Abstract. We consider convolving a Gaussian of a varying scale ϵ against a Borel measure μ on Euclidean δ-dimensional space. The L^q norm of the result is differentiable in ϵ. We calculate this derivative and show how the upper order of its growth relates to the lower Rényi dimension of μ. We assume q is strictly between 1 and ∞ and that μ is finite with compact support.

Consider choosing a sequence ϵ_n of scales for the Gaussians

$$g_\epsilon(\mathbf{x}) = \epsilon^{-\delta} e^{-(|\mathbf{x}|/\epsilon)^2}.$$

Let $\|f\|_q$ denote the L^q norm for Lebesgue measure. The differences

$$\left| \left\| g_{\epsilon_{n+1}} * \mu \right\|_q - \left\| g_{\epsilon_n} * \mu \right\|_q \right|$$

between the norms at adjacent scales ϵ_n and ϵ_{n-1} can be made to grow more slowly than any positive power of n by setting the ϵ_n by a power rule. The correct exponent in the power rule is determined by the lower Rényi dimension.

We calculate and find bounds on the derivative of the Gaussian kernel versions of the correlation integral. We show that a Gaussian kernel version of the Rényi entropy sum is continuous.