ON ANTI-INVERSE RINGS

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Following B. Cerović [1], a ring $R (\neq 0)$ is called an anti-inverse ring if every element $x$ in $R$ has an anti-inverse $x^*: x^*xx^* = x$ and $xx^*x = x$. If $R$ is an anti-inverse ring, then so is every non-zero homomorphic image of $R$, and $x^2 = x^{*2} = (xx^*)^2 = (x^*x)^2$ and $x = x^{*2}xx^* = x^3$ for any $x \in R$; in particular, $R$ is a strongly regular ring.

The present objective is to prove neatly the following proposition which covers all the results in [1].

**Proposition.** The following are equivalent:

1. $R$ is an anti-inverse ring.
2. $R$ is a subdirect sum of $GF(2)'$s and $GF(3)'$s.
3. $R$ satisfies the polynomial identity $x^3 - x = 0$.

**Proof.** Obviously, (2) $\Rightarrow$ (3) $\Rightarrow$ (1). It remains therefore to prove that (1) implies (2). Without loss of generality, we may assume that $R$ is subdirectly irreducible. Then, we can easily see that the strongly regular ring $R$ is a division ring. Now, let $x$ be an arbitrary non-zero element of $R$. Then, $x^2 = 1$ and $0 = (xx^* - x^*x)^2 = 2(x^2 - x^4) = 2(x^2 - 1)$. Hence, if $R$ is not of characteristic 2 then $x = \pm 1$, and so $R = GF(3)$. On the other hand, if $R$ is of characteristic 2 then $0 = x^4 - 1 = (x - 1)^2$ implies $x = 1$, and so $R = GF(2)$.

**REFERENCES**