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ON CHARACTERIZATIONS OF SOME COVERING PROPERTIES IN L-FUZZY TOPOLOGICAL SPACES

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Abstract. In this work, various characterizations of some weaker and stronger forms of L-fuzzy compactness are obtained in L-fuzzy topological spaces, where L is a fuzzy lattice.

1. Introduction

The notion of compactness is one of the most important concepts in general topology. Therefore, the problem of generalization of the classical compactness to fuzzy topological spaces has been intensively discussed over the past 30 years. Many papers on fuzzy compactness have been published and various kinds of fuzzy compactness have been presented and studied. Among these compactness', the fuzzy compactness in L-fuzzy topological spaces introduced by Warner and McLean [10] and extended to arbitrary fuzzy sets by Kudri [4] possesses more satisfactory properties than others. Good extensions of some weaker and stronger fuzzy covering properties (e.g. almost, near, S-closed, strong) were introduced and studied by Kudri and Warner. As a result, it is sufficient for an adequate compactness theory in L-fuzzy topology, being a good extension [10], defined on arbitrary L-fuzzy sets [4], proposed other fuzzy covering properties [5,6], and with a general Tychonoff theorem.

In the present paper, we first provide a different description for the L-fuzzy compactness and its some weaker and stronger forms (almost, near, strong compactness and S-closedness) and then obtain characterizations of these fuzzy covering properties by α-nets.

2. Preliminaries

We assume that the reader is familiar with the usual notations and most concepts of fuzzy topology and lattice theory.

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Throughout this paper X will be a non-empty ordinary set and $L = L(\leqslant, \vee, \wedge, ')$ will denote a fuzzy lattice, i.e. a completely distributive lattice with a smallest element $\mathbf{0}$ and a largest element $\mathbf{1}$ $(0 \neq 1)$ and with an order reversing involution $a \to a'$ $(a \in L)$. We shall denote by L^X the lattice of all L-fuzzy subsets of X. The interior and closure of an L-fuzzy set f will be denoted respectively by $\mathrm{int}(f)$ and $\mathrm{cl}(f)$. $Pr(L)$ and $M(L)$ will denote respectively the set of all the prime elements and the set of all the coprime elements of L [3].

Let (X, τ) be an L-fuzzy topological space (for short *L-fts*) and let D be a directed set. A net in (X, τ) is a function $S\colon D \to M(L^X)$. For $m \in D$, we shall denote $S(m)$ by S_m and the net S by $(S_m)_{m \in D}$. The net S is called a net contained in f if and only if $S_m \leqslant f$ for each $m \in D$, i.e. $h(S_m) \leqslant f(\operatorname{Supp} S_m)$ for each $m \in D$ [4].

Let $\alpha \in M(L)$. A net $(S_m)_{m \in D}$ is called an α-net if and only if for each $\gamma \in \beta^*(\alpha)$ the net $h(S) = (h(S_m))_{m \in D}$ is eventually greater than γ. If $h(S_m) = \alpha$ for all $m \in D$, then we shall say that $(S_m)_{m \in D}$ is a constant α-net [4].

3. Some characterizations

In this section, we present various characterizations of the covering properties (compactness [4], almost compactness [5], near compactness [5], strong compactness [6] and S-closedness [6]) in L-fuzzy topological spaces.

LEMMA 3.1. *Let (X, τ) be an L-fts and let $p \in pr(L)$. Then the family*

$$\phi_p(\tau) = \{\, f^{-1}(\{t \in L : t \nleqslant p\}) : f \in \tau \,\}$$

is an ordinary topology on X.

Proof. (i) $\emptyset$, $X \in \phi_p(\tau)$ because $\mathbf{0}, \mathbf{1} \in \tau$.

(ii) Let $f^{-1}(\{t \in L : t \nleqslant p\})$, $g^{-1}(\{t \in L : t \nleqslant p\}) \in \phi_p(\tau)$ where $f, g \in \tau$. Then $f \wedge g \in \tau$. Since $p \in pr(L)$, by Lemma 3.2 in [1], we have

$$f^{-1}(\{t \in L : t \nleqslant p\}) \cap g^{-1}(\{t \in L : t \nleqslant p\}) = (f \wedge g)^{-1}(\{t \in L : t \nleqslant p\}).$$

Hence, $f^{-1}(\{t \in L : t \nleqslant p\}) \cap g^{-1}(\{t \in L : t \nleqslant p\}) \in \phi_p(\tau)$.

(iii) Let $f_i^{-1}(\{t \in L : t \nleqslant p\}) \in \phi_p(\tau)$ where $f_i \in \tau$, $i \in I$. Then $\bigvee_{i \in I} f_i \in \tau$. On the other hand, by Lemma 3.3 in [1], we have $(\bigvee_{i \in I} f_i)^{-1}(\{t \in L : t \nleqslant p\}) = \bigcup_{i \in I} f_i^{-1}(\{t \in L : t \nleqslant p\})$ and therefore $\bigcup_{i \in I} f_i^{-1}(\{t \in L : t \nleqslant p\}) \in \phi_p(\tau)$.

Consequently, $\phi_p(\tau)$ is an ordinary topology on X. ■

The next theorem shows that the compactness in an L-fuzzy topological space (X, τ) is characterized by the compactness in the ordinary topological spaces $(X, \phi_p(\tau))$, where $p \in pr(L)$.

THEOREM 3.2. *Let (X, τ) be an L-fts and $g \in L^X$. The L-fuzzy set g is compact if and only if for every $p \in pr(L)$, $G_p = \{x \in X : g(x) \geqslant p'\}$ is compact in the ordinary topological space $(X, \phi_p(\tau))$.*

Proof. *Necessity.* Let $p \in pr(L)$ and let $(A_i)_{i\in J}$ be an open covering of G_p, where $A_i = f_i^{-1}(\{t \in L : t \nleqslant p\})$ and $f_i \in \tau$ for each $i \in J$. Then , $G_p \subseteq \bigcup_{i\in J} f_i^{-1}(\{t \in L : t \nleqslant p\})$, i.e., $(\bigvee_{i\in J} f_i)(x) \nleqslant p$ for all $x \in X$ with $g(x) \geqslant p'$. Due to the compactness of g , there is a finite subset F of J such that $(\bigvee_{i\in F} f_i)(x) \nleqslant p$ for all $x \in X$ with $g(x) \geqslant p'$, i.e., for all $x \in G_p$. Hence, $G_p \subseteq \bigcup_{i\in F} A_i$ and so G_p is compact in $(X, \phi_p(\tau))$.

Sufficiency. Let $p \in pr(L)$ and let $(f_i)_{i\in J}$ be a p-level open cover of g. Then, $(\bigvee_{i\in J} f_i)(x) \nleqslant p$ for all $x \in X$ with $g(x) \geqslant p'$. Then, $G_p \subseteq \bigcup_{i\in J} f_i^{-1}(\{t \in L : t \nleqslant p\})$ and $f_i^{-1}(\{t \in L : t \nleqslant p\}) \in \phi_p(\tau)$ for each $i \in J$. By the compactness of G_p in $(X, \phi_p(\tau))$, there is a finite subset F of J such that $G_p \subseteq \bigcup_{i\in F} f_i^{-1}(\{t \in L : t \nleqslant p\})$ which implies that $(\bigvee_{i\in F} f_i)(x) \nleqslant p$ for all $x \in X$ with $g(x) \geqslant p'$. Hence, g is compact in (X, τ). ■

COROLLARY 3.3. *An L-fuzzy topological space (X, τ) is compact if and only if for every $p \in pr(L)$ the ordinary topological space $(X, \phi_p(\tau))$ is compact.*

Proof. This follows directly from the previous theorem. ■

The next theorem provides a different description for the compactness in L-fuzzy topological spaces.

THEOREM 3.4. *Let (X, τ) be an L-fts and $g \in L^X$. The L-fuzzy set g is compact if and only if for every $p \in pr(L)$ and every collection $(f_i)_{i\in J}$ of open L-fuzzy sets with $(\bigvee_{i\in J} f_i \vee g')(x) \nleqslant p$ for all $x \in X$, there is a finite subset F of J such that $(\bigvee_{i\in F} f_i \vee g')(x) \nleqslant p$ for all $x \in X$.*

Proof. *Necessity.* Let $p \in pr(L)$ and let $(f_i)_{i\in J}$ be a collection of open L-fuzzy sets with $(\bigvee_{i\in J} f_i \vee g')(x) \nleqslant p$ for all $x \in X$. Then, $(\bigvee_{i\in J} f_i)(x) \nleqslant p$ for all $x \in X$ with $g(x) \geqslant p'$. Since g is compact, there is a finite subset F of J such that $(\bigvee_{i\in F} f_i)(x) \nleqslant p$ for all $x \in X$ with $g(x) \geqslant p'$.

Take an arbitrary $x \in X$. If $g'(x) \leqslant p$ then $g'(x) \vee (\bigvee_{i\in F} f_i)(x) = (\bigvee_{i\in F} f_i \vee g')(x) \nleqslant p$ because $(\bigvee_{i\in F} f_i)(x) \nleqslant p$. If $g'(x) \nleqslant p$ then we have $g'(x) \vee (\bigvee_{i\in F} f_i)(x) = (\bigvee_{i\in F} f_i \vee g')(x) \nleqslant p$. Thus, we have $(\bigvee_{i\in F} f_i \vee g')(x) \nleqslant p$ for all $x \in X$.

Sufficiency. Let $p \in pr(L)$ and let $(f_i)_{i\in J}$ be a p-level open cover of g. Then, $(\bigvee_{i\in J} f_i)(x) \nleqslant p$ for all $x \in X$ with $g(x) \geqslant p'$. Hence, $(\bigvee_{i\in J} f_i \vee g')(x) \nleqslant p$ for all $x \in X$. From the hypothesis, there is a finite subset F of J such that $(\bigvee_{i\in F} f_i \vee g')(x) \nleqslant p$ for all $x \in X$. Then, $(\bigvee_{i\in F} f_i)(x) \nleqslant p$ for all $x \in X$ with $g'(x) \leqslant p$. Hence, g is compact. ■

As stated in the next theorems similar descriptions are valid for the strong, almost and near compactness' and S-closedness in L-fuzzy topological spaces.

THEOREM 3.5. *Let (X, τ) be an L-fts and $g \in L^X$. The L-fuzzy set g is strong compact if and only if for every $p \in pr(L)$ and every collection $(f_i)_{i\in J}$ of pre-open L-fuzzy sets with $(\bigvee_{i\in J} f_i \vee g')(x) \nleqslant p$ for all $x \in X$, there is a finite subset F of J such that $(\bigvee_{i\in F} f_i \vee g')(x) \nleqslant p$ for all $x \in X$.*

Proof. This is very similar to the proof of the previous theorem. ■

THEOREM 3.6. *Let (X,τ) be an L-fts and $g \in L^X$. The L-fuzzy set g is almost compact if and only if for every $p \in pr(L)$ and every collection $(f_i)_{i\in J}$ of open L-fuzzy sets with $(\bigvee_{i\in J} f_i \vee g')(x) \nleqslant p$ for all $x \in X$, there is a finite subset F of J such that $(\bigvee_{i\in F} \mathrm{cl}(f_i) \vee g')(x) \nleqslant p$ for all $x \in X$.*

Proof. This is very similar to the proof of Theorem 3.4. ■

THEOREM 3.7. *Let (X,τ) be an L-fts and $g \in L^X$. The L-fuzzy set g is nearly compact if and only if for every $p \in pr(L)$ and every collection $(f_i)_{i\in J}$ of open L-fuzzy sets with $(\bigvee_{i\in J} f_i \vee g')(x) \nleqslant p$ for all $x \in X$, there is a finite subset F of J such that $(\bigvee_{i\in F} \mathrm{int}(\mathrm{cl}(f_i)) \vee g')(x) \nleqslant p$ for all $x \in X$.*

Proof. This is very similar to the proof of Theorem 3.4. ■

THEOREM 3.8. *Let (X,τ) be an L-fts and $g \in L^X$. The L-fuzzy set g is S-closed if and only if for every $p \in pr(L)$ and every collection $(f_i)_{i\in J}$ of semi-open L-fuzzy sets with $(\bigvee_{i\in J} f_i \vee g')(x) \nleqslant p$ for all $x \in X$, there is a finite subset F of J such that $(\bigvee_{i\in F} \mathrm{cl}(f_i) \vee g')(x) \nleqslant p$ for all $x \in X$.*

Proof. This is very similar to the proof of Theorem 3.4. ■

With the next four theorems we obtain characterizations of the almost, near and strong compactness' and S-closedness in terms of closed (pre-closed, semi-closed) L-fuzzy sets.

THEOREM 3.9. *Let (X,τ) be an L-fts and $g \in L^X$. The L-fuzzy set g is almost compact if and only if for every $\alpha \in M(L)$ and every collection $(h_i)_{i\in J}$ of closed L-fuzzy sets with $(\bigwedge_{i\in J} h_i)(x) \ngeqslant \alpha$ for all $x \in X$ with $g(x) \geqslant \alpha$, there is a finite subset F of J such that $(\bigwedge_{i\in F} \mathrm{int}(h_i))(x) \ngeqslant \alpha$ for all $x \in X$ with $g(x) \geqslant \alpha$.*

Proof. Using the fact $(\mathrm{int}(h))' = \mathrm{cl}(h')$ for every $h \in L^X$ [7], this follows easily from the definition of almost compactness. ■

THEOREM 3.10. *Let (X,τ) be an L-fts and $g \in L^X$. The L-fuzzy set g is nearly compact if and only if for every $\alpha \in M(L)$ and every collection $(h_i)_{i\in J}$ of closed L-fuzzy sets with $(\bigwedge_{i\in J} h_i)(x) \ngeqslant \alpha$ for all $x \in X$ with $g(x) \geqslant \alpha$, there is a finite subset F of J such that $(\bigwedge_{i\in F} \mathrm{cl}(\mathrm{int}(h_i)))(x) \ngeqslant \alpha$ for all $x \in X$ with $g(x) \geqslant \alpha$.*

Proof. Using the fact $(\mathrm{int}(h))' = \mathrm{cl}(h')$ and $(\mathrm{cl}(h))' = \mathrm{int}(h')$ for every $h \in L^X$ [7], this follows easily from the definition of nearly compactness. ■

THEOREM 3.11. *Let (X,τ) be an L-fts and $g \in L^X$. The L-fuzzy set g is S-closed if and only if for every $\alpha \in M(L)$ and every collection $(h_i)_{i\in J}$ of semi-closed L-fuzzy sets with $(\bigwedge_{i\in J} h_i)(x) \ngeqslant \alpha$ for all $x \in X$ with $g(x) \geqslant \alpha$, there is a finite subset F of J such that $(\bigwedge_{i\in F} \mathrm{int}(h_i))(x) \ngeqslant \alpha$ for all $x \in X$ with $g(x) \geqslant \alpha$.*

Proof. Using the fact $(\mathrm{int}(h))' = \mathrm{cl}(h')$ for every $h \in L^X$ [7], this follows easily from the definition of S-closedness. ■

THEOREM 3.12. *Let (X,τ) be an L-fts and $g \in L^X$. The L-fuzzy set g is strong compact if and only if for every $\alpha \in M(L)$ and every collection $(h_i)_{i\in J}$ of* *pre-closed L-fuzzy sets with* $(\bigwedge_{i\in J} h_i)(x) \not\geqslant \alpha$ *for all* $x \in X$ *with* $g(x) \geqslant \alpha$, *there is a finite subset* F *of* J *such that* $(\bigwedge_{i\in F} \mathrm{int}(h_i))(x) \not\geqslant \alpha$ *for all* $x \in X$ *with* $g(x) \geqslant \alpha$.

Proof. This follows immediately from the definition of strong compactness. ■

Now we define θ-cluster point, δ-cluster point, semi-θ-cluster point and pre-cluster point of a net $(S_m)_{m\in D}$ and then characterize almost compactness, near compactness, S-closedness and strong compactness by α-nets.

DEFINITION 3.13. Let (X, τ) be an L-fts, let x_α be an L-fuzzy point in $M(L^X)$ and let $S = (S_m)_{m\in D}$ be a net. The L-fuzzy point x_α is called a:

(i) *θ-cluster point* of S iff for each closed L-fuzzy set f with $f(x) \not\geqslant \alpha$ and for all $n \in D$, there is $m \in D$ such that $m \geqslant n$ and $S_m \not\leqslant \mathrm{int}(f)$, i.e. $h(S_m) \not\leqslant \mathrm{int}(f)(\mathrm{Supp}\, S_m)$.

(ii) *δ-cluster point* of S iff for each closed L-fuzzy set f with $f(x) \not\geqslant \alpha$ and for all $n \in D$, there is $m \in D$ such that $m \geqslant n$ and $S_m \not\leqslant \mathrm{cl}(\mathrm{int}(f))$.

(iii) *semi-θ-cluster point* of S iff for each semi-closed L-fuzzy set f with $f(x) \not\geqslant \alpha$ and for all $n \in D$, there is $m \in D$ such that $m \geqslant n$ and $S_m \not\leqslant \mathrm{int}(f)$.

(iv) *pre-cluster point* of S iff for each pre-closed L-fuzzy set f with $f(x) \not\geqslant \alpha$ and for all $n \in D$, there is $m \in D$ such that $m \geqslant n$ and $S_m \not\leqslant f$.

THEOREM 3.14. *Let* (X, τ) *be an L-fts and* $g \in L^X$. *The L-fuzzy set* g *is almost compact if and only if every constant* α*-net contained in* g *has a* θ*-cluster point* $x_\alpha \in M(L^X)$, *with height* α, *contained in* g, *for each* $\alpha \in M(L)$.

Proof. Necessity. Let $\alpha \in M(L)$ and let $(S_m)_{m\in D}$ be a constant α-net contained in g without any θ-cluster point with height α contained in g. Then, for each $x \in X$ with $g(x) \geqslant \alpha$, x_α is not a θ-cluster point of $(S_m)_{m\in D}$, i.e. there are $n_x \in D$ and a closed L-fuzzy set f_x with $f_x(x) \not\geqslant \alpha$ and $S_m \leqslant \mathrm{pint}(f_x)$ for each $m \geqslant n_x$.

Let $x^1, \ldots, x^k$ be elements of X with $g(x^i) \geqslant \alpha$ for each $i \in \{1, \ldots, k\}$. Then, there are $n_{x_1}, \ldots, n_{x_k} \in D$ and closed L-fuzzy sets f_{x_i} with $f_{x_i}(x^i) \not\geqslant \alpha$ and $S_m \leqslant \mathrm{int}(f_{x_i})$ for each $m \geqslant n_{x_i}$ and for each $i \in \{1, \ldots, k\}$. Since D is a directed set, there is $n_0 \in D$ such that $n_0 \geqslant n_{x_i}$ for every $i \in \{1, \ldots, k\}$ and $S_m \leqslant \mathrm{int}(f_{x_i})$ for $i \in \{1, \ldots, k\}$ and for each $m \geqslant n_0$.

Now consider the family $\Gamma = (f_x)_{x\in X}$ with $g(x) \geqslant \alpha$. Then $(\bigwedge_{f_x\in\Gamma} f_x)(y) \not\geqslant \alpha$ for all $y \in X$ with $g(y) \geqslant \alpha$ because $f_y(y) \not\geqslant \alpha$. We also have that for any finite subfamily $\Lambda = \{f_{x_1}, \ldots, f_{x_k}\}$ of Γ, there is $y \in X$ with $g(y) \geqslant \alpha$ and $(\bigwedge_{i=1}^k f_{x_i})(y) \geqslant \alpha$ since $S_m \leqslant \bigwedge_{i=1}^k \mathrm{int}(f_{x_i})$ for each $m \geqslant n_0$ because $S_m \leqslant \mathrm{int}(f_{x_i})$ for each $i \in \{1, \ldots, k\}$ and for each $m \geqslant n_0$. Hence, by the previous theorem, g is not almost compact.

Sufficiency. Suppose that g is not almost compact. Then, by the previous theorem, there exist $\alpha \in M(L)$ and a collection $\Gamma = (f_i)_{i\in J}$ of closed L-fuzzy sets with $(\bigwedge_{i\in J} f_i)(x) \not\geqslant \alpha$ for all $x \in X$ with $g(x) \geqslant \alpha$, but for any finite subfamily β of Γ there is $x \in X$ with $g(x) \geqslant \alpha$ and $(\bigwedge_{i\in\beta} \mathrm{int}(f_i))(x) \geqslant \alpha$.

Consider the family of all finite subsets of Γ, $2^{(\Gamma)}$, with the order $\beta_1 \leqslant \beta_2$ if and only if $\beta_1 \subseteq \beta_2$. Then $2^{(\Gamma)}$ is a directed set. So, writing x_α as S_β for every $\beta \in 2^{(\Gamma)}$, $(S_\beta)_{\beta \in 2^{(\Gamma)}}$ is a constant α-net contained in g because the height of S_β for all $\beta \in 2^{(\Gamma)}$ is α and $S_\beta \leqslant g$ for all $\beta \in 2^{(\Gamma)}$, i.e. $g(x) \geqslant \alpha$. $(S_\beta)_{\beta \in 2^{(\Gamma)}}$ also satisfies the condition that for each closed L-fuzzy set $f_i \in \beta$ we have $x_\alpha = S_\beta \leqslant \mathrm{int}(f_i)$.

Let $y \in X$ with $g(y) \geqslant \alpha$. Then, $(\bigwedge_{i \in J} f_i)(y) \ngeqslant \alpha$, i.e. there exists $j \in J$ with $f_j(y) \ngeqslant \alpha$. Let $\beta_0 = \{f_j\}$. So, for any $\beta \geqslant \beta_0$, $S_\beta \leqslant \bigwedge_{f_i \in \beta} \mathrm{int}(f_i) \leqslant \bigwedge_{f_i \in \beta_0} \mathrm{int}(f_i) = \mathrm{int}(f_j)$. Thus, we got a closed L-fuzzy set f_j with $f_j(y) \ngeqslant \alpha$ and $\beta_0 \in 2^{(\Gamma)}$ such that for any $\beta \geqslant \beta_0$ $S_\beta \leqslant \mathrm{int}(f_j)$, that means that $y_\alpha \in M(L^X)$ is not a θ-cluster point of $(S_\beta)_{\beta \in 2^{(\Gamma)}}$ for all $y \in X$ with $g(y) \geqslant \alpha$. Hence the constant α-net $(S_\beta)_{\beta \in 2^{(\Gamma)}}$ has no θ-cluster point with height α, contained in g. ∎

Corollary 3.15. *An L-fts (X, τ) is almost compact if and only if every constant α-net in (X, τ) has a θ-cluster with height α.*

Proof. This follows immediately from the previous theorem. ∎

Theorem 3.16. *Let (X, τ) be an L-fts and $g \in L^X$. The L-fuzzy set g is nearly compact if and only if every constant α-net contained in g has a δ-cluster point $x_\alpha \in M(L^X)$, with height α, contained in g, for each $\alpha \in M(L)$.*

Proof. Using Theorem 3.10, this is very similar to the proof of Theorem 3.14. ∎

Corollary 3.17. *An L-fts (X, τ) is nearly compact if and only if every constant α-net in (X, τ) has a δ-cluster with height α.*

Proof. This is an immediate of the previous theorem. ∎

Theorem 3.18. *Let (X, τ) be an L-fts and $g \in L^X$. The L-fuzzy set g is S-closed if and only if every constant α-net contained in g has a semi-θ-cluster point $x_\alpha \in M(L^X)$, with height α, contained in g, for each $\alpha \in M(L)$.*

Proof. Using Theorem 3.11, this is very similar to the proof of Theorem 3.14. ∎

Corollary 3.19. *An L-fts (X, τ) is S-closed if and only if every constant α-net in (X, τ) has a semi-θ-cluster with height α.*

Proof. This follows immediately from the previous theorem. ∎

Theorem 3.20. *Let (X, τ) be an L-fts and $g \in L^X$. The L-fuzzy set g is strong compact if and only if every constant α-net contained in g has a pre-cluster point $x_\alpha \in M(L^X)$, with height α, contained in g, for each $\alpha \in M(L)$.*

Proof. Using Theorem 3.12, this is very similar to the proof of Theorem 3.14. ∎

Corollary 3.21. *An L-fts (X, τ) is strong compact if and only if every constant α-net in (X, τ) has a pre-cluster with height α.*

Proof. This follows immediately from the previous theorem. ∎

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