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A NOTE ON LIMITING BEHAVIOR OF SOLUTIONS OF ONE-DIMENSIONAL NONLINEAR STOCHASTIC WAVE EQUATION WITH FUNCTIONALS OF THE WHITE NOISE AS INITIAL DATA

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Abstract. The limitting behaviour of solutions to stochastic wave equations with singularities represented by stochastic terms is considered. In cases when the initial data are the white noise process or its functionals it is proved that the triviality effect appears.

1. Introduction

The modeling of many systems by differential equations requires random parameters. In such cases very often the white noise procces apperas. The appearance of the white noise process in the equations is the reason for treating those problems in the framework of the Colombeau theory. Indeed, it is well known that the white noise process can not be defined in a classical way but only as a generalized stochastic process. But working with generalized stochastic processes involves distribution spaces which are not suitable for multiplication and thus for dealing with nonlinear stochastic PDEs. In order to overcome the multiplication problem we use the theory of Colombeau generalized function spaces (see [3], [6]). This is also done in papers [19], [20], [23] and in a similar way in paper [1].

In this paper we consider the wave equations with the white noise process or its functionals as the initial data. We are interested in the limiting behaviour of solutions. Studying the limiting behavior of generalized solutions to (not only stochastic) PDEs is very important but usually not trivial. Here will answer some of the questions which can arise.

At the beginning of the paper some results given by Oberguggenberger and Ruso about the existence of solutions and their limiting behaviour in case when the initial data is simply the white noise process will be exposed. After that, we will consider some problems of the same kind but with the functional of the white noise

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process as the initial data. More results about limiting behavior of solutions to this equation with some other initial data will be exposed in the paper [22] which is still in preparation.

We remark that the case when the nonlinear part is not Lipschitz is considered in [14] and [15]. It is out of scope of this paper.

We study the Cauchy problem

$$(\partial_t^2 - \partial_x^2)U(x,t) = F(U(x,t)), \qquad U(x,0) = H(x) = h(\dot{W}(x))$$

in the framework of algebra of Colombeau stochastic generalized processes. We suppose that nonlinear part F is smooth and together with all derivatives growing at most polynomially at infinity, globally Lipschitz, bounded function and that H is a Colombeau generalized stochastic process, a functional of the white noise process $\dot{W}$.

2. Preliminaries

We recall some fundamentals of stochastic analysis we will use later for solving equations in the framework of Colombeau generalized functions.

Let (Ω, Σ, μ) be a probability space. A weakly measurable mapping $X\colon \Omega \to \mathcal{D}'(\mathbb{R}^d)$ is called a generalized stochastic process on $\mathbb{R}^d$. The space of generalized stochastic processes will be denoted by $\mathcal{D}'_\Omega(\mathbb{R}^d)$. The characteristic functional of a process X is $C_X(\varphi) = \int e^{i\langle X(\omega),\varphi\rangle}\, d\mu(\omega)$, $\varphi \in \mathcal{D}(\mathbb{R}^d)$.

Take the probability space Ω to be the space of tempered distributions $\mathcal{S}'(\mathbb{R}^d)$ and Σ to be the Borel σ-algebra generated by the weak topology. Then there is a unique probability measure μ on (Ω, Σ) such that

$$\int e^{i\langle X(\omega),\varphi\rangle}\, d\mu(\omega) = e^{-\frac{1}{2}\|\varphi\|^2_{L^2(\mathbb{R}^d)}}, \;\; \varphi \in \mathcal{S}(\mathbb{R}^d).$$

This result follows from the Bochner-Minlos theorem (see [7] or [8]).

White noise $\dot{W}\colon \Omega \to \mathcal{D}'(\mathbb{R}^d)$ is the identity mapping $\dot{W}(\omega) = \omega$, i.e., $\langle \dot{W}(\omega), \varphi\rangle = \langle \omega, \varphi\rangle$, $\varphi \in \mathcal{D}(\mathbb{R}^d)$. It is a generalized Gaussian process with mean zero and variance $V(\dot{W}(\varphi)) = E(\dot{W}(\varphi)^2) = \|\varphi\|^2_{L^2(\mathbb{R}^d)}$, where E denotes expectation. Its covariance is the bilinear functional $E(\dot{W}(\varphi)\dot{W}(\psi)) = \int_{\mathbb{R}^d} \varphi(y)\psi(y)\, dy$ represented by Dirac's measure on the diagonal $\mathbb{R}^d \times \mathbb{R}^d$, showing the singular nature of white noise. It is well known that the white noise on $\mathbb{R}^d$ can be viewed as the d-fold distributional derivative of (generalized) Wiener process.

Let φ_ε be a mollifier given by

$$\varphi_\varepsilon(y) = \frac{1}{\varepsilon^d}\varphi\left(\frac{y}{\varepsilon}\right), \;\; \varphi \in \mathcal{D}(\mathbb{R}^d), \;\; \int \varphi(y)\, dy = 1.$$

A net φ_ε is called a nonnegative model delta net.

Smoothed white noise process on $\mathbb{R}^d$ is defined as $\dot{W}_\varepsilon(x) = \langle \dot{W}(y), \varphi_\varepsilon(x-y)\rangle$, where $\dot{W}\colon \Omega \to \mathcal{D}'(\mathbb{R}^d)$ is white noise process on $\mathbb{R}^d$ and φ_ε is a nonnegative model delta net.

Finally, we introduce a positive noise process. The analysis of positive noise viewed as a Wick exponential is done in detail in [8]. Here we consider the smoothed positive noise process. We confine ourselves to the one-dimensional case since that is the case investigated in applications. Smoothed positive noise process $W_\varepsilon^+(x)$ on $\mathbb{R}$ is defined as

$$W_\varepsilon^+(x) = \exp\Big(\dot{W}_\varepsilon(x) - \frac{1}{2}\,\|\varphi_\varepsilon\|_{L^2}^2\Big), \tag{1}$$

where $\dot{W}_\varepsilon$ is the smoothed white noise process on $\mathbb{R}$ and φ_ε is a nonnegative model delta net. We will show later that this is the process with mean value 1 and variance $V(W_\varepsilon^+) = e^{\sigma_\varepsilon^2} - 1$, where $\sigma_\varepsilon^2 = \|\varphi_\varepsilon\|_{L^2}^2$.

Let us now recall some basic facts from the Colombeau theory that we will need here. For O, an open subset of $\mathbb{R}^n$, we define $\mathcal{E}(O)$ as the space of all mappings $G : (0,1) \times O \to \mathbb{C}$ such that $G(\varepsilon, \cdot) = G_\varepsilon \in C^\infty(O)$, $\varepsilon > 0$.

$\mathcal{E}_M([0,T) \times \mathbb{R}^n)$ is the space of all $G_\varepsilon \in \mathcal{E}([0,T) \times \mathbb{R}^n)$ with the property that for all $T > 0$ and $\alpha \in \mathbb{N}_0^n$ there exists $N \in \mathbb{N}$ such that $\|\partial^\alpha G_\varepsilon\|_{L^\infty}$ has a moderate bound, i.e., $\|\partial^\alpha G_\varepsilon\|_{L^\infty([0,T)\times\mathbb{R}^n)} = \mathcal{O}(\varepsilon^{-N})$.

$\mathcal{N}([0,T) \times \mathbb{R}^n)$ is the space of all $G_\varepsilon \in \mathcal{E}([0,T) \times \mathbb{R}^n)$ with the property that for all $T > 0$, $\alpha \in \mathbb{N}_0^n$ and $a \in \mathbb{R}$, $\|\partial^\alpha G_\varepsilon\|_{L^\infty}$ is negligible, i.e., $\|\partial^\alpha G_\varepsilon\|_{L^\infty([0,T)\times\mathbb{R}^n)} = \mathcal{O}(\varepsilon^a)$.

Spaces $\mathcal{E}_M([0,T) \times \mathbb{R}^n)$ and $\mathcal{N}([0,T) \times \mathbb{R}^n)$ are algebras and $\mathcal{N}([0,T) \times \mathbb{R}^n)$ is an ideal of $\mathcal{E}_M([0,T) \times \mathbb{R}^n)$. The factor algebra

$$\mathcal{G}([0,T) \times \mathbb{R}^n) = \mathcal{E}_M([0,T) \times \mathbb{R}^n)/\mathcal{N}([0,T) \times \mathbb{R}^n)$$

is called the algebra of Colombeau generalized functions of bounded type.

Similarly we define algebras $\mathcal{E}_M(\mathbb{R}^n)$, $\mathcal{N}(\mathbb{R}^n)$ and $\mathcal{G}(\mathbb{R}^n)$. Their elements do not depend on time t. Let Q denote $[0,T) \times O$ or O.

Definition 1. A Colombeau generalized stochastic process on a probability space (Ω, Σ, μ) is a mapping $U : \Omega \to \mathcal{G}(Q)$ such that there exists a function $\tilde{U} : (0,1) \times Q \times \Omega \to \mathbb{R}$ with the following properties:

1) For fixed $\varepsilon \in (0,1)$, the mapping $(x,\omega) \mapsto \tilde{U}(\varepsilon, x, \omega)$ is jointly measurable in $Q \times \Omega$.

2) $\varepsilon \mapsto \tilde{U}(\varepsilon, \cdot, \omega)$ belongs to $\mathcal{E}_M(Q)$ almost surely in $\omega \in \Omega$, and it is a representative of $U(\omega)$.

By $\mathcal{G}^\Omega(Q)$ we denote the algebra of Colombeau generalized stochastic processes.

For simplicity, the variable ε will be written as a subindex and ω will usually be omitted; that is, instead of $\tilde{U}(\varepsilon, \cdot, \omega)$ we simply write $U_\varepsilon(\cdot)$.

Since $\mathcal{G}^\Omega(\mathbb{R}^n)$ is a differential algebra, Colombeau generalized stochastic processes can be multiplied and have (generalized) derivatives of arbitrary order. In addition, this fact makes superposition of Colombeau generalized stochastic process with polynomially bounded functions (or generalized function) possible.

To any generalized stochastic process one can assign a Colombeau generalized stochastic process with a representative satisfying certain growing property. For example, one can define the positive noise wieved as a Colombeau stochastic process in the following way.

EXAMPLE 1. The positive noise process $W^+(x) \in \mathcal{G}^\Omega(\mathbb{R})$ is defined by

$$W^+_\varepsilon(x) = \exp\left(\dot{W}_\varepsilon(x) - \frac{1}{2}\|\varphi_\varepsilon\|^2_{L^2}\right) \in \mathcal{E}^\Omega_M(\mathbb{R}),$$

where $\dot{W}_\varepsilon \in \mathcal{E}^\Omega_M(\mathbb{R}) = \dot{W} * \varphi_\varepsilon(x)$ is the smoothed white noise process (φ_ε is a nonnegative model delta net).

3. One-dimensional nonlinear stochastic wave equations with Lipschitz nonlinearities

It is well known that Oberguggenberger and Russo (cf. [19]) showed the existence and uniqueness of generalized solution for n-dimensional stochastic wave equations in case when $n = 1, 2, 3$. In particular, in case $n = 1$, they considered the limiting behavior of the solution when the initial value is a white noise process. Here we will present those results, as well as results we obtained which are related to the limiting behavior of generalized solutions in case when the initial data are functionals of the white noise process having certain properties (for instance, the initial data can be the positive noise process). As the reader will see, in that case the phenomenon known as triviality effect appears which means that the solution of the nonlinear equation in some sense behaives at infinity as the solutions of the linear one. Oberguggenberger, Russo, Albeverio and some other authors also have investigated the triviality effect.

3.1. Existence and uniqueness of solutions

We study the Cauchy problem

$$(\partial_t^2 - \partial_x^2)U(x,t) = F(U(x,t)) + H(x,t), \;\; U(x,0) = I(x), \;\; \partial_t U(x,0) = J(x), \quad (2,3)$$

where F is a smooth and globally Lipschitz function with all derivatives growing at most polynomially at infinity and H, I and J are generalized stochastic processes on $\mathbb{R}^2$, respectively on $\mathbb{R}$.

Te following result is valid in cases $n = 1, 2, 3$, but for our purpose $n = 1$ is enough.

THEOREM 1. [19] *Let (Ω, Σ, μ) be a fixed probability space, F be as above, $H \in \mathcal{G}^\Omega(\mathbb{R}^2)$ and $I, J \in \mathcal{G}^\Omega(\mathbb{R})$. Then there is an almost surely unique solution $U \in \mathcal{G}^\Omega(\mathbb{R}^2)$ to problem (2)–(3).*

3.2. White noise as initial data and limiting behaviour of the solutions—previous results

In the sequel, we study the Cauchy problem

$$(\partial_t^2 - \partial_x^2)U(x,t) = F(U(x,t)), \;\; U(x,0) = H = h(\dot{W})(x), \;\; \partial_t U(x,0) = 0$$

in the algebra of Colombeau stochastic generalized processes. We suppose that the nonlinear part F is smooth with all derivatives growing at most polynomially at infinity, globally Lipschitz, and that $H \in \mathcal{G}^\Omega(\mathbb{R})$ is a Colombeau generalized stochastic process, a functional of the white noise process $\dot{W} \in \mathcal{G}^\Omega(\mathbb{R})$. In the previous section one saw that there exists an almost surely unique solution $U \in \mathcal{G}^\Omega(\mathbb{R}^2)$ to the problem above.

We start with the Cauchy problem

$$(\partial_t^2 - \partial_x^2)U(x,t) = F(U(x,t)), \;\; U(x,0) = \dot{W}(x), \;\; \partial_t U(x,0) = 0, \tag{4,5}$$

where $\dot{W}$ is the white noise process on $\mathbb{R}$, considered as an element of $\mathcal{G}^\Omega(\mathbb{R})$ and F is smooth with all derivatives growing at most polynomially at infinity, globally Lipschitz and such that it has a limit at infinity $\lim_{|y|\to\infty} F(y) = L$.

If $V \in \mathcal{G}^\Omega(\mathbb{R}^2)$ is the solution to the free equation

$$(\partial_t^2 - \partial_x^2)V(x,t) = 0, \;\; V(x,0) = \dot{W}(x), \;\; \partial_t V(x,0) = 0, \tag{6,7}$$

then it is given by the representative $V_\varepsilon(x,t) = \frac{1}{2}(\dot{W}_\varepsilon(x+t) + \dot{W}_\varepsilon(x-t))$, where $\dot{W}_\varepsilon(x,t) \in \mathcal{E}^\Omega(\mathbb{R})$ is a representative of the white noise process.

For each fixed (x,t), functions $V_\varepsilon(x,t)$ are mean zero Gaussian variables on the white noise probability space (Ω, Σ, μ). The variance of the $V_\varepsilon(x,t)$ tends to infinity as ε tends to zero. That implies that every sequence tending to 0 has a subsequence $\varepsilon_k \to 0$ such that $|V_{\varepsilon_k}(x,t,\omega)| \to \infty$ almost surely in $\omega \in \Omega$ for all $(x,t) \in \mathbb{R}^2$.

Now, we return to the problem (4)-(5) and define the function $M = \frac{t^2}{2}\, L$, where L is, as we supposed, the limit of function F at infinity. In the form of representatives problem (4)-(5) reads

$$(\partial_t^2 - \partial_x^2)U_\varepsilon(x,t) = F(U_\varepsilon(x,t)), \;\; U_\varepsilon(x,0) = \dot{W}_\varepsilon(x), \;\; \partial_t U_\varepsilon(x,0) = 0. \tag{8,9}$$

The following result holds.

Theorem 2. [19] *Let F be smooth with all derivatives growing at most polynomially at infinity, globally Lipschitz and such that it has a limit at infinity, denoted by L. Every sequence tending to zero has a subsequence $\varepsilon_k \to 0$ such that for all compact sets $K \subset \mathbb{R}^2$*

$$\lim_{k\to\infty} \|U_{\varepsilon_k} - V_{\varepsilon_k} - M\|_{L^1(K)} = 0$$

μ-almost surely, where $M = \frac{t^2}{2}\, L$.

Remark 1. In space dimension $n = 1$ the problem $(\partial_t^2 - \partial_x^2)U(x,t) = F(U(x,t))$, $U(x,0) = 0$, $\partial_t U(x,0) = \dot{W}(x)$, admits a classical, almost surely continuous solution. That follows from d'Alembert's formula and the fact that by integration of the white noise process one obtains a Wiener process. The Colombeau generalized solution is almost surely associated with that classical solution.

3.3. Positive noise process as initial data and the limiting behaviour of the solutions

The positive noise process $W^+(x) \in \mathcal{G}^\Omega(\mathbb{R})$ is defined by

$$W_\varepsilon^+(x) = \exp\left(\dot{W}_\varepsilon(x) - \frac{1}{2}\|\varphi_\varepsilon\|_{L^2}^2\right) \in \mathcal{E}_M^\Omega(\mathbb{R}),$$

where $\dot{W}_\varepsilon \in \mathcal{E}_M^\Omega(\mathbb{R}) = \dot{W} * \varphi_\varepsilon(x)$ is the smoothed white noise process (φ_ε is a nonnegative model delta net).

PROPOSITION 1. *Smoothed positive noise is a process with mean value 1 and variance* $V(W_\varepsilon^+) = e^{\sigma_\varepsilon^2} - 1$, *where* $\sigma_\varepsilon^2 = \|\varphi_\varepsilon\|_{L^2}^2$.

Proof. For smoothed white noise $\dot{W}_\varepsilon = \dot{W} * \varphi_\varepsilon$ we have that the mean value and the variance are $E(\dot{W}_\varepsilon) = 0$, $V(\dot{W}_\varepsilon) = \sigma_\varepsilon^2$. Using that fact we obtain

$$
\begin{aligned}
E\left(W_\varepsilon^+\right) &= E\Big(\exp\Big(\dot{W}_\varepsilon(x) - \frac{1}{2}\|\varphi_\varepsilon\|_{L^2}^2\Big)\Big) \\
&= \int \frac{1}{\sqrt{2\pi}\sigma_\varepsilon} e^{-\frac{1}{2\sigma_\varepsilon^2}y^2} e^{(y-\frac{1}{2}\sigma_\varepsilon^2)}\,dy = \frac{1}{\sqrt{2\pi}\sigma_\varepsilon}\int e^{-\frac{1}{2\sigma_\varepsilon^2}(y-\sigma_\varepsilon^2)^2}\,dy = 1.
\end{aligned}
$$

Similarly, we have

$$
\begin{aligned}
E((W_\varepsilon^+)^2) &= E(\exp(2\dot{W}_\varepsilon(x) - \|\varphi_\varepsilon\|_{L^2}^2)) \\
&= \int \frac{1}{\sqrt{2\pi}\sigma_\varepsilon} e^{-\frac{1}{2\sigma_\varepsilon^2}y^2} e^{(2y-\sigma_\varepsilon^2)}\,dy = e^{\sigma_\varepsilon^2}\int \frac{1}{\sqrt{2\pi}\sigma_\varepsilon} e^{-\frac{1}{2\sigma_\varepsilon^2}(y-2\sigma_\varepsilon^2)^2}\,dy = e^{\sigma_\varepsilon^2}.
\end{aligned}
$$

Therefore, $V(W_\varepsilon^+) = E((W_\varepsilon^+)^2) - (E(W_\varepsilon^+))^2 = e^{\sigma_\varepsilon^2} - 1$. Thus, the proof is completed. ∎

We consider the Cauchy problem

$$
(\partial_t^2 - \partial_x^2)U(x,t) = F(U(x,t)),\ \ (x,t) \in \mathbb{R}^2,\ \ U(x,0) = W^+(x), \tag{10,11}
$$

where $W^+(x) \in \mathcal{G}^\Omega(\mathbb{R})$ is defined by (1). We suppose that function F is smooth with all derivatives growing at most polynomially at infinity, globally Lipschitz, bounded and $F(0) = 0$.

If W^+ is of log-type, an almost surely unique solution $U \in \mathcal{G}^\Omega(\mathbb{R}^2)$ to problem (10)-(11) exists.

Let $V \in \mathcal{G}^\Omega(\mathbb{R}^2)$ be a solution to the free equation

$$
(\partial_t^2 - \partial_x^2)V(x,t) = 0,\ \ (x,t) \in \mathbb{R}^2,\ \ V(x,t) = W^+(x). \tag{12,13}
$$

It is given by its representative

$$
V_\varepsilon(x,t) = \frac{1}{2}(W_\varepsilon^+(x-t) + W_\varepsilon^+(x+t)). \tag{14}
$$

LEMMA 1. *Let F be as above and $V \in \mathcal{G}^\Omega(\mathbb{R}^2)$ be the solution to the problem (12)-(13). Then, $E\,|F(V_\varepsilon(x,t))| \to 0$, as $\varepsilon \to 0$, for almost every x and $t > 0$.*

Proof. Since the smoothed white noise is a Gaussian stochastic process, for small ε and $t > 0$ we have

$$
\begin{aligned}
&E\,|F(V_\varepsilon(x,t))| \\
&= E\left|F\left(\frac{1}{2}\exp\left(\dot{W}_\varepsilon(x-t) - \frac{1}{2}\sigma_\varepsilon^2\right) + \frac{1}{2}\exp\left(\dot{W}_\varepsilon(x+t) - \frac{1}{2}\sigma_\varepsilon^2\right)\right)\right| \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{2\pi\sigma_\varepsilon^2}\left|F\left(\frac{1}{2}e^{y_1 - \frac{\sigma_\varepsilon^2}{2}} + \frac{1}{2}e^{y_2 - \frac{\sigma_\varepsilon^2}{2}}\right)\right| e^{-\frac{y_1^2}{2\sigma_\varepsilon^2} - \frac{y_2^2}{2\sigma_\varepsilon^2}}\,dy_1\,dy_2
\end{aligned}
$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{2\pi}\left|F\left(\frac{1}{2}e^{\sigma_\varepsilon y_1 - \frac{\sigma_\varepsilon^2}{2}} + \frac{1}{2}e^{\sigma_\varepsilon y_2 - \frac{\sigma_\varepsilon^2}{2}}\right)\right| e^{-\frac{1}{2}(y_1^2+y_2^2)}\, dy_1\, dy_2.$$

Since F is bounded, Lebesgue's theorem immediately implies $E\,|F(V_\varepsilon(x,t))| \to F(0) = 0$, as $\varepsilon \to 0$, for almost every x and $t > 0$. ∎

Now, we can formulate the triviality result.

THEOREM 3. *Under the assumptions above, let $U \in \mathcal{G}^\Omega(\mathbb{R}^2)$ be the solution to the problem (10)-(11) and $V \in \mathcal{G}^\Omega(\mathbb{R}^2)$ be the solution to the problem (12)-(13). Then*

$$E\,\|U_\varepsilon - V_\varepsilon\|_{L^1(K_\tau)} \to 0,\ \textit{as } \varepsilon \to 0, \tag{15}$$

where, for fixed $r > 0$ and $\tau > 0$, K_τ is defined by

$$K_\tau = \left\{(x,t) \in \mathbb{R}^2\ ;\ 0 \le t \le \tau,\ |x| \le r - t\right\}. \tag{16}$$

Proof. Taking the expectation in the estimate

$$\|U_\varepsilon - V_\varepsilon\|_{L^1(K_\tau)} \le \tau\,\|F'\|_{L^\infty(\mathbb{R})}\int_0^\tau \|U_\varepsilon - V_\varepsilon\|_{L^1(K_t)}\, dt + \tau^2\,\|F(V_\varepsilon)\|_{L^1(K_\tau)}$$

similarly as before, we obtain

$$E\,\|U_\varepsilon - V_\varepsilon\|_{L^1(K_\tau)} \le \tau C_F \int_0^\tau E\,\|U_\varepsilon - V_\varepsilon\|_{L^1(K_t)}\, dt + \tau^2 E\,\|F(V_\varepsilon)\|_{L^1(K_\tau)},$$

where C_F is the Lipschitz constant for the function F.

But, we have that

$$E\,\|F(V_\varepsilon)\|_{L^1(K_\tau)} = \iint_{K_\tau} E\,|F(V_\varepsilon(x,t))|\, dx\, dt.$$

According to Lemma 1, the last term in the right-hand side tends to 0 as ε tends to zero. Therefore, $\tau^2 E\,\|F(V_\varepsilon)\|_{L^1(K_\tau)} \to 0$, as $\varepsilon \to 0$. Gronwall's inequality implies the assertion. ∎

REMARK 2. The same result holds in more general case, namely, if we take for the initial data in our Cauchy problem any $H = [h(\dot{W}_\varepsilon)] \in \mathcal{G}^\Omega(\mathbb{R})$ such that h is smooth function with all derivatives growing at most polynomially at infinity vanishing at zero.

One can go step further and take for the initial data in the Cauchy problem we consider some other functionals of the white noise process, also wieved as elements of certain Colombeau algebras. This analysis is done in the paper [22]. In all those cases the triviality effect appears.

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