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SOME CLOSURE PROPERTIES OF THE NONDETERMINISTIC REGULAR TRANSLATIONS

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Abstract. In this paper we examine closure properties of the class of nondeterministic reguler translation with respect to the operations of union, intersection and complemnt. it is shown that the class of nondtereministic regular translations in closed under the operation of union, and not closed under the operation of complementation and intersection.

Definition 1. A *finite transducer* M is a 6-tuple $(Q, \Sigma, \Delta, \delta, q_0, F)$, where

1) Q is a finite set of states.
2) Σ is a finite input alphabet.
3) Δ is a finite output alphabet.
4) δ as a mapping from $Q \times (\Sigma \cup \{e\})$ to finite subset of $q \times \Delta^*$.
5) $q_0 \in Q$ is the initial state.
6) $F \subseteq Q$ is the set of final states.

Definition 2. A *configuration* of M is a triple (q, x, y), where

1) $q \in Q$ is the current state of the finite control.
2) $x \in \Sigma^*$ is the input string remaining on the input tape, with the left-most symbol of x under the input head.
3) $y \in \Delta^*$ is the output string emitted up to this moment.

Definition 3. A *binary relation* on configurations is denoted $\vdash$ or $\underset{M}{\vdash}$.

It is introduced to reflest a move by M. For all $q \in Q$, $a \in \Sigma \cup \{e\}$, $x \in \Sigma^*$, and $y \in \Delta^*$ such that $\delta(p, a)$ contains (q, z) we write

$$(p, ax, y) \vdash (q, x, yz).$$

The symbols $\overset{+}{\vdash}$ and $\overset{*}{\vdash}$ are transitive extensions of the relation $\vdash$, respectively.

We say that y is an output for x if $(q_0,\, x,\, e) \overset{*}{\vdash} (q,\, e,\, y)$ for some q in F.

Definition 4. The *reguler translation* defined by M, denoted $\tau(M)$, is

$$\{(x,y) \,|\, (q_0,\, x,\, e) \overset{*}{\vdash} (q,\, e,\, y) \text{ for some } q \text{ in } F\}.$$

Notice that before an output string y can be considered a translation of an input string x, the finite transducer M must taje the input x from the initial state to a final state.

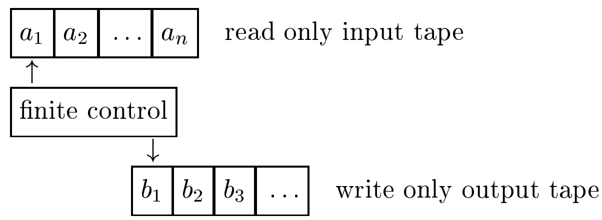

Fig. 1. Finite transducer

Definition 5. For a finite transducer $M = (Q,\, \Sigma,\, \Delta,\, \delta,\, q_0,\, F)$ we define input automaton $M_u = (Q,\, \Sigma,\, \delta_1,\, q_0,\, F)$, where δ_1 is a mapping from $Q \times (\Sigma \cup \{e\})$ to finite subset of Q, and δ_1 is defined in the following way:

for all $p,\, q \in Q,\; a \in \Sigma \cup \{e\},\; y \in \Delta^*$

$\delta_1(q,\, a) \ni p$ iff there exists y such that $\delta(q,\, a) \ni (p,\, y)$.

It is easy to see that the language $L(M_u)$ defined by the input automaton M_u is determined in the following way: $L(M_u) = \{x | (x,y) \in \tau(M)\}$.

Regular translation $\tau(M)$ can be treated as a binary relation. This relation is the subset of $\Sigma^* \times \Delta^*$. It leads to the consideration of set operations. The operation of complementation is defined as $(\tau(M))^c = (\Sigma^* \times \Delta^*) \backslash \tau(M)$. Closure properties of the class of nondeterministic regular translation with respect to the operations of union, intersection, and complement, are discussed further.

THEOREM 1. *The class of nondeterministic regular translations is closed under the operation of union.*

Proof. Let us denote by $M_i = (Q_i,\, \Sigma,\, \Delta,\, \delta_i,\, q_i,\, F_i)\,(i = 1,2)$ two arbitrary finite transducers. We shall denote by $T_i = \tau(M_i)\,(i = 1,2)$ the corresponding regular transducers. Without loss of generality, we shall suppose that $Q_1 \cap Q_2 = \varnothing$. Let us construct a finite transducer $M = (Q,\, \Sigma,\, \Delta,\, \delta,\, q_0,\, F)$, so that the corresponding regular translation T will be the union of T_1 and T_2. Let us consider a start state $q_0 \notin Q_1 \cup Q_2$ and $Q = Q_1 \cup Q_2 \cup \{q_0\}$. We define mapping δ in the following way:

$$\delta(q_0,\, a) = \delta_1(q_1,\, a) \cup \delta_2(q_2,\, a) \text{ for all } a \in \Sigma \cup \{e\},$$

$$\delta(q,\, a) = \delta_1(q_1,\, a) \text{ for all } q \in Q_1 \text{ and all } a \in \Sigma \cup \{e\},$$

$$\delta(q,\, a) = \delta_2(q_1,\, a) \text{ for all } q \in Q_2 \text{ and all } a \in \Sigma \cup \{e\},$$

The set final states is determined by $F = F_1 \cup F_2$.

It is easy to show by induction on $i \geq 1$ that $(q_0,\, x,\, e) \overset{i}{\underset{M}{\vdash}} (q,\, e,\, y)$ af and only if $q \in Q_1$ and $(p_1,\, x,\, e) \overset{i}{\underset{M}{\vdash}} (q,\, e,\, y)$ or

$q \in Q_2$ and $q_2,\, x,\, e) \overset{i}{\underset{M_2}{\vdash}} (q,\, e,\, y)$.

So it is shown that $(x, y) \in T$ if and only if $(x, y) \in T_1 \cup T_2$.

Theorem 2. *The class of nondeterministic regular translations is not closed under the operation of complementation.*

Proof: Let M be an arbitrary finite transducer, and let $T = \tau(M)$ be the corresponding regular translation. We shall suppose the contrary, i.e. that the complement of T, denoted T^c, is again a regular translation, and we shall show that this assumption is not true. Let us suppose that the complement of a regular translation is again a regular translation. As shown in the previous theorem, the union of two regular translations is again a regular translation. It would imply that the intersection $T_1 \cap T_2$ of two regular translations is also a regular translation because $T_1 \cap T_2 = (T_1^c \cup T_2^c)^c$. Further on, for regular translations T_1 and T_2, translation T defined as $T = (T_1 \cap T_2^c) \cup (T_1^c \cap T_2)$ would be regular, too. Using the et property $(A \cap B^c) \cup (A^c \cap B) = \varnothing$ if and only if $A = B$, we can see that the translation $T = (T_1 \cap T_2^c) \cup (T_1^c \cap T_2)$ is empty if and only if $T_1 = T_2$. As it is shown by [**3**] the equivalence problem is algorithmicaly unsolvable for arbitrary regular translations T_1 and T_2. These facts imply, that for an arbitrary regular translation, the emptiness problem is algorithmicaly unsolvable. On the other side, from the definition of the finite transducer, whichstates that it is an automaton that can emit an output word y only if the input word x is accepted by the input automaton M_u it follows that the translation T is empty if and only if the input automaton language $L(M_u) = \varnothing$. The emptiness problem is algorithicaly solvable for finite automaton language, see [**2**]. hence, the emptiness problem is algorithmicaly solvable for regular translations, too. These leads to the contradiction. Hence, the assumption is not true. So, it is proved that regular translations are not closed under the operation of complementation.

Theorem 3. *The class of nondeterministic regular translation is not closed under the operation of intersection.*

Proof. We shall suppose that the class of regular translations is closed under the operation of intersection. As shown previously, the emptiness problem is algorithmicaly solvable for an arbitrary nondeterministic regular translation. The algorithmic solvability of the emptiness problem of the intersection of two arbitrary regular translations dollows from this fact and the above assumption. But in [**4**] it is shown that the emptiness problem of the intersectin of the two nondeterministic regular translations is algorithmicaly unsolvable. Hence, the assumption is not true. So it is shown that the intersection of the two regular translations it not necesarily regular translation.

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