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ON COLC TOPOLOGIES

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Abstract. In this paper we introduce the concept of coLC topologies and discuss some of their basic properties. We relate this concept to classes of functions between topological spaces.

1. Introduction

A topological space whose Lindelöf subsets are closed is called an LC-space by Mukherji and Sarkar [17] and by Gauld, Mršević, Reilly and Vamanamurthy [7]. LC-spaces are also known as L-closed spaces [8,9,10,15]. They specialize KC-spaces, i.e. spaces in which compact subsets are closed [18] and Hausdorff P-spaces, i.e. spaces in which F_σ-sets are closed [16]. LC-spaces have been of some interest during recent years [1,2,3,4].

In 1984, Gauld, Mršević, Reilly and Vamanamurthy [7] defined the coLindelöf topology of τ on X. Let (X,τ) be a topological space. Then $\ell(\tau) = \{\emptyset\} \cup \{G \in \tau : X - G$ is Lindelöf in $(X,\tau)\}$ is a topology on X with $\ell(\tau) \subseteq \tau$, and it is called the coLindelöf topology of τ on X. This topology is analogous to the cocompact topology $c(\tau)$ which is considered by Gauld [5] and [6] and by Kohli [11].

The purpose of the present paper is to introduce the concept of the coLC topology of τ on X denoted by $\ell c(\tau)$ and to study its basic properties. In Section 2, by giving an example, it is observed that X with the coLC topology of τ on X is not necessarily an LC-space. We investigate the conditions under which the topological space $(X, \ell c(\tau))$ is an LC-space where (X,τ) is a topological space. We relate $\ell c(\tau)$ to the cocompact topology of (X,τ) and to the coLindelöf topology $\ell(\tau)$ of (X,τ). We also investigate relationship between the coLC topology of the subspace topology and the subspace topology of $\ell c(\tau)$ for any subset of X. In the last section we give some continuity results. Our terminology is standard. The closure and interior of a subset Y of a space (X,τ) are denoted by τclY and $\tau intY$, respectively. Denote the relative topology on Y by τ_Y. Given a topological space (X,τ) and a subset $Y \subseteq X$, $\ell c(\tau_Y)$ and $\ell c(\tau)_Y$ denote the coLC topology of the relative topology τ_Y on X and the relative topology induced by the coLC topology $\ell c(\tau)$ of τ on X, respectively.

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2. CoLC topologies

Firstly we give the following results which will be needed in the sequel.

Proposition 1. *The union of any two closed LC-subspaces of (X,τ) is an LC-subspace.*

Proof. Let $K_1, K_2 \subseteq X$ be closed, LC-subspaces of (X,τ). Let K be a Lindelöf subset of $(K_1 \cup K_2, \tau_{K_1 \cup K_2})$. Then $K \cap K_1$ is a Lindelöf subset of (K_1, τ_{K_1}). Since K_1 is an LC-subspace of (X,τ), $K \cap K_1$ is closed in (K_1, τ_{K_1}). The set $K \cap K_1$ is closed in X since K_1 is closed in (X,τ). Similarly $K \cap K_2$ is closed in (X,τ). Thus $(K \cap K_1) \cup (K \cap K_2) = K$ is closed in (X,τ). This implies that K is closed in $(K_1 \cup K_2, \tau_{K_1 \cup K_2})$. Hence $K_1 \cup K_2$ is an LC-subspace of (X,τ). ∎

Corollary 2. *Let X be a topological space. If X can be expressed as the union of two closed LC-subspaces, then X is an LC-space.*

Definition 3. Let (X,τ) be a topological space. The collection $\ell c(\tau) = \{\emptyset\} \cup \{U \in \tau : X - U \text{ is an } LC\text{-subspace in } (X,\tau)\}$ is a topology on X with $\ell c(\tau) \subseteq \tau$, called the co$LC$ topology of τ on X.

It is easy to see that $\ell c(\tau)$ is a topology on X since the LC property is hereditary [17] and by Proposition 1.

The next example shows that $(X, \ell c(\tau))$ is not necessarily an LC-space, even if (X,τ) is Lindelöf.

Example 4. Let X be an uncountable set and τ be the countable complement topology on X. The coLC topology of τ on X is τ, that is $\tau = \ell c(\tau)$. Since (X,τ) is not an LC-space [17], $(X, \ell c(\tau))$ is not an LC-space.

Proposition 5. *If (X,τ) is an LC-space, then $\tau = \ell c(\tau)$.*

Proof. By definition $\ell c(\tau) \subseteq \tau$. Now take $U \in \tau$. Then $X - U$ is an LC-subspace of (X,τ) since X is an LC-space and the LC property is hereditary. Hence $U \in \ell c(\tau)$. ∎

Example 4 shows that the reverse implication of Proposition 5 does not hold in general.

Proposition 6. *Let (X,τ) be a topological space. If $\tau = \ell c(\tau)$ and (X,τ) has no Lindelöf-dense subset, then (X,τ) is an LC-space.*

Proof. Let L be a Lindelöf subset of (X,τ). Then L is Lindelöf in the subspace τclL of (X,τ). Now $X - \tau clL \neq \emptyset$ by hypothesis. Since $X - \tau clL \in \tau = \ell c(\tau)$ we have that $X - (X - \tau clL) = \tau clL$ is an LC-subspace of (X,τ) and hence L is closed in τclL. Since τclL is closed in (X,τ), L is closed in (X,τ). Hence (X,τ) is an LC-space. ∎

In the above theorem the condition that (X,τ) has no Lindelöf-dense subset can not be removed as the Example 4 shows.

Corollary 7. *Let (X,τ) be a topological space. If $(X,\ell c(\tau))$ has no Lindelöf-dense subset then $(X,\ell c(\tau))$ is an LC-space.*

Proof. Firstly we show that (X,τ) is an LC-space. Let L be a Lindelöf subset of (X,τ). Then L is Lindelöf in the subspace τclL of (X,τ). We have $\tau clL \subseteq \ell c(\tau)clL$ and $\ell c(\tau)clL \neq X$ by hypothesis. Since $X-\ell c(\tau)clL \in \ell c(\tau)$, $\ell c(\tau)clL$ is an LC-subspace of (X,τ). Then τclL is an LC-subspace of (X,τ). Hence L is closed in τclL. Since τclL is closed in (X,τ), L is closed in (X,τ). Thus (X,τ) is an LC-space. We obtain $\tau = \ell c(\tau)$ from Proposition 5 and hence $(X,\ell c(\tau))$ is an LC-space. ∎

Theorem 8. *For a space (X,τ) the following are equivalent:*

(1) *(X,τ) is an LC-space.*

(2) *$(X,\ell c(\tau))$ is an LC-space.*

Proof. (1)⇒(2) It follows from Proposition 5.

(2)⇒(1) Let L be a Lindelöf subset of (X,τ). All Lindelöf subsets of (X,τ) are Lindelöf in $(X,\ell c(\tau))$ since $\ell c(\tau)\subseteq\tau$. Thus L is Lindelöf in $(X,\ell c(\tau))$ and so L is closed in $(X,\ell c(\tau))$ since $(X,\ell c(\tau))$ is an LC-space. Hence L is closed in (X,τ). ∎

Remark 9. Let (X,τ) be a topological space which is not countable discrete. If $(X,\ell c(\tau))$ is a separable space, then $(X,\ell c(\tau))$ and (X,τ) are not LC-spaces. Hence if $(X,\ell c(\tau))$ is second countable, then $(X,\ell c(\tau))$ and (X,τ) are not LC-spaces.

Proposition 10. *If $(X,\ell c(\tau))$ is Hausdorff, then (X,τ) is an LC-space.*

Proof. Let $x,y\in X$ and $x\neq y$. Since $(X,\ell c(\tau))$ is Hausdorff, there are $U,V\in\ell c(\tau)$ such that $x\in U$, $y\in V$ and $U\cap V=\emptyset$. Thus $X=(X-U)\cup(X-V)$ and hence X can be written as the union of two closed, LC-subspaces of (X,τ). By Corollary 2 (X,τ) is an LC-space. ∎

Corollary 11. *Let (X,τ) be a topological space. If $(X,\ell c(\tau))$ is Hausdorff, then $(X,\ell c(\tau))$ is an LC-space.*

The following example shows that if (X,τ) is Hausdorff, then (X,τ) is not necessarily an LC-space.

Example 12. Let $\mathbf{R}$ be the set of real numbers and τ be the usual topology on $\mathbf{R}$. Then $(\mathbf{R},\tau)$ is Hausdorff. The subset $\mathbf{Q}$ of rational numbers is Lindelöf but not closed. Hence $(\mathbf{R},\tau)$ is not an LC-space.

Proposition 13. *Let (X,τ) be a topological space. $(X,\ell c(\tau))$ is an LC-space if it can be expressed as the union of two closed sets which are not equal to X.*

Proof. Let $X=K_1\cup K_2$ and let $K_1\neq X$, $K_2\neq X$ be closed in $(X,\ell c(\tau))$. Then K_1 and K_2 are LC-subspaces of (X,τ) by Definition 3. Therefore (X,τ) is an LC-space by Corollary 2, since K_1 and K_2 are closed in (X,τ). Hence $(X,\ell c(\tau))$ is an LC-space by Theorem 8. ∎

Corollary 14. *If $(X, \ell c(\tau))$ is not connected, then $(X, \ell c(\tau))$ is an LC-space.*

Theorem 15. *Let (X, τ) be a topological space. Then (X, τ) is an LC-space and not connected if and only if $(X, \ell c(\tau))$ is not connected.*

Proof. ($\Rightarrow$) Let (X, τ) be an LC-space and non-connected. Then $\tau = \ell c(\tau)$ by Proposition 5. Hence $(X, \ell c(\tau))$ is not connected.

($\Leftarrow$) (X, τ) is not connected since $(X, \ell c(\tau))$ is non-connected and $\ell c(\tau) \subseteq \tau$. By Corollary 14, $(X, \ell c(\tau))$ is an LC-space since $(X, \ell c(\tau))$ is non-connected. Hence (X, τ) is an LC-space by Theorem 8. ■

The following four generalizations of LC-spaces and Theorem are given by Dontchev, Ganster and Kanibir [2].

Definition 16. A topological space (X, τ) is called:

(1) an L_1-space if every Lindelöf F_σ-set is closed,

(2) an L_2-space if $\tau cl L$ is Lindelöf whenever $L \subseteq X$ is Lindelöf,

(3) an L_3-space if every Lindelöf subset L is an F_σ-set,

(4) an L_4-space if, whenever $L \subseteq X$ is Lindelöf, then there is a Lindelöf F_σ-set F with $L \subseteq F \subseteq \tau cl L$.

Theorem 17. *(1) If (X, τ) is an LC-space, then (X, τ) is an L_i-space, $i = 1, 2, 3, 4$.*

(2) Every L_2-space is an L_4-space and every L_3-space is an L_4-space.

Proposition 18. *Let (X, τ) be a topological space. If $(X, \ell c(\tau))$ has no Lindelöf-dense subset, then (X, τ) and $(X, \ell c(\tau))$ are L_i-spaces, $i = 1, 2, 3, 4$.*

Proof. $(X, \ell c(\tau))$ is an LC-space by Corollary 7 since $(X, \ell c(\tau))$ has no Lindelöf-dense set. Thus (X, τ) is an LC-space. By Theorem 17(1) (X, τ) and $(X, \ell c(\tau))$ are L_i-spaces, $i = 1, 2, 3, 4$. ■

Proposition 19. *Let (X, τ) be a topological space. If $(X, \ell c(\tau))$ is an L_2-space but not an LC-space, then $(X, \ell c(\tau))$ is a Lindelöf space.*

Proof. Let $(X, \ell c(\tau))$ be L_2 and not an LC-space. Then there is a Lindelöf-dense set L in $(X, \ell c(\tau))$ by Corollary 7 and $\ell c(\tau) cl L = X$. Hence $(X, \ell c(\tau))$ is a Lindelöf space since $(X, \ell c(\tau))$ is an L_2-space. ■

A space (X, τ) satisfying the hypothesis of the previous Proposition is given by Example 4.

Proposition 20. *If (X, τ) is T_1, then $(X, \ell c(\tau))$ is T_1.*

Proof. Let $x \in X$. Since (X, τ) is T_1 and $\{x\}$ is an LC-subspace of (X, τ), $X - \{x\} \in \ell c(\tau)$. Hence $\{x\}$ is closed in $(X, \ell c(\tau))$. Thus $(X, \ell c(\tau))$ is T_1. ■

Example 12 shows that if (X, τ) is Hausdorff then $(X, \ell c(\tau))$ is not necessarily Hausdorff. $(\mathbf{R}, \tau)$ is Hausdorff, but is not an LC-space. Hence $(\mathbf{R}, \ell c(\tau))$ is not Hausdorff by Proposition 10.

Recall that if (X,τ) is a topological space, then the cocompact topology of τ on X is $c(\tau)=\{\emptyset\}\cup\{U\in\tau : X-U \text{ is compact in } (X,\tau)\}$ [5].

Proposition 21. *Let (X,τ) be a topological space.*

(1) If (X,τ) is an LC-space, then $\ell(\tau)\subseteq \ell c(\tau)$ and $c(\tau)\subseteq \ell c(\tau)$.

(2) If (X,τ) is a Lindelöf space, then $\ell c(\tau)\subseteq \ell(\tau)$.

(3) If (X,τ) is a compact space, then $\ell c(\tau)\subseteq c(\tau)$.

Proof. (1) Let (X,τ) be an LC-space. Then $\tau=\ell c(\tau)$ by Proposition 4. Since $\ell(\tau)\subseteq\tau$ and $c(\tau)\subseteq\tau$, $\ell(\tau)\subseteq \ell c(\tau)$ and $c(\tau)\subseteq \ell c(\tau)$. This completes the proof.

(2) Let (X,τ) be a Lindelöf space. Then $\tau=\ell(\tau)$ by Corollary 1 of [7]. Since $\ell c(\tau)\subseteq\tau$, $\ell c(\tau)\subseteq \ell(\tau)$. This completes the proof.

(3) Let (X,τ) be a compact space. Then $c(\tau)=\tau$ by Corollary 3 of [5]. Hence $\ell c(\tau)\subseteq\tau$. ∎

Note that the reverse inclusions of (1), (2) and (3) are false in general.

Example 22. Let X be an uncountable set and let τ be the discrete topology on X. Since X is an LC-space, $\tau=\ell c(\tau)$ by Proposition 5. It is clear that $\tau\neq\ell(\tau)$ and $\tau\neq c(\tau)$. Hence $\ell c(\tau)\nsubseteq \ell(\tau)$ and $\ell c(\tau)\nsubseteq c(\tau)$.

Example 23. Let X be the set of real numbers with the usual topology, $Y=\{0\}\cup\{\frac{1}{n} : n=1,2,3,\dots\}$ and let τ be the topology induced on the set Y by the topology of X. Since (Y,τ) is compact, it is Lindelöf. Hence $\tau=c(\tau)=\ell(\tau)$. We take $\{\frac{1}{2}\}\in\tau$ and $A=\{1,\frac{1}{3},\frac{1}{4},\dots\}$. Then $A\subseteq K=Y-\{\frac{1}{2}\}$ is a Lindelöf subset of (K,τ_K), but not closed in (K,τ_K). Therefore K is not an LC-subspace of (Y,τ). So $\{\frac{1}{2}\}\notin \ell c(\tau)$. Hence $c(\tau)\nsubseteq \ell c(\tau)$ and $\ell(\tau)\nsubseteq \ell c(\tau)$.

Proposition 24. *Let L be a subset of the topological space (X,τ). Then $\ell c(\tau)_L\subseteq \ell c(\tau_L)$.*

Proof. Suppose $U\in \ell c(\tau)_L$. Then there is a subset $A\in \ell c(\tau)$ with $U=A\cap L$. Since $A\in\ell c(\tau)$, $X-A$ is an LC-subspace of (X,τ). Thus $L-U$ is an LC-subspace of $(X-A,\tau_{X-A})$. Hence $L-U$ is an LC-subspace of (X,τ) since $(\tau_{X-A})_{L-U}=\tau_{L-U}$. Further $U\in\tau_L$, so $U\in \ell c(\tau_L)$. Hence $\ell c(\tau)_L\subseteq \ell c(\tau_L)$. ∎

Theorem 25. *Let (X,τ) be a topological space and $Y\subseteq X$. If $\tau=\ell c(\tau)$, then $\tau_Y=\ell c(\tau_Y)=\ell c(\tau)_Y$.*

Proof. Firstly we will show that $\tau_Y\subseteq \ell c(\tau_Y)$. Let $U_Y\in\tau_Y$ and $U_Y=U\cap Y$, $U\in\tau$. Since $\tau=\ell c(\tau)$, $X-U$ is an LC-subspace of (X,τ). Therefore $Y-U_Y$ is an LC-subspace of (X,τ) because of $Y-U_Y\subseteq X-U$. Since $\tau_{Y-U_Y}=(\tau_Y)_{Y-U_Y}$, $Y-U_Y$ is an LC-subspace of (Y,τ_Y). Hence $\tau_Y\subseteq \ell c(\tau_Y)$.

We observe that $\ell c(\tau_Y)\subseteq\tau_Y$. Thus $\tau_Y=\ell c(\tau_Y)$. We have $\tau_Y=\ell c(\tau)_Y$ from the hypothesis. Hence $\tau_Y=\ell c(\tau_Y)=\ell c(\tau)_Y$. ∎

Note that, if (X,τ) is an LC-space, then Proposition 5 and Theorem 25 imply that $\tau_Y=\ell c(\tau_Y)=\ell c(\tau)_Y$ for every $Y\subseteq X$.

Note that the $\tau = \ell c(\tau)$ condition in Theorem 25 can not be removed as the following example shows. This example also shows that the reverse inclusion of Proposition 24 is false in general.

Example 26. Let $X = \{a, b, c, d\}$, $Y = \{a, b\}$ and $\tau = \{\emptyset, X, \{a\}, \{b, c, d\}\}$. Then $\ell c(\tau) = \{\emptyset\} \cup \{U \in \tau : X - U$ is an LC-subspace in $(X, \tau)\} = \{\emptyset, X, \{b, c, d\}\}$ and $\tau_Y = \{\emptyset, Y, \{a\}, \{b\}\}$. Therefore $\ell c(\tau)_Y = \{\emptyset, Y, \{b\}\}$ and $\ell c(\tau_Y) = \{\emptyset, Y, \{a\}, \{b\}\}$. Hence $\ell c(\tau_Y) \neq \ell c(\tau)_Y$.

3. Some continuity results

Now we give some definitions. Let P denote a property, not necessarily topological, possessed by certain subsets of a topological space.

Definition 27. [13] Let X be a topological space and $A \subseteq X$. Then

(1) A is a P-set if A possesses property P; and

(2) A has P-complement if $X - A$ possesses property P.

Definition 28. Let $f : X \to Y$ be a function from a topological space X into a topological space Y. Then f is said to be

(1) P-continuous [12] if for each $x \in X$ and each open set V containing $f(x)$ and having P-complement, there is an open set U containing x such that $f(U) \subseteq V$; and

(2) P^*-continuous [14] if for each $x \in X$ and set B with $f(x) \in intB$, whenever B has P-complement, there is an open set U containing x such that $f(U) \subseteq B$.

The class of P^*-continuous functions constitutes a subclass of the class of P-continuous functions, and they coincide with each other if the property P implies the property of being a closed set.

In the above definition if P denotes the property of being a closed LC-subspace, then we have the following definition.

Definition 29. A function $f : X \to Y$ is called ℓc-continuous if for each point $x \in X$ and each open set V containing $f(x)$ and having LC-complement there is an open set U containing x such that $f(U) \subseteq V$.

It is immediate from these definitions that every continuous function is ℓc-continuous. However, the reverse implication does not hold in general.

For example, let $X = Y = \{a, b, c, d\}$, $\vartheta = \{Y, \emptyset, \{b, c, d\}\}$, $\tau = \{X, \emptyset, \{a\}, \{b, c, d\}\}$. Let f be the identity mapping from (X, ϑ) onto (Y, τ). Then f is ℓc-continuous but it is not continuous.

Theorem 30. *Let $f : X \to Y$ be an ℓc-continuous function such that $f(X)$ is contained in a closed LC-subspace of Y. Then f is continuous.*

Proof. Let L be a closed LC-subspace of Y containing $f(X)$ and V be any open subset of Y. Then $V \cup (Y - L)$ is an open subset of Y having LC-complement. Now the result follows since f is ℓc-continuous and $f^{-1}(V) = f^{-1}(V \cup (Y - L))$. ∎

The proof of the following result is straightforward, so we omit it.

Theorem 31. *Let $f\colon X \to Y$ be a function from a topological space X into a topological space Y. The following statements are equivalent.*

(1) f is ℓc-continuous,

(2) for every A of Y having LC-complement, $f^{-1}(intA) \subseteq intf^{-1}(A)$.

There are more results that are related to ℓc-continuous functions that are similar to section 3 of [14].

The following Theorem is the ℓc-continuity version of a result given for ℓ-continuous functions in [7]. Its proof follows from the definitions.

Theorem 32. *The function $f : (X, \vartheta) \to (Y, \tau)$ is ℓc-continuous if and only if $f : (X, \vartheta) \to (Y, \ell c(\tau))$ is continuous.*

Now we give a stronger version of ℓc-continuity similar to strong ℓ-continuity in [7].

Definition 33. A function $f : (X, \vartheta) \to (Y, \tau)$ is called strongly ℓc-continuous if $f : (X, \ell c(\vartheta)) \to (Y, \ell c(\tau))$ is continuous.

It is clear from Theorem 32 that strong ℓc-continuity implies ℓc-continuity. On the other hand, ℓc-continuity does not imply strong ℓc-continuity in general.

Example 34. Let $X = \{a, b, c, d, e\}$, $\vartheta = \{X, \emptyset, \{a, b, c\}, \{d, e\}\}$. Let $Y = \{a, b, c, d\}$, $\tau = \{Y, \emptyset, \{a, b, c\}, \{d\}\}$. Let $f\colon (X, \vartheta) \to (Y, \tau)$ be a function defined by $f(a) = a$, $f(b) = b$, $f(c) = c$, $f(d) = d$, $f(e) = d$. Then f is ℓc-continuous but it is not strongly ℓc-continuous.

The following result is immediate from Theorem 32 and Definition 33.

Proposition 35. *If $f : (X, \vartheta) \to (Y, \tau)$ is ℓc-continuous and $g : (Y, \tau) \to (Z, \upsilon)$ is strongly ℓc-continuous then $g \circ f$ is ℓc-continuous.*

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