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SOME CURVATURE CONDITIONS OF THE TYPE 2×4 ON THE SUBMANIFOLDS SATISFYING CHEN'S EQUALITY

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Abstract. Submanifolds of the Euclidean spaces satisfying equality in the basic Chen's inequality have, as is known, many interesting properties. In this paper, we discuss on such submanifolds the curvature conditions of the form $E_2 \cdot F_4 = 0$, where E_2 is the Ricci or the Einstein curvature operator, F_4 is any of the standard curvature operators R, Z, P, K, C, and E_2 acts on F_4 as a derivation.

1. Introduction

This paper is a continuation of the paper [21] by M. Petrović-Torgašev and the present author. Hence, in this paper all definitions, denotations and the properties from [21] are accepted. Only some necessary definitions and statements from the mentioned previous paper are briefly repeated.

Let M^n be an n-dimensional submanifold of a Euclidean space E^m of dimension $m = n + p$ $(n \geq 2, p \geq 1)$, which we suppose to be *ideal in the sense of Chen* (see [5]), thus which satisfies the so called *the basic Chen's equality.*

Let R, S, G and τ be respectively the Riemann-Christoffel curvature tensor, the Ricci tensor, the Einstein tensor and the scalar curvature of M^n. The corresponding curvature operators in the tangent space $T(M^n)$ are denoted by $R(X,Y)$ etc.

Besides, we shall consider the next, standard curvature operators on M^n: the *concircular curvature operator* $Z(X,Y)$ $(n \geq 2)$, the *projective curvature operator* $P(X,Y)$ $(n \geq 2)$, the *Weyl conformal curvature operator* $C(X,Y)$ $(n \geq 3)$, the *conharmonic curvature operator* $K(X,Y)$ $(n \geq 3)$, each of them acting in the tangent space $T_x(M^n)$, where x is a fixed or variable point of M^n.

In the next simple proposition we collect the conditions for a manifold M^n satisfying Chen's equality to be flat with respect to different curvature operators.

Proposition. ([15]) *If $n \geq 2$, then M^n is flat if and only if $n = 2$ and $\sigma = ab$, or $n \geq 3$ and $\sigma = a = b = 0$.*

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If $n \geq 3$, then M^n is of constant curvature if and only if it is flat.

If $n \geq 3$, then M^n is conformally flat if and only if $n = 3$, or $n \geq 4$ and $\sigma = ab$.

If $n \geq 3$, then M^n if conharmonically flat if and only if it is flat, or $n = 3$ and $\sigma = ab + \mu^2$.

In the sequel, for a manifold M^n, we shall consider the curvature conditions of the form $E \cdot F = 0$, where $E = E_2$ is one of the operators S, G, $F = F_4 \in \mathcal{M} = \{R, Z, P, C, K\}$, and $E_2 \cdot F_4$ is defined as

$$(E_2 \cdot F_4(X,Y))U = E_2(F_4(X,Y)U) - F_4(E_2X,Y)U \\ - F_4(X,E_2Y)U - F_4(X,Y)(E_2U),$$

where $X, Y, U \in T_x(M^n)$.

Recall that semisymmetric manifolds are defined by relation $R \cdot R = 0$, meaning that

$$(R(X,Y) \cdot R)(U,V)W = R(X,Y)(R(U,V)W) - R(R(X,Y)U,V)W- \\ - R(U,R(X,Y)V)W - R(U,V)(R(X,Y)W) = 0,$$

for all tangent vector fields X, Y, U, V, W on M^n.

The semisymmetric manifolds and similar curvature conditions have been investigated in many papers (see for instance, [1–3], [7–10], [14–15], [18–21], [23–34] etc).

2. Main results

Throughout this section, we shall suppose that a submanifold M^n in an Euclidean space E^m ($m = n + p$, $p \geq 1$, $n \geq 2$), satisfies the basic Chen's equality, and we investigate in such submanifolds several curvature conditions of the form $E_2 \cdot F_4 = 0$, where E_2 equals S or G, and F_4 is one of the operators R, Z, P, K, C. In general case the operators $S \cdot F_4$ and $G \cdot F_4$ do not coincide, so we have to discuss the corresponding curvature conditions separately.

In the simplest case $n = 2$, we get that $R = \frac{\tau}{2}B$, $Z = P = 0$, $S = \frac{\tau}{2}\,I$, $G = 0$, so that $G \cdot R = G \cdot Z = G \cdot P = S \cdot Z = S \cdot P = 0$. But, as is easily seen, $S \cdot R = 0$ if and only if $R_1 = 0$. So, the case $n = 2$ is mostly trivial and we shall usually suppose that $n \geq 3$.

If F is one of the operators R, Z, K, C, and $(E_2 \cdot F)_{ijk} = (E_2 \cdot F(e_i, e_j))e_k$ ($i, j, k = 1, \dots, n$), then, by linearity, it is easily seen that $E_2 \cdot F = 0$ if and only if $(E_2 \cdot F)_{ijk} = 0$ for any choice of indices $i, j, k = 1, \dots, n$. Moreover, if $n \geq 3$, then by a straightforward calculation, one can see that $E_2 \cdot F = 0$ if and only if the next system of equations holds: $(E_2 \cdot F)_{121} = (E_2 \cdot F)_{122} = (E_2 \cdot F)_{131} = (E_2 \cdot F)_{133} = (E_2 \cdot F)_{232} = (E_2 \cdot F)_{233} = (E_2 \cdot F)_{343} = 0$. So we obtain a system of equations which completely describes the curvature condition $E_2 \cdot F = 0$:

$$\alpha\, F_1 = \beta\, F_1 = \alpha\, F_2 = \beta\, F_3 = \gamma\, F_2 = \gamma\, F_3 = \gamma\, F_0 = 0. \tag{1}$$

Note that the last equation exists only for $n \geq 4$.

THEOREM 1. *If $n \geq 2$, then $S \cdot R = 0$ if and only if M^n is a totally geodesic plane, or $n = 2$ and $R_1 = 0$.*

Proof. If M^n is a totally geodesic plane, or $n = 2$ and $R_1 = 0$, then obviously $S \cdot R = 0$.

Next, suppose that $n \geq 2$ and $S \cdot R = 0$.

If $n = 2$ then $\alpha = -R_1$, and equation $(S \cdot R)_{121} = 0$ gives $R_1 = 0$.

If $n \geq 4$, then equation $(S \cdot R)_{343} = 0$ gives $\mu = 0$, and consequently $\alpha = \beta = -R_1$. Then by equation $(S \cdot R)_{121} = 0$ we obtain $R_1 = 0$, $\sigma = ab = -a^2$, and immediately $\sigma = a = b = 0$, thus M^n is a totally geodesic plane.

Finally, assume that $n = 3$. Then $\alpha = a\,\mu - R_1$, $\beta = b\,\mu - R_1$, $\gamma = \mu^2$, and equations $(S \cdot R)_{133} = 0$ and $(S \cdot R)_{233} = 0$ give $a\,\mu = b\,\mu = 0$, and consequently $\alpha = -R_1$, $\beta = -R_1$. But then equation $(S \cdot R)_{121} = 0$ also gives $R_1 = 0$. If $\mu \neq 0$, then $a = b = 0$, a contradiction. If $\mu = 0$, then $a = -b$, and $\sigma = ab$ again gives $\sigma = a = b = 0$, thus M^3 is a totally geodesic plane. ■

In the proof of Theorem 2, we shall need the next lemma which is proved in [21].

LEMMA 1. *If $n \geq 3$ and $Z_1 = 0$, then M^n is a totally geodesic plane.*

THEOREM 2. *If $n \geq 3$, then $S \cdot Z = 0$ if and only if M^n is a totally geodesic plane.*

Proof. If M^n $(n \geq 3)$ is a totally geodesic plane, then obviously $S \cdot Z = 0$.

Next, assume that $n \geq 3$, $S \cdot Z = 0$ and M^n is not totally geodesic. Then by Lemma 1, $Z_1 \neq 0$, and by equations $(S \cdot Z)_{121} = 0$ and $(S \cdot Z)_{122} = 0$ we obtain $\alpha = \beta = 0$, so that $a = \pm b$, $\sigma = ab + (n-2)\,a\mu$. If now $\mu = 0$, then $\sigma = -a^2$, and immediately M^n is a totally geodesic plane, a contradiction. If $a = b$, then $\sigma = (2n-3)\,a^2$, and by equation $(S \cdot Z)_{133} = 0$ we obtain $\gamma\, Z_2 = 0$. But since then

$$\gamma = 4(n-2)\,a^2, \quad Z_2 = \frac{2(n^2 - 7n + 8)}{n(n-1)}\,a^2,$$

we find that $a = 0$, thus $\sigma = a = b = 0$, and M^n is totally geodesic, again a contradiction. ■

THEOREM 3. *If $n \geq 3$, then the following conditions are equivalent*: (1°) $S \cdot C = 0$; (2°) $G \cdot C = 0$; (3°) *M^n is conformally flat.*

Proof. The implications (3°) $\Longrightarrow$ (1°) and (3°) $\Longrightarrow$ (2°) are obvious.

If $n = 3$, then M^3 is conformally flat, so in the sequel we can assume that $n \geq 4$.

(1°) $\Longrightarrow$ (3°). Assume that $S \cdot C = 0$ and M^n is not conformally flat, thus $C_1 \neq 0$ and $C_2 \neq 0$. Then by equations $(S \cdot C)_{121} = 0$, $(S \cdot C)_{122} = 0$ and

$(S \cdot C)_{232} = 0$, we get $\alpha = \beta = \gamma = 0$, so M^n is Ricci flat. But then it must be totally geodesic, which is a contradiction.

(2°) $\implies$ (3°). Assume that $n \geq 4$, $G \cdot C = 0$ and M^n is not conformally flat, thus $R_1 \neq 0$. Then equations $(G \cdot C)_{121} = 0$, $(G \cdot C)_{122} = 0$ and $(G \cdot C)_{343} = 0$ immediately give $\alpha_0 = \beta_0 = \gamma_0 = 0$. Hence, M^n is Einstein, which yields that M^n is a totally geodesic plane, a contradiction. ■

In the proof of Theorem 4, we shall need another lemma which is also proved in [21].

Lemma 2. *If* $n \geq 3$ *and* $K_1 = K_2 = 0$*, then* M^n *is conharmonically flat.*

Theorem 4. *If* $n \geq 3$*, then the following conditions are equivalent*: (1°) $S \cdot K = 0$; (2°) $G \cdot K = 0$; (3°) M^n *is conharmonically flat.*

Proof. The condition (3°) obviously implies (1°) and (2°).

(1°) $\implies$ (3°). Assume that $n \geq 3$ and $S \cdot K = 0$.

If $n \geq 4$, then by equation $(S \cdot K)_{343} = 0$ we get $\mu = 0$, hence $K_2 = -R_1/(n-2)$, $\alpha = \beta = -R_1$, while equation $(S \cdot K)_{131} = 0$ gives $R_1 = 0$, so that M^n is a totally geodesic plane.

Let $n = 3$. Then

$$K_1 = K_2 = -R_1 + \mu^2, \quad \alpha = a\,\mu - R_1, \quad \beta = b\,\mu - R_1, \quad \gamma = \mu^2.$$

If $K_1 \neq 0$, then by the first, second and the fifth equation we get $\alpha = \beta = \gamma$, and hence $\mu = 0$, $R_1 = 0$. But this means that M^3 is totally geodesic. If $K_1 = 0$, then by Lemma 2, M^3 is conharmonically flat.

(2°) $\implies$ (3°). Suppose that $G \cdot K = 0$. If $K_2 \neq 0$, then by equations $(G \cdot K)_{131} = 0$, $(G \cdot K)_{232} = 0$, $(G \cdot K)_{133} = 0$, we get $\alpha_0 = \beta_0 = \gamma_0 = 0$, so that M^n is Einstein. But then it must be totally geodesic, which contradicts to $K_2 \neq 0$. Hence, $K_2 = 0$, i.e. $R_1 = (n-2)\,\mu^2$. If in addition, $K_1 = 0$, then M^n is conharmonically flat by Lemma 2. Assuming that $K_1 \neq 0$, by equations $(G \cdot K)_{121} = 0$, $(G \cdot K)_{122} = 0$, we find $\alpha_0 = \beta_0 = 0$, thus

$$(n-2)\,a\,\mu - R_1 = (n-2)\,b\,\mu - R_1 = \frac{\tau}{n}. \tag{2}$$

From the first equation in (2) we get $a = \pm b$. If $\mu = 0$, then $R_1 = 0$, and M^n is totally geodesic, which contradicts to $K_1 \neq 0$. Hence $\mu \neq 0$, which implies $a = b$. Then $R_1 = -2(n-2)\,a^2$, and equation (2) easily get $\mu = 0$, which is a contradiction again.

This completes the proof. ■

Theorem 5. *If* $n \geq 3$*, then following conditions are equivalent*: (1°) $G \cdot R = 0$; (2°) $G \cdot Z = 0$; (3°) M^n *is a totally geodesic plane.*

Proof. The condition (3°) obviously implies (1°) and (2°).

(1°) $\implies$ (3°). Assume that $n \geq 3$ and $G \cdot R = 0$.

If $\mu = 0$, then $R_1 = \sigma + a^2$, $\tau = -2R_1$, $\alpha_0 = -\dfrac{n-2}{n} R_1$, so that equation $(G \cdot R)_{121} = 0$ immediately gives $\sigma = a = 0$. Then $b = 0$ too, so M^n is a totally geodesic plane. Next, assume that $\mu \neq 0$, and M^n is not totally geodesic. Note that γ_0 can be written as

$$\gamma_0 = \frac{2\sigma + (n-2)\,a^2 + (n-2)\,b^2 + 2(n-3)\,ab}{n}.$$

Since $\sigma \geq 0$, and the expression $(n-2)\,a^2 + (n-2)\,b^2 + 2(n-3)\,ab \geq 0$ $(n \geq 3)$, and equals zero if and only if $a = b = 0$, we have that $\gamma_0 > 0$ since M^n is not totally geodesic. Now, using the equations $(G \cdot R)_{133} = 0$, $(G \cdot R)_{233} = 0$, we find $a\,\mu = b\,\mu = 0$, hence by adding $(a+b)\,\mu = \mu^2 = 0$, thus $\mu = 0$, which is a contradiction.

(2°) $\implies$ (3°). Now assume that $n \geq 3$, $G \cdot Z = 0$, and M^n is not totally geodesic. Then $\gamma_0 > 0$, and using the equations $(G \cdot Z)_{133} = 0$, $(G \cdot Z)_{233} = 0$, we find that $Z_2 = Z_3 = 0$. Hence, $a = \pm b$. If $\mu = 0$, then $\tau = -2R_1 = -2(\sigma + a^2)$,

$$\alpha_0 = -\frac{n-2}{n}\,R_1, \quad Z_1 = \frac{(n-2)(n+1)}{n(n-1)}\,R_1,$$

and by equation $(G \cdot Z)_{121} = 0$ we get $R_1 = 0$, thus $Z_1 = 0$. But this by Lemma 1 contradicts to assumption that M^n is not totally geodesic. If $a = b \neq 0$, then

$$\begin{aligned}
&\alpha_0 = -(n-2)(n-3)\,a^2, \quad \sigma = (n^2 - 5n + 5)\,a^2,\\
&R_1 = (n-1)(n-4)\,a^2, \quad \tau = 2n(n-1)\,a^2,\\
&Z_1 = (n-2)(n-3)\,a^2.
\end{aligned}$$

But then equation $(G \cdot Z)_{121} = 0$ gives $\alpha_0 Z_1 = 0$, thus $(n-3)\,a = 0$. Since $a \neq 0$, we then get $n = 3$, $Z_1 = 0$, and Lemma 1 gives that M^3 is totally geodesic, again a contradiction.

This completes the proof. ∎

Further, we shall consider conditions of the form $E_2 \cdot P = 0$, where E_2 equals S or G. If $n = 2$ then $P = 0$ so that $S \cdot P = G \cdot P = 0$.

If $n \geq 3$, then by a straightforward calculation, it can be seen that $E_2 \cdot P = 0$, thus $(E_2 \cdot P)_{ijk} = (E_2 \cdot P(e_i, e_j))e_k = 0$ for any choice of indices $i, j, k = 1, \ldots, n$ if and only if the following conditions hold: $(E_2 \cdot P)_{121} = (E_2 \cdot P)_{122} = (E_2 \cdot P)_{131} = (E_2 \cdot P)_{133} = (E_2 \cdot P)_{232} = (E_2 \cdot P)_{233} = (E_2 \cdot P)_{343} = 0$. Hence, $E_2 \cdot P = 0$ if and only if the following system of equations holds:

$$\begin{cases} \alpha\,P_1 = 0,\ \beta\,\widetilde{P}_1 = 0,\ \alpha\,P_2 = 0,\ \gamma\,\widetilde{P}_2 = 0,\\ \beta\,P_3 = 0,\ \gamma\,\widetilde{P}_3 = 0,\ \gamma\,P_0 = 0\ (n \geq 4). \end{cases} \tag{3}$$

Discussing this system in the particular cases $E_2 = S$ and $E_2 = G$, we get the next two theorems.

Theorem 6. *If $n \geq 3$, then $S \cdot P = 0$ if and only if one of the following cases occurs*: (1°) *M^n is totally geodesic*; (2°) *$n = 3$, $a = b \neq 0$ and $\sigma = 3a^2$.*

Proof. It is easy to prove that $S \cdot P = 0$ if for instance $n = 3$, $a = b \neq 0$ and $\sigma = 3\,a^2$.

Conversely, assume that $n \geq 3$ and $S \cdot P = 0$.

If $n \geq 4$, then equation $\gamma\, P_0 = 0$ gives that $\mu = 0$, so that equation $\alpha\, P_1 = 0$ implies $R_1 = 0$, and M^n is a totally geodesic plane.

Next, suppose that $n = 3$. Then the first and the second equation give that $R_1 = \pm a\,\mu = \pm b\,\mu$, and consequently $a = \pm b$. If $\mu = 0$, then $R_1 = 0$, and M^n is totally geodesic. If $a = b$, then the first equation implies $(\sigma + a^2)(\sigma - 3a^2) = 0$. If $\sigma = a = 0$, then M^3 is totally geodesic. Otherwise, we have $\sigma = 3a^2$, so we find the case (2°). ■

Theorem 7. *If $n \geq 3$, then $G \cdot P = 0$ if and only if M^n is a totally geodesic plane.*

Proof. Assume that $n \geq 3$, $G \cdot P = 0$ and M^n is not totally geodesic.

If $n \geq 4$, then $\gamma_0 > 0$, and the seventh equation gives $\mu = 0$. Then $\tau = -2R_1$, $\alpha_0 = -\dfrac{n-2}{n}\,R_1$, while the first equation gives $R_1 = 0$, which contradicts to M^n is not totally geodesic.

Finally, assume that $n = 3$. From the fourth and the sixth equations, we obviously get $a = \pm b$. If $\mu = 0$, then the first equation gives $R_1 = 0$, contradicting to M^3 is not totally geodesic. If $a = b$, then $\alpha_0 = -(\sigma + a^2)/3$, $R_1 + a\,\mu = \sigma + a^2$, and the first equation again gives $\sigma = a = 0$, thus again the same contradiction.

This ends the proof. ■

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