lewinter, M.*: Graphs with all diametral paths through distant central vertices. Math. Comput. Modelling *17* (1993), 35–41.
[4] *Gliviak, F.*: Two classes of graphs related to extremal eccentricities. Math. Bohem. *122* (1997), 231–241.
[5] *Ore, O.*: Diameters in graphs. J. Combin. Theory *5* (1968), 75–81.

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