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ON THE EMBEDDING OF PROPOSITIONAL MODELS

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Abstract. We consider the problem of isomorpical embedding for propositional models (where propositional letters are represented by propositional letters and, more generally, by propositional formulae) and prove some general theorems which parallel to those due to Los [1] and Keisler [2]. As a consequence of the proved theorems we obtain necessary and sufficient condions for embedding each model α of the language P in some model β of the set $\mathcal{F}$ of propositional formulae in the language Q. In the second part of the paper, in the case P, Q are finite and $\mathcal{F}$ is empty we prove that such embedding can be characterised in some other ways.

1. A *Propositional language* is any non-empty set of symbols which are called *propositional letters.* We suppose that each propositional language is indexed by some well-odered set. Let P be a propositional language, or simply language. By model of P we mean each mapping α of the forme $\alpha: P \to \{\top, \bot\}$. If P, Q are languages and $\alpha: P \to \{\top, \top\}$, $\beta: Q \to \{\top, \bot\}$ their models, then β is an *extension* of α iff then following conditions are satisfied:

(i) $P \subseteq Q$.

(ii) Restriction of the mapping β to P equals α, i.e. $\beta\,|_P = \alpha$. If β is an extension of α, then it is easy to see that for each formula F in P the following equivalence

$$\alpha \models F \text{ iff } \beta \models F \tag{1}$$

holds.

Further, let $f: P \to Q$ be an 1–1 mapping such that $\beta \circ f$ is an extension of α, where α, β are models of the languages P, Q respectively. Then we say that α is *f-embedded* in β. In other words:

$$\alpha \textit{ is } f\textit{-embedded in } \beta \textit{ iff } \beta \circ f = a$$

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[1]Where $\circ$ is defined as follows: $(\beta \circ f)(x) \stackrel{\text{def}}{=} \beta(f(x))$

If α if f-embedded in β and $F(u_1, \dots, u_n)$, $u_i \in P$ is any formula in P, then the following equivalence

$$\alpha \models F(u_1, \dots, u_n) \text{ iff } \beta \models F(f(u_1), \dots, f(u_n)) \tag{2}$$

holds, what can easily be proved.

If α, β are models of the languages P, Q raspectively, then we say that α is isomorphically embedable in β iff there exists an 1–1 mapping $f\colon P \to Q$ such that α is f-embedded in β. For example,

$$\alpha = \begin{pmatrix} p_0 & p_1 & \cdots & p_n & \cdots \\ \bot & \bot & \cdots & \bot & \cdots \end{pmatrix}_{n<\omega} \quad \beta = \begin{pmatrix} p_0 & p_1 & \cdots & q_{2n} & q_{2n+1} & \cdots \\ \top & \bot & \cdots & \top & \bot & \cdots \end{pmatrix}_{n<\omega}$$

then α is isomorphically embeddable in β. One embedding is:

$$f = \begin{pmatrix} p_0 & p_1 & \cdots & p_n & \cdots \\ q_1 & q_3 & \cdots & q_{2n+1} & \cdots \end{pmatrix}_{n<\omega}$$

Let now $P = \{p_i \mid i \in I\}$, $Q = \{q_j \mid j \in J\}$ be languages, α and β respectively their models, and let $\alpha(p_i) = \alpha_i (i \in I)$, $\beta(q_j) = \beta_j (j \in J)$. It is easy to see that the following lemma holds.

LEMMA 1. *β is an extension of α iff β is a model of the set P^α, where*

$$P^\alpha \stackrel{\text{def}}{=} \{p_i^{\alpha^i} \mid i \in I\}$$

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The set P^α is the diagram of α and it parallels to the notion of diagram in the predicate logic.

Let futher α be a model of P and let $\mathcal{F}$ be a set propositional formulae in the language Q, $P \subseteq Q$. Similar to the predicate case the following problem often arises: Decide wheather it is possible to extend α to some model β of the set $\mathcal{F}$. The sufficient and necessary conditions for this gives the following theorem.

THEOREM 1. *The model α can be extended to some model β of the set $\mathcal{F}$ iff α is a model for each consequence A of $\mathcal{F}$ which is of the form*

$$u_1^{a_1} \vee u_2^{a_2} \vee \cdots \vee u_k^{a_k} \tag{3}$$

where $a_i \in \{\top, \bot\}$, $u_i \in P$, such that $i \neq j \Rightarrow u_i \neq u_j$, i.e. iff for each formula of the form (3) *the following condition*

$$\mathcal{F} \vdash A \to \alpha \models A \tag{4}$$

[2] $u^\top$, $u^\bot$ designate the formulae u, $\neg u$ respectively.

holds.

PROOF. *Only if:* Suppose that β is an extension of α which is a model of $\mathcal{F}$. Further, let A be any formula in P such that $\mathcal{F} \vdash A$, then $\beta \models A$. Using (1) we immediately conclude $\alpha \models A$.

If: Suppose that (4) holds for each formula A of the form (3) and that α cannot be extended to a model of $\mathcal{F}$. That means that the set $\mathcal{F} \cup P^{\alpha}$ has no model. Using the compactness theorem we conclude that the set $\mathcal{F} \cup \mathcal{K}$, where $\mathcal{K}$ is some finite subset of P, has no model. Let

$$\mathcal{K} = \{p_{i_1}^{\alpha_{i_1}}, \dots, p_{i_k}^{\alpha_{i_k}}\}$$

Then the set

$$F \cup \{p_{i_1}^{\alpha_{i_1}} \vee \dots \vee p_{i_k}^{\alpha_{i_k}}\}$$

has no model. Therefore

$$\mathcal{F} \vdash \neg(p_{i_1}^{\alpha_{i_2}} \vee \dots \vee p_{i_k}^{\alpha_{i_k}}\}$$

i.e.

$$\mathcal{F} \vdash p_{i_1}^{\neg\alpha_{i_1}} \vee \dots \vee p_{i_k}^{\neg\alpha_{i_k}}\}$$

The formula

$$p_{i_1}^{\neg\alpha_{i_1}} \vee \dots \vee p_{i_k}^{\neg\alpha_{i_k}}$$

is obviously of the form (3), but it is not true on α, what contradicts (4).

We now generalise the notion of isomorphic embedding in the following way. Let P, Q be languages and $For(Q)$ the set of all propositional formulae in Q. Further, let α, β be models of P, Q respectively and let f be an 1–1 mapping from P to $For(Q)$[3]. We say that α is *f-embedded* in β iff $\beta \circ f = \alpha$. If α is f-embedded in β and $P = Q$, than β is an *f-extension* of α. In the case f is identity mapping, the notion of f-extension reduces to the notion of extension defined in the first part of the paper.

We give an example. Let $P = \{p_1, p_2, p_3, p_4\}$, $Q = \{q_1, q_2, q_3\}$ and let

$$\alpha = \begin{pmatrix} p_1 & p_2 & p_3 & p_4 \\ \bot & \top & \top & \bot \end{pmatrix}, \qquad \beta = \begin{pmatrix} q_1 & q_2 & q_3 \\ \top & \bot & \top \end{pmatrix}$$

If f is the following mapping

$$f = \begin{pmatrix} p_1 & p_2 & p_3 & p_4 \\ \neg q_1 & q_1 \vee q_2 & q_3 \Rightarrow q_1 & q_2 \vee \neg q_1 \end{pmatrix}$$

then α is f-embedded in β, for

$$\beta(\neg q_1) = \bot, \quad \beta(q_1 \vee q_2) = \top, \quad \beta(q_3 \Rightarrow q_1) = \top, \quad \beta(q_2 \vee \neg q_1) = \bot$$

[3]This implies that $\overline{\overline{P}} \leq \overline{\overline{For(Q)}}$.

therefrom it follows immediately $\beta \circ f = \alpha$.

Let α, β be models of P, Q respectively. We say that α *is ismorphically embeddable* in β (in the generalised sense) iff there exists an 1–1 maping $f\colon P \to For(Q)$ which is an f-embedding, i.e. such that the equality $\beta \circ f = \alpha$ holds.

If α is f-embedded in β, where α, β are models of P, Q respectively, then the equivalence (2) remains true for each formula $F(u_1, \dots, u_n)$ in P.

The following lemma parallels to lemma 1.

LEMMA 2. *Let $P = \{p_i \mid i \in I\}$, $Q = \{q_j \mid j \in J\}$ be languages, α and β their models, where $\alpha(p_i) = \alpha_i (i \in I)$ $\beta(q_j) = \beta_j$ $(j \in J)$. Further, let $f\colon P \to For(Q)$ be an 1–1 mapping, where $f(p_i) = P_i (i \in I)$. Then α is f-embedded in β iff β is a model of the set $f(P)^\alpha$, where*

$$f(P)^\alpha \stackrel{\text{def}}{=} \{P_i^{\alpha_i} \mid i \in I\}$$

We prove the following theorem which is a generalisation of theorem 1.

THEOREM 2. *Let $P = \{p_i \mid i \in I\}$, $Q = \{q_j \mid j \in J\}$ be languages, α a model of P, where $\alpha(p_i) = \alpha_i$, and let $\mathcal{F}$ be a set of forumlae in Q. α can be f-embedded in some model β of the set $\mathcal{F}$, where f is an 1–1 mapping from P to $For(Q)$, iff for each formula A in Q which is of the form*

$$U_1^{a_1} \vee U_2^{a_2} \vee \dots \vee U_k^{a_k} \qquad (U_i \in f(p), \qquad U_i \neq U_j \text{ if } i \neq j) \tag{5}$$

the following condition

$$\mathcal{F} \vdash A \to \alpha \models f^{-1}(A) \tag{6}$$

holds, where $f^{-1}(A)$ is the formula

$$f^{-1}(U_1)^{a_1} \vee f^{-1}(U_2)^{a_1} \vee \dots \vee f^{-1}(U_k)^{a_k}. \tag{7}$$

PROOF. *Only if:* Suppose that α is f-embedded in β, where β is a model of $\mathcal{F}$. Futher, let A be a formula in Q which is built up from the formulae in $f(P)$, i.e. A is of the form $A(U_1, \dots, U_k)$, where $U_i \in f(P)$. If $\mathcal{F} \vdash A(U_1, \dots, U_k)$, then $\beta \models A(U_1, \dots, U_k)$ therefrom, using (2), we immediately conclude $\alpha \models A(f^{-1}(U_1), \dots, f^{-1}(U_k))$.

If: Suppose now that (6) holds for each formula A of the form (5) and that α cannot be f-embedded in a model of $\mathcal{F}$, where $f\colon P \to For(Q)$ is a given 1–1 mapping. This means that the set $\mathcal{F} \cup f(P)^\alpha$ has no model. Then there exists a finite subset $\mathcal{K} \subseteq f(P)^\alpha$ such that $\mathcal{F} \cup \mathcal{K}$ has no model. Let

$$\mathcal{K} = \{f(p_{i_1})^{\alpha_{i_1}}, \dots, f(p_{i_k})^{\alpha_{i_k}}\}$$

Similar to the proof of theorem 1 we deduce

$$\mathcal{F} \vdash f(p_{i_1})^{\neg\alpha_{i_1}} \vee \cdots \vee f(p_{i_k})^{\neg\alpha_{i_k}}$$

The formula

$$f(p_{i_1})^{\neg\alpha_{i_1}} \vee \cdots \vee f(p_{i_k})^{\neg\alpha_{i_k}}$$

is obviously of the form (5) but its unverse image

$$p_{i_1}^{\neg\alpha_{i_1}} \vee \cdots \vee p_{i_k}^{\neg\alpha_{i_k}}$$

is not true on α, what contradicts (6).

Using the preceding theorem it is easy to obtain the following result.

THEOREM 3. *Each model α can be f-embedded in some model β of the set of formulae $\mathcal{F}$ iff there is no formula A of the form* (5) *which is a consequence of $\mathcal{F}$.*

PROOF. From the proceding theorem it follows that each model α can be f-embedded in some model β of $\mathcal{F}$ iff for each formula A of the form (5) the following condition

$$(\forall\alpha)(\mathcal{F} \vdash A \rightarrow \alpha \models f^{-1}(A)), \text{ i.e, } \mathcal{F} \vdash A \rightarrow \models f^{-1}(A)$$

holds. But the formula $f^{-1}(A)$ is of the form

$$u_1^a \vee \cdots \vee u_k^a \qquad (u_i \in P,\ u_i \neq u_j \text{ if } i \neq j)$$

and it cannot be a tautology. Thus, $\mathcal{F} \vdash A$ is not posible if A is of the form (5).

2. Let now $P = \{p_1, \ldots, p_n\}$, $Q = \{q_1, \ldots, q_m\}$ be finite languages, $\mathcal{F} = \emptyset$ and let f be the 1–1 maping

$$f = \begin{pmatrix} p_1 & p_2 & \cdots & p_n \\ A_1 & A_2 & \cdots & A_n \end{pmatrix}$$

where $A_i \in \mathit{For}(Q)$. Obviously, each model α of P can be f-embedded in some model β of Q iff the sequence

$$(A_1, A_2, \ldots, A_n) \tag{9}$$

can take each value $(\alpha_1, \alpha_2, \ldots, \alpha_n) \in \{\top, \bot\}^n$, i.e. iff

$$(\forall\alpha_1, \ldots, \alpha_n \in \{\top, \bot\})(\exists\beta\colon Q \rightarrow \{\top, \bot\})\beta A_1 = \alpha_1, \ldots, \beta A_n = \alpha_n \tag{10}$$

It is easy to see that (10) implies the condition $n \leq m$. Further if (10) holds for the sequence (9) with $n = m$, then it also holds for each subsequence of (9). Therefore it suffices to consider the case $m = n$. Thus, let $P = \{p_1, \ldots, p_n\}$, $Q + \{q_1, \ldots, q_n\}$ and let f be 1–1 mapping from P to $\mathit{For}(Q)$ determined by (8). Each model α of P can be f-embedded in some model of Q iff the condition (10) holds. Obviously, there are just $2^n!$ n-tuples[4] (9) satisfying (10). Namely, the sequence (9) has n members and as it must take each value, each permutation of the set $\{\top, \bot\}^n$ determines one sequence (9) satisfying the condition (10). For example, if $n = 2$, the number of such sequences is $2^2!$, i.e. 24. It is easy to see that all possible sets $\{A_1, A_2\}$ are the following:

$$\{q_1, q_2\},\ \{q_1, \neg q_2\},\ \{\neg q_1, q_2\},\ \{\neg q_1, \neg q_2\},\ \{q_1, q_1 \Leftrightarrow q_2\},\ \{q_1, \neg(q_1 \Leftrightarrow q_2)\}$$

$$\begin{aligned} \{\neg q_1, q_1 \Leftrightarrow q_2\},\ \{\neg q_1, \neg(q_1 \Leftrightarrow q_2)\},\ \{q_2, q_1 \Leftrightarrow q_2\},\ \{q_2, \neg(q_1 \Leftrightarrow q_2)\}, \\ \{\neg q_2, q_1 \Leftrightarrow q_2\}\ \{\neg q_2, \neg(q_1 \Leftrightarrow q_2)\}. \end{aligned} \tag{11}$$

Therefrom we immediately obtain all 24 ordered pairs (A_1, A_2).

Generally, for a given permutation

$$(\alpha_{11}, \alpha_{12}, \dots, \alpha_{1n}),\ (\alpha_{21}, \alpha_{22}, \dots \alpha_{2n}), \dots,\ (\alpha_{2^n 1}, \dots, \alpha_{2^n n}) \tag{12}$$

of the set $\{\top, \bot\}^n$ any formula A_i in (9) is determined by[5]

$$\bigvee_{(\lambda_1, \dots, \lambda_n) \in \{\top, \bot\}^n} a_{\lambda_1 \cdots \lambda_n} q_1^{\lambda_1} q_2^{\lambda_2} \cdots q_n^{\lambda_n}$$

where the sequence $(a_{\lambda_1 \dots \lambda_n})_{(\lambda_1, \dots, \lambda_n) \leq (\top, \dots, \top)}$ equals to $(\alpha_{ji})_{j \leq 2^n}$. In what follows we are going to give some other sufficient and necessary conditions for f-embedding each model $\alpha: P \to \{\top, \bot\}$ in some model $\beta: Q \to \{\top, \bot\}$. First of all we give some definitions.

Let A, B be propositional formulae in some given language and $\mathcal{F}_1$, $\mathcal{F}_2$ be sets of formulae. Then we defined

$$A\, equ\, B \ \text{ iff} \models A \Leftrightarrow B \tag{$D1$}$$

$$\begin{aligned} \mathcal{F}_1\, equ\, \mathcal{F} \ \text{ iff } \ & (\forall F_1 \in \mathcal{F}_1)(\exists F_2 \in \mathcal{F}_2) F_1\, equ\, F_2 \\ & (\forall F_2 \in \mathcal{F}_2)(\exists F_1 \in \mathcal{F}_1) F_2\, equ\, F_1 \end{aligned} \tag{$D2$}$$

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[4]That means, $2^n!$ n-tuples which are not equivalent to each other, i.e. which do not have equivalent corresponding cordinates.

[5]Throughout the paper instead of

$$(\dots(A_1^{a_1} \wedge A_2^{a_2}) \wedge \dots \wedge A_n^{a_n}),$$

where $A_1, \dots, A_n$ are formulae and $a_1, \dots, a_n \in \{\top, \bot\}$, we write

$$A_1^{a_1} A_2^{a_2} \dots A_n^{a_n}$$

We note that *equ* is an equivalence relation for formulae, i.e. for sets of formulae. Further, if elements of $\mathcal{F}_1$, $\mathcal{F}_2$ respectively are nonequivalent formulae, then the condition $\mathcal{F}_1\, equ\, \mathcal{F}_2$ implies $\overline{\overline{\mathcal{F}_1}} = \overline{\overline{\mathcal{F}_2}}$. We now prove the folowing theorem.

THEOREM 4. *Let $A_1, \ldots, A_n$ be formulae in $Q = \{q_1, \ldots, q_n\}$. Then the condition* (10) *is equivalent to*

$$\begin{aligned}\{A_1^{\alpha_1} A_2^{\alpha_2} \cdots A_n^{\alpha_n} \mid (\alpha_1, \ldots, \alpha_n) \in \{\top, \bot\}^n\}\, equ\, \{q_1^{\beta_1} q_2^{\beta_2} \cdots q_n^{\beta_n} \mid \\ \mid (\beta_1, \ldots, \beta_n) \in \{\top, \bot\}^n\}\end{aligned} \tag{14}$$

PROOF. The implication (14) $\to$ (10) follows immediately. To prove (10) $\to$ (14) we first note that (10) is equivalent to

$$(\forall \alpha_1, \ldots, \alpha_n \in \{\top, \bot\})(\exists_1 \beta \colon Q \to \{\top, \bot\} \beta A_1 = \alpha_1, \ldots, \beta A_n = \alpha_n \tag{15}$$

what can easily be proved. Suppose now (10) i.e. (15) and let $(\alpha_1, \ldots, \alpha_n)$ be an element of $\{\top, \bot\}^n$. By disjunctive normal form we have

$$A_1^{\alpha_1} A_2^{\alpha_2} \cdots A_n^{\alpha_n}\, equ \bigvee_{(\beta_2, \ldots, \beta_n) \in I} q_1^{\beta_1} q_2^{\beta_2} \cdots q_n^{\beta_n} \tag{16}$$

where $I \subseteq \{\top, \bot\}^n$. If $\overline{\overline{I}} \geq 2$, then there would be at least two values

$$(\beta_1, \ldots, \beta_n),\ (\bar{\beta}_1, \ldots, \bar{\beta}_n) \in I$$

such that for the corresponding models β, $\bar{\beta}$, say the following eqalities

$$\beta(A_1^{\alpha_1} A_2^{\alpha_2} \cdots A_n^{\alpha_n}) = \top, \qquad \bar{\beta}(A_1^{\alpha_1} A_2^{\alpha_2} \cdots A_n^{\alpha_n}) = \top$$

hold what contradicts (15). So $\overline{\overline{I}} = 1$, i.e. for some $(\beta_1, \ldots, \beta_n) \in \{\top, \bot\}^n$

$$A_1^{\alpha_1} A_2^{\alpha_2} \cdots A_n^{\alpha_n}\, equ\, q_1^{\beta_1} q_2^{\beta_2} \cdots q_n^{\beta_n}$$

Thus, we have just proved

$$\begin{gathered}(\forall \alpha_1, \ldots, \alpha_n \in \{\top, \bot\})(\exists \beta_1, \ldots, \\ \beta_n \in \{\top, \bot\} A_1^{\alpha_1} A_2^{\alpha_2} \cdots A_n^{\alpha_n}\, equ\, q_1^{\beta_1} q_2^{\beta_2} \cdots q_n^{\beta_n}\end{gathered} \tag{17}$$

It remains to prove

$$\begin{gathered}(\forall \beta_2, \ldots \beta_n \in \{\top, \bot\})(\exists \alpha_1, \ldots, \\ \alpha_n \in \{\top, \bot\}) A_1^{\alpha_1} A_2^{\alpha_2} \cdots A_n^{\alpha_n}\, equ\, q_1^{\beta_1} q_2^{\beta_2} \cdots q_n^{\beta_n}.\end{gathered} \tag{18}$$

Let $(\beta_1, \ldots, \beta_n)$ be an element of $\{\top, \bot\}^n$ and β be the following model

$$\beta = \begin{pmatrix} q_1 q_2 \cdots q_n \\ \beta_1 \beta_2 \cdots \beta_n \end{pmatrix} \tag{19}$$

Defining $\alpha_1, \ldots, \alpha_n$ as $\beta A_1, \ldots, \beta A_n$ respectively we immediately conclude

$$\begin{aligned} &\models q_1^{\beta_1} q_2^{\beta_2} \cdots q_n^{\beta_n} \Rightarrow A_1^{\alpha_1} A_2^{\alpha_2} \cdots A_n^{\alpha_n}, \text{i.e.} \\ &\models q_1^{\beta_1} q_2^{\beta_2} \cdots q_n^{\beta_n} \Rightarrow A_1^{\beta A_1} A_2^{\beta A_2} \ldots A_n^{\beta A_n} \end{aligned} \tag{20}$$

Using (15) it is easy to see that conversly

$$\begin{aligned} &\models A_1^{\alpha_1} A_2^{\alpha_2} \cdots A_n^{\alpha_n} \Rightarrow q_1^{\beta_1} q_2^{\beta_2} \cdots q_n^{\beta_n}, \text{i.e.} \\ &\models A_1^{\beta A_1} A_2^{\beta A_2} \cdots A_n^{\beta A_n} \Rightarrow q_1^{\beta_1} q_2^{\beta_2} \cdots q_n^{\beta_n} \end{aligned} \tag{21}$$

From (20) and (21) we obtain

$$\models q_1^{\beta_1} q_2^{\beta_2} \cdots q_n^{\beta_n} \Leftrightarrow A_1^{\beta A_1} A_2^{\beta A_2} \cdots A_n^{\beta A_n} \tag{22}$$

where β is defined by (19), wherefrom (18) follows immediately. The proof of the theorem is complete.

We now give another characterisation of the sequence (9) so that each model $\alpha: P \to \{\top, \bot\}$ can be f-embedded in some model $\beta: Q \to \{\top, \bot\}$.

THEOREM 5. *Each formula F in $Q = \{q_1, \ldots, q_2\}$ can be expressed in terms of $A_1, \ldots, A_n$ in the unique way, i.e. there exitst the unique 2^n-tuple*

$$(f_1, f_2, \ldots, f_{2^n}) \in \{\top, \bot\}^{2^n}$$

such that the equivalence

$$F \mathit{equ}\, f_1 A_1^{\top} A_2^{\top} \cdots A_n^{\bot} \vee f_2 A_1^{\top} A_2^{\top} \cdots A_n^{\bot} \vee \cdots \vee f_{2^n} A_1^{\bot} A_2^{\bot} \cdots A_n^{\bot} \tag{23}$$

holds iff the equivalence (14) *holds.*

PROOF. If (14) holds, then using the fact that each formula $F(q_1, \ldots, q_n)$ is equivalent to some formula of the form

$$a_1 q_1^{\top} q_2^{\top} \cdots q_n^{\top} \vee a_2 q_1^{\top} q_2^{\top} \cdots q_n^{\bot} \vee \cdots \vee a_{2^n} q_1^{\bot} q_2^{\bot} \cdots q_n^{\bot}$$

the eqivalence (23) follows immediately, where $(f_1, f_2, \ldots, f_{2^n})$ is a permutation of $(a_1, a_2, \ldots, a_{2^n})$.

Suppose now that each formula $F(q_1, \ldots, q_n)$ can be expressed, in the uniqe way, in terms of $A_1, \ldots, A_n$. By uniqueness it follows that no formula of the form

$$A_1^{\alpha_1} \cdots A_n^{\alpha_n}$$

is a contradiction. Thus for each $\alpha_1, \dots, \alpha_n \in \{\top, \bot\}$ there exists $\beta\colon Q \to \{\top, \bot\}^n$ such that

$$\beta(A_1^{\alpha_1} \cdots A_n^{\alpha_n}) = \top, \quad \text{i.e.} \quad \beta A_1 = \alpha_1, \dots, \beta A_n = \alpha_n.$$

Therefrom we conclude that (10) holds and thus (14) holds what follows by theorem 4.

Using the preceding two theorems we immediately obtain the following consequence.

Consequence. Let $P = \{p_1, \dots, p_n\}$, $Q = \{q_1, \dots, q_n\}$ be languages and

$$f = \begin{pmatrix} p_1 p_2 \cdots p_n \\ A_1 A_2 \cdots A_n \end{pmatrix}$$

an 1–1 mapping from P to For (Q). Then each model α of P can be f-embedded in some model β of Q iff the sequence $(A_1, \dots, A_n)$ satisfies the following condition:

Each formula F in Q can be expressed in the unique way in terms of $A_1, \dots, A_n$, i.e. there exists a unique 2^n-tuple $(f_1, f_2, \dots, f_{2^n})$, such that the equivalence (23) *holds.*

Problem. In the paper we give some caracterisations for f-embedding each model α of P in some model β of Q, when P, Q are finite languages. The problem is how to caracterise the same thing in the case P, Q are infinite.

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