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ON A RUSCHEWEYH TYPE GENERALIZATION OF THE PASCU CLASS OF ANALYTIC FUNCTIONS

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Abstract. New classes $M_a(\alpha;h), R_a(\alpha,h)$ and $I_a(\alpha;h)$ of analytic functions are defined and studied. results of this paper generalizee mainly results of Padmanabham and Manjini [**5**], Padmanabhan and Parvatham [**4**] and Pascu [**5**].

Let $E = \{z \in \mathbf{C} : | z |< 1\}$ be the open unit disc in $\mathbf{C}$ and $H(E)$ be the class of functions $f(z)$ holomorphic in E. Let $A = \{f \in H(E); f(0) = 0 = f'(0) - 1\}$. By $f * g$ we denote the Hadamard product or convolution of $f, g \in H(E)$; that is if $f(z) = \sum\limits_{j=0}^{\infty} a_j z^j$ and $g(z) = \sum\limits_{j=0}^{\infty} b_j z^j$ then $(f * g)(z) = \sum\limits_{j=0}^{\infty} a_j b_j z^j$.

Let g and G be two functions in $H(E)$. Then $g(z)$ is said to be subordinate to $G(z)$ (writen $g(z) \prec G(z)$) if G is univalent, $g(0) = G(0)$ and $g(E) \subset G(E)$. Let $k_a(z) = z(1 - z)^{-a}$, where a is any real number. In the sequal $h \in H(E)$ is a convex univalent function in E with $h(0) = 1$ and Re $h(z) > 0$ in E.

In this paper we define certain new classes of functions holomorphic in E with Montel's normalizations and study these classes in detail. To establish results of this paper connected with these new classes, we require the following two theorems. Theorem A is due to Eenigenburg, Miller, Mocanu and Reade [**2**] and Theorem B may be found in [**4**].

THEOREM A. *Let $\beta, \gamma \in \mathbf{C}$ and $h \in H(E)$ be convex univalent in E with $h(0) = 1$ and $Re(\beta h(z) + \gamma) > 0$ in E, and let $p(z) = 1 + p_1 z + \cdots \in H(E)$. Then $p(z) + \frac{zp'(z)}{\beta p(z)+\gamma} \prec h(z)$ implies $p(z) \prec h(z)$.*

THEOREM B. *Let $\beta, \gamma \in \mathbf{C}$, $k \in H(E)$ be convex univalent in E with $h(0) = 1$ and $Re(\beta h(z) + \gamma) > 0$ in E. Let $q \in H(E)$ with $q(0) = 1$ and $q(z) \prec h(z)$ in E. If $p(z) = 1 + p_1 z + \cdots \in H(E)$ then $p(z) + \frac{zp'(z)}{\beta q(z)+\gamma} \prec h(z)$ implies $p(z) \prec h(z)$.*

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First let us define a new class $M_a(\alpha; h)$ of holomorphic functions in E and study its properties.

Definition 1. Let $M_a(\alpha; h)$ denote the class of functions f with

$$(k_a * f)'(z)(k_a * f)(z) \neq 0 \text{ in } E\text{–}\{0\} \quad \text{satisfying}$$

$$\frac{\alpha z(z(k_a * f)'(z))' + (1-\alpha)z(k_a * f)'(z)}{\alpha z(k_a * f)'(z) + (1-\alpha)(k_a * f)(z)} \prec h(z) \text{ for } \alpha \geq 0.$$

Note 1. When $\alpha = 0$ this class coincides with the class $S_a(h)$ studied in [**4**] and when $\alpha = 1$ this is the same class as $K_a(h)$ in [**3**]. Also $M_1(\alpha; (1-z)(1+z)^{-1})$ is the class introduced by Pascu and Podaru [**6**].

Theorem 1. *For* $0 < \alpha \leq 1$ *we have* $M_a(\alpha; h) \subset M_a(0; h) = S_a(h)$.

Proof. Let $f \in M_a(\alpha; h)$ and $p(z) = \frac{z(k_a * f)'(z)}{(k_a * f)(z)}$. Then

$$\begin{aligned} &\alpha z(z(k_a * f)'(z))' + (1-\alpha)z(k_a * f)'(z) \\ &\quad = \alpha z(k_a * f)(z)p'(z) + \alpha z p(z)(k_a * f)'(z) + (1-\alpha)p(z)(k_a * f)(z) \\ &\quad = (\alpha z p'(z) + p(z)(ap(z) + (1-\alpha)))(k_a * f)(z); \\ &az(k_a * f)'(z) + (1-\alpha)(k_a * f)(z) = (ap(z) + (1-\alpha))(k_a * f)(z) \end{aligned}$$

Hence

$$\begin{aligned} \frac{\alpha z(z(k_a * f)'(z))' + (1-\alpha)z(k_a * f)'(z)}{\alpha z(k_a * f)'(z) + (1-\alpha)(k_a * f)(z)} &= \frac{\alpha z p'(z) + p(z)(\alpha p(z) + (1-\alpha))}{\alpha p(z) + (1-\alpha)} \\ &= \frac{zp'(z)}{p(z) + (\alpha^{-1} - 1)} + p(z) \prec h(z) \end{aligned}$$

because $f \in M_a(\alpha; h)$. Since $0 < \alpha \leq 1$, an application of Theorem A gives $p(z) \prec h(z)$ in E which implies $f \in S_a(h)$.

Theorem 2. *Let* $f \in M_a(\alpha; h)$. *Then for* $0 < \alpha \leq 1$ *we have*

$$F(z) = \alpha^{-1} z^{1-1/\alpha} \int_0^z t^{1/\alpha - 2} f(t)dt \in M_a(\alpha; h).$$

Proof. Differentiating $F(z) = \alpha^{-1} z^{1-1/\alpha} \int_0^z t^{1/\alpha-2} f(t)dt$ with respect to z and simplifying we get $\alpha z F'(z) + (1-\alpha)F(z) = f(z)$. This, by convolution with $k_a(z)$, gives

$$\alpha z(k_a * F)'(z) + (1-\alpha)(k_a * F)(z) = (k_a * f)(z)$$

where we used the fact that $k_a * zF'(z) = z(k_a * F)'(z)$. Taking logarithmic derivative with respect to z and multiplying by z we get

$$\frac{\alpha z(z(k_a * F)'(z))' + (1-\alpha)z(k_a * F)'(z)}{\alpha z(k_a * F)'(z) + (1-\alpha)(k_a * F)(z)} = \frac{z(k_a * f)'(z)}{(k_a * f)(z)}.$$

The member on the right hand side is subordinate to $h(z)$ since $f \in M_a(\alpha;h) \subset S_a(h)$ by the previous theorem. Also $F(z) = \gamma(z) * f(z)$ where $\gamma_\alpha(z) = \sum_{n=1}^{\infty} \frac{1/\alpha}{1/\alpha+n-1} z^n$. Since $(k_a * f)(z) \neq 0$, $(k_a * f)'(z) \neq 0$ in $E - \{0\}$ and $\alpha > 0$ we have $(k_a * F)(z) = \gamma_a(z)(k_a * f)(z) \neq 0$, hence $(k_a F)'(z) \neq 0$ in $E - \{0\}$. Thus $F \in M_a(\alpha;h)$.

We now obtain an estimate for the modulii of the coefficients, $\mid a_n \mid$ where $f(z) = z + \sum_2^{\infty} a_n z^n \in M_a(\alpha;h)$

Theorem 3. *Let* $f(z) = z + \sum_{i=2}^{\infty} a_i z^i$ *be in* $M_a(\alpha;h)$ *Then*

$$\mid a_i \mid \leq \frac{\mid h_1 \mid (1+ \mid h_1 \mid) \cdots (i-2+ \mid h_1 \mid)}{((i-1)\alpha+1)a(a+1)\cdots(a+i-2)}, \quad i \geq 2), \tag{1}$$

where $h(z) = 1 + h_1 z + \cdots$.

Proof. Let $\dfrac{\alpha z(z(k_a * f)'(z))' + (1-\alpha)z(k_a * f)'(z)}{\alpha z(k_a * f)'(z) + (1-\alpha)(k_a * f)(z)} = p(z) = 1 + p_1 z + \cdots$.

Since $f \in M_a(\alpha;h), p(z) \prec h(z) = 1 + h_1 z + h_2 z^2 + \cdots$. It is well known that $\mid p_i \mid \leq \mid h_1 \mid$ for all $i \geq 2$. Now,

$$\alpha z(z(k_a * f)'(z))' + (1-\alpha)z(k_a * f)'(z) = p(z)(\alpha z(k_a * f)'(z) + (-\alpha)(k_a * f)(z)) \tag{2}$$

and $k_\alpha(z) = z(1-z)^{-a} = z + \sum_{i=2}^{\infty} b_i z^i$ where $b_i = a(a+1)\cdots(a+i-2)/(i-1)!$. By actual computation we have

$$\alpha z(z(k_a * f)'(z))' + (1-\alpha)z(k_a * f)'(z) = z + \sum_{i=2}^{\infty} i((i-1)\alpha+1)a_i b_i z^i;$$

$$p(z)(\alpha z k_a * f)'(z) + (1-\alpha)(k_a * f)(z)) =$$
$$=(1 + p_1 z + \cdots)\left(z + \sum_{i=2}^{\infty}((i-1)\alpha+1)a_i b_i z^i\right).$$

Now comparing the coefficients on either side of (2) we get

$$i((i-1)\alpha+1)a_i b_i = p_{i=1} + p_{i-2}a_2 b_2(\alpha+1) + p_{i-3}a_3 b_3(2\alpha+1)$$
$$+ \cdots + p_1 a_{i-1} b_{i-1}((i-2)\alpha+1) + a_i b_i((i-1)\alpha+1)).$$

Let (1) be true for all $i = 2, \ldots, n-1$. In other words

$$\mid a_i b_i \mid \leq \frac{\mid h_1 \mid (1+ \mid h_1 \mid) \cdots (1+ \mid h_1 \mid (i-2)^{-1})}{(i-1)((i-1)\alpha+1)} \text{ for } i = 2,3,\ldots,n-1. \tag{3}$$

Now for $i = n$ we have

$$n(n-1)\alpha+1)a_nb_n = p_{n-1} + p_{n-2}a_2b_2(\alpha+1) + p_{n-3}a_3b_3(2\alpha+1) = \cdots$$
$$\cdots + p_1a_{n-1}b_{n-1}((n-2)\alpha+1) + a_nb_n((n-1)\alpha+1);$$

$$(n-1)((n-1)\alpha+1)a_nb_n = p_{n-1} + p_{n-2}a_2b_2(\alpha+1) + \cdots$$
$$\cdots + p_1a_{n-1}b_{n-1}((n-2)\alpha+1);$$

$$(n-1)((n-1)\alpha+1) \mid a_nb_n \mid \leq \mid h_1 \mid + \mid h_1 \mid (\alpha+1) \mid a_2b_2 \mid + \cdots$$
$$\cdots + \mid h_1 \mid ((n-2)\alpha+1) \mid a_{n-1}b_{n-1} \mid \leq \mid h_1 \mid + \mid h_1 \mid^2 /2 \cdot (1+ \mid h_1 \mid) + \cdots$$
$$\cdots + \mid h_1 \mid^2 /(n-2) \cdot (1+ \mid h_1 \mid) \cdots (1+ \mid h_1 \mid /(n-3))$$
$$= \mid h_1 \mid (1+ \mid h_1 \mid)(1+ \mid h_1 \mid /2) \cdots (1+ \mid h_1 \mid /(n-2)).$$

Hence

$$\mid a_nb_n \mid \leq \frac{\mid h_1 \mid (1+ \mid h_1 \mid)(1+ \mid h_1 \mid /2) \cdots (1+ \mid h_1 \mid /(n-2))}{(n-1)((n-1)\alpha+1)}$$

which means (3) is true for $i = n$ provided it is true for $i = 2, 3, \cdots, n-1$. It is easy to see that (1) is true for $i = 2$ and hence it is true for all $i \geq 2$.

Remark. The results above generalize many results found in [**3, 4**] and [**6**].

Now we define another new class of functions $R_a(\alpha; h)$ which generalizes both the class $C(\alpha; h)$ of Pascu [**5**] and the class $C_a(h)$ in [**4**].

Definition 2. Let $R_a(\alpha; h)$ denote the class of functions $f \in A$ such that $\frac{\alpha z(z(k_a*f)'(z))' + (1-\alpha)z(k_a*f)'(z)}{\alpha z(k_a*\varphi)'(z) + (1-\alpha)(k_a*\varphi)(z)} \prec h(z)$ for some $\varphi \in M_a(\alpha; h)$ and $\alpha \geq 0$.

Here we prove an inclusion relation and also the fact that this class is closed under a certain integral operator.

THEOREM 4. *$R_a(\alpha; h) \subset R_a(O; h) = C_a(h)$ for $0 < \alpha < 1$.*

Proof. Let $f \in M_a(\alpha; h)$. Seting

$$p(z) = \frac{z(k_a*f)'(z)}{(k_a*\varphi)(z)} \text{ and } q(z) = \frac{z(k_a*\varphi)'(z)}{(k_a*\varphi)(z)},$$

we have

$$\frac{\alpha z(z(k_a*f)'(z))' + (1-\alpha)z(k_a*f)'(z)}{\alpha z(k_a*\varphi)'(z) + (1-\alpha)(k_a*\varphi)(z)}$$
$$= \frac{azp'(z) + p(z)\left(\frac{\alpha z(k_a*\varphi)(z)}{(k_a*\varphi)'(z)} + 1 - \alpha\right)}{\frac{\alpha z(k_a*\varphi)'(z)}{(k_a*\varphi)'(z)} + (1-\alpha)}$$
$$= p(z) + \frac{zp'(z)}{\frac{z(k*\varphi)'(z)}{(k_a*\varphi)(z)} + \left(\frac{1}{\alpha} - 1\right)} \prec h(z)$$

since $f \in R_a(\alpha;h)$. Here $q(z) \prec h(z)$ by Theorem 1. Since $\alpha \leq 1$ an application of Theorem B yields $p(z) \prec h(z)$ there by establishing the theorem.

THEOREM 5. *$F(z) = \alpha^{-1} z^{1-1/\alpha} \int_0^z t^{1/\alpha-2} f(t)dt \in R_a(\alpha;h)$ whenever $f \in R_a(\alpha;h)$, for $0 < \alpha \leq 1$.*

Proof. Differentiating F with respect to z we have

$$\alpha z F'(z) + (1-\alpha)F(z) = f(z).$$

This on convolution with $k_a(z)$ yields

$$\alpha z(k_a * F)'(z) + (1-\alpha)(k_a * F)(z) = (k_a * f)(z),$$

where we used the identity $(k_a * zF')(z) = z(k_a * F)'(z)$. Again differentiating with respect to z we get

$$\alpha z(z(k_a * F)'(z))' + (1-\alpha)z(k_a * F)'(z) = z(k_a * f)'(z), \quad f \in R_a, (\alpha;h). \tag{4}$$

Hence there exist a $\varphi \in M_a(\alpha;h)$ such that

$$\frac{\alpha z(z(k_a * f)'(z))' + (1-\alpha)z(k_a * f)'(z)}{\alpha z(k_a * \varphi)'(z) + (1-\alpha)z(k_a * \varphi)(z)} \prec h(z) \text{ in } E.$$

Then by Theorem 2, Φ defined by

$$\Phi(z) = \alpha^{-1} z^{1-1/\alpha} \int_0^z t^{1/\alpha-2} \varphi(t)dt \tag{5}$$

is in $M_a(\alpha;h)$ for $1 \geq \alpha > 0$. Differentiating (5) with recpect to z and convoluting the result with $k_a(z)$ yields after simplication

$$\alpha z(k_a * \Phi)'(z) + (1-\alpha)(k_a * \Phi)(z) = (k_a * \varphi)(z). \tag{6}$$

Finally (4) and (6) together yield,

$$\frac{\alpha z(z(k_a * F)'(z))' + (1-\alpha)z(k_a * F)'(z)}{\alpha z(k_a * \varphi)'(z) + (1-\alpha)(k_a * \varphi)(z)} = \frac{z(k_a * f)(z)}{(k_a * \varphi)(z)}.$$

Since $f \in R_a(\alpha;h)$ by Theorem 4 we get $\frac{z(k_a * f)'(z)}{(k_a * \varphi)(z)} \prec h(z)$ for $z \in E,\ 0 < \alpha \leq 1$. In the same way as in Theorem 2 we can show that $(k_a * F)'(z) \neq 0,\ (k_a * F)(z) \neq 0$ in $E - \{0\}$ from the fact that $(k_a * f)'(z) \neq 0,\ (k_a * f)(z) \neq 0$ in $E - \{0\}$ for $\alpha > 0$. Thus we get $F \in R_a(\alpha;h)$.

Remark. Theorem 4 and Theorem 5 generalize results in [**4**] and [**5**].

Now let us establish a representation theorem for function belonging to the class $R_a(\alpha;h)$.

THEOREM 6. *A functions f belongs to $R_a(\alpha;h)$ if and only there exists a function $G \in H(E)$ with $G(0) = 0$ such that $zG'(z)/G(z) \prec h(z)$ in E and an analytic function $p(z)$ with $p(0) = 1$ and $p(z) \prec h(z)$ in E such that*

$$(k_a * f)'(z) = \alpha^{-1} z^{-1/\alpha} \int_0^z p(t)G(t)t^{1/\alpha-2}dt, \quad \textit{if } \alpha \neq 0;$$

$$(k_a * f)'(z) = p(z)G(z)/z, \quad \textit{if } \alpha = 0.$$

Proof. $f(z) \in R_a(\alpha; h)$ means that there exists, a $\varphi \in M_a(\alpha; h)$ such that

$$\frac{\alpha z(z(k_a * f)'(z))' + (1-\alpha)z(k_a * f)'(z)}{\alpha z(k_a * \varphi)'(z) + (1-\alpha)(k_a * \varphi)(z)} \prec h(z).$$

Since $\varphi(z) \in M_a(\alpha; h)$, there exists a $G(z)$ such that

$$\frac{zG'(z)}{G(z)} = \frac{\alpha z(z(k_a * \varphi)'(z))' + (1-\alpha)z(k_a * \varphi)'(z)}{\alpha z(k_a * \varphi)'(z) + (1-\alpha)(k_a * \varphi)(z)} \prec h(z).$$

This on integration gives $G(z) = \alpha z(k_a * \varphi)'(z) + (1-\alpha)(k_a * \varphi)(z)$ and so $\alpha z(z(k_a * f)'(z))' + (1-\alpha)z(k_a * f)'(z) = p(z)G(z)$ where $p(z) \prec h(z)$. In $\alpha \neq o$, multiplying by $\alpha^{-1}z^{1/\alpha-2}$ and integrating we get

$$(k_a * f)'(z) = a^{-1}z^{-1/\alpha}\int_0^s p(t)G(t)t^{1/\alpha-2}dt.$$

Conversely, it is easy to see that if $f(z)$ has the above integral representation then $f(z) \in R_a(\alpha; h)$. For $\alpha = 0$ let $G(z) = (k_a * \varphi)(z)$. Then $(k_a * f)'(z) = p(z)G(z)/z$, where $p(z) \prec h(z)$ and the converse is trivially true.

Finally we define a new class $I_a(\alpha; h)$ which coincides with $C_a(h)$ of [**4**] when $\alpha = 0$. In particular if $\varphi(z)$ coincides with $f(z)$ then $I_a(\alpha; h)$ is nothing but $K_a^\alpha(h)$ studied in [**3**].

Definition 3. Let $I_a(\alpha; h)$ denote the class of functions $f \in A$ such that

$$\frac{\alpha(z(k_a * f)'(z))'}{(k_a * \varphi)'(z)} + (1-\alpha)\frac{z(k_a * f)'(z)}{(k_a * \varphi)(z)} \prec h(z)$$

for some $\varphi \in S_a(h)$ and $\alpha \geq 0$ in E.

Remark. Though for $a = 1$ the class $I_a(\alpha; h)$ coincides with $C_a^\alpha(h)$ studied in [**4**] for other values of $a, I_a(\alpha; h)$ is certainly different from $C_a^\alpha(h)$.

THEOREM 7. *We have the folloving inclusion relation* $I_a(\alpha; h) \subset I_a(0; h) = C_a(h)$.

Proof. Let $f \in I_a(\alpha; h)$. Setting

$$p(z) = \frac{z(k_a * f)'(z)}{(k_a * \varphi)(z)} \text{ and } q(z) = \frac{z(k_a * \varphi)'(z)}{(k_a * \varphi)(z)}$$

we have

$$\begin{aligned}&\frac{\alpha(z(k_a * f)'(z))'}{(k_a * \varphi)'(z)} + (1-\alpha)\frac{z(k_a * f)'(z)}{(k_a * \varphi)(z)} = \\ &= \alpha(zp'(z)/q(z) + p(z)) + (1-\alpha)p(z) = p(z) + azp'(z)/q(z).\end{aligned}$$

Since $f(z) \in I_a(\alpha, h)$ we have $p(z) + azp'(z)/q(z) \prec h(z)$ and $q(z) \prec h(z)$. Now an application of Theorem B yields that $p(z) \prec h(z)$ which imlies $f \in I_a(\alpha; h)$.

THEOREM 8. *For $\alpha > \beta \geq 0$ we have $I_a(\alpha; h) \subset I_a(\beta; h)$.*

Proof. The case $\alpha = 0$ has already been proved in Theorem 7. Hence we can assume $\beta \neq 0$. Suppose that $f(z) \in I_a(\alpha; h)$. Then,

$$\frac{\alpha(z(k_a * f)'(z))'}{(k_a * \varphi)(z)} + (1-\alpha)\frac{z(k_a * f)'(z)}{(k_a * \varphi)(z)} \prec h(z).$$

Let z_1 be an arbitrary point in E. Then,

$$\frac{\alpha(z_1(k_a * f)'(z_1))'}{(k_a * \varphi)'(z_1)} + (1-\alpha)\frac{(z_1(k_a * f)'(z_1))}{(k_a * \varphi)(z_1)} \in h(E). \tag{7}$$

Because of Theorem 7 we have $\frac{z(k_a * f)'(z)}{(k_a * \varphi)(z)} \prec h(z)$. Hence,

$$\frac{z_1(k * f)'(z_1)}{(k_a * \varphi)(z_1)} \in h(E). \tag{8}$$

Also

$$\begin{aligned}\frac{\beta(z(k_a * f)'(z))'}{(k_a * \varphi)'(z)} &+ (1-\beta)\frac{z(k * f)'(z)}{(k_a * \varphi)(z)} = \\ &= \left(1 - \frac{\beta}{\alpha}\right)\frac{z(k_a * f)'(z)}{(k_a * \varphi)(z)} + \frac{\beta}{\alpha}\frac{\alpha(z(k_a * f)'(z))'}{(k_a * \varphi)'(z)} + (1-\alpha)\frac{z(k_a * f)'(z)}{(k_a * \varphi)(z)},\end{aligned}$$

Since $\beta/\alpha < 1$ and $h(E)$ is convex we have

$$\frac{\beta(z_1(k_a * f)'(z_1))'}{(k_a * \varphi)'(z_1)} + (1-\beta)\frac{s_1(k_a * f)'(z_1)}{(k_a * \varphi)(z_1)} \in h(E) \text{ by (7) and (8).}$$

Thus it follows

$$\frac{\beta(z(k_a * f)'(z))'}{(k_a * \varphi)'(z)} + (1-\beta)\frac{z(k_a * f)'(z)}{(k_a * \varphi)(z)} \prec h(z)$$

which means $f(z) \in I_a(\beta; h)$.

Now, let us state a representation Theorem for functions belonging to the class $I_a(\alpha; h)$ without a proof since it follows the idea of the proof of Theorem 6.

THEOREM 9. *A function $f(z)$ belongs to $I_a(\alpha; h)$ if and only if there exists a function $G(z) \in H(E)$ with $G(0) = 0$ such that $zG'(z)/G(z) \prec h(z)$ in E and an analytic function $p(z)$ with $p(0) = 1$ and $p(z) \prec h(z)$ in E such that*

$$\begin{aligned}&(k_a * f)'(z) = \alpha^{-1} z^{-1/\alpha+1} \int_0^z p(t) G^{1/\alpha}(t) G'(t)\, dt \quad \textit{if } \alpha \neq 0 \textit{ and} \\ &(k_a * f)'(z) = p(z)G(z)/z, \quad \textit{if } \alpha = 0,\end{aligned}$$

Remark. If $a = 1$ and $h(z) = (1-z)/(1+z)$. Theorem 7, Theorem 8 and Theorem 9 reduce to Theorem 1, Theorem 2 and Theorem 3 respectively of Chichra [**1**].

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