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GENERALIZED RANDOM PROCESSES ON THE ZEMANIAN SPACE $\mathcal{A}$

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Abstract. We give several representation theorems for the generalized random processes whose sample functions are generalized functions from Zemanian space $\mathcal{A}'$. Using these representations we give the characterizations of a sequence of generalized random processes on $\mathcal{A}$ that converges almost surely ($\mathcal{A}'$).

1. Introduction

The generalized random processes (g.r.p.) were studied by several authors [**1**,**3**,**4**,**5**,**7**,**8**,**10**,**11**,**12**,**13**] and a variety of different viewpoints have been taken to define g.r.p.. In this paper we follow the aproach of [**1**,**4**,**7**,**10**,**12**,**13**]. In [**1**,**3**,**7**,**8**,**12**,**13**] spaces $\mathcal{D}$ and $K\{M_p\}$ were taken to be the spaces of test functions and in [**1**,**7**,**8**,**12**,**13**] representation theorems for g.r.p. were given. In [**7**,**8**] several types of convergences of g.r.p. were defined and investigated, and representation theorems for expectation and conditional expectation of g.r.p. were given as well.

For a space of test functions we take the space $\mathcal{A}$, whose elements have orthonormal expansions. The space $\mathcal{A}$ and its dual space $\mathcal{A}'$ were introduced in [**14**]. Our construction of the spaces $\mathcal{A}$ and $\mathcal{A}'$ is different from [**14**], and details are given in [**11**].

Since elements from $\mathcal{A}$ and $\mathcal{A}'$ have orthonormal expansions we are able to give several representation theorems for g.r.p. on $\mathcal{A}$. In Theorem 3.1 we give the representation of a g.r.p. as an infinite series on a set of arbitrary large probability. In Theorem 3.2 we give the conditions under which this representation is valid on a set of probability one. In Theorems 3.3, 3.4, 3.5, we use generalized diferential self-adjoint operator to represent a g.r.p..

In Section 4 we investigate almost sure convergence of g.r.p. on $\mathcal{A}$. In Theorem 4.1 we give the necessary and sufficient conditions for almost sure convergence

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of a sequence of g.r.p., and in Theorems 4.2, 4.3 and 4.4 we give the characterization of such convergence using the representations obtained in Theorems 3.1, 3.4 and 3.5.

Our approach to the notion of almost sure convergence is motivated by the papers of Kitchens [**7**,**8**].

2. Spaces $\mathcal{A}$ and $\mathcal{A}'$

We shall use the notation from [**14**]. Let I be an open interval of the real line $\mathbf{R}$ and $L^2(I)$ be the space of the equivalence classes of square integrable functions with values in the set of complex numbers $\mathbf{C}$ with the usual norm. Denote by $C^\infty(I)$ the set of infinitely differentiable (smooth) functions, by $\mathbf{N}$ the set $\{1,2,\dots\}$ and let $\mathbf{N}_0 = \mathbf{N} \cup \{0\}$. Let $\mathcal{R}$ be a linear differential self-adjoint operator of the form $\mathcal{R} = \theta_0 D^{n_1}\theta_1 \dots D^{n_\nu}\theta_\nu$ where $D = d/dx$, $n_k \in \mathbf{N}_0$, $k = 1,2,\dots,\nu$; θ_k, $k = 0,1,\dots,\nu$, are smooth functions without zeros on I. We suppose that there exists a sequence of real numbers $\{\lambda_n, n \in \mathbf{N}_0\}$, and a sequence of smooth functions $\{\psi_n, n \in \mathbf{N}_0\}$ such that $\mathcal{R}\psi_n = \lambda_n\psi_n$, $n \in \mathbf{N}_0$. Furthermore, suppose that the sequence $\{|\lambda_n|, n \in \mathbf{N}_0\}$ monotonically tends to infinity and that $\{\psi_n, n \in \mathbf{N}_0\}$ is a complete orthonormal system (o.n.s.). We can enumerate ψ_n and λ_n so that $|\lambda_0| \le |\lambda_1| \le |\lambda_2| \le \dots$. Put $\widetilde{\lambda}_n = \lambda_n$ if $\lambda_n \ne 0$, and $\widetilde{\lambda}_n = 1$ if $\lambda_n = 0$, $n \in \mathbf{N}_0$. The sequence $\{\widetilde{\lambda}_n, n \in \mathbf{N}_0\}$ is nondecreasing and $|\widetilde{\lambda}_n| \to \infty$, $n \to \infty$. Let $\mathcal{R}^{k+1} = \mathcal{R}(\mathcal{R}^k)$, $k \in \mathbf{N}_0$, $\mathcal{R}^0 = \Im$, $\Im$ is the identity operator. In [**11**] the scale of spaces $\mathcal{A}_k$, $k \in \mathbf{N}_0$ is defined in the following way.

$$\mathcal{A}_k = \left\{\varphi \in L^2(I) : \varphi = \sum_{m=0}^{\infty} a_m\psi_m,\ \|\varphi\|_k = \sum_{m=0}^{\infty} |a_m|^2 \widetilde{\lambda}_m^{2k} < \infty\right\}, \quad k \in \mathbf{N}_0.$$

Put

$$\mathcal{A} = \bigcap_{k=0}^{\infty} \mathcal{A}_k = \left\{\varphi \in L^2(I) : \varphi = \sum_{m=0}^{\infty} a_m\psi_m,\ \forall k \in \mathbf{N},\ \sum_{m=0}^{\infty} |a_m|^2 \widetilde{\lambda}_m^{2k} < \infty\right\}.$$

The set $S_r = \left\{\varphi = \sum_{m=0}^{s}(a_m + ib_m)\psi_m : s \in \mathbf{N}_0,\ a_m, b_m \in \mathbf{Q},\ m \in \mathbf{N}_0\right\}$, ($\mathbf{Q}$ is the set of rational numbers), is a countable dense set in each $\mathcal{A}_k$, $k \in \mathbf{N}_0$, and hence in $\mathcal{A}$.

Let $\mathcal{A}'$, $(\mathcal{A}'_k)$ be the dual space of the space $\mathcal{A}$, $(\mathcal{A}_k)$. We have $\mathcal{A}' = \bigcup_{k=0}^{\infty} \mathcal{A}'_k$.

From [**14**, ch. 9.3. and 9.6.] it follows that $(\mathcal{R}^k\psi_m, \varphi) = (\psi_m, \mathcal{R}^k\varphi)$, $n, k \in \mathbf{N}_0$, $\varphi \in \mathcal{A}$, where for $\varphi \in \mathcal{A}$, $f \in \mathcal{A}'$, $(f, \varphi) = \langle f, \overline{\varphi}\rangle$.

3. Generalized random processes on $\Omega \times \mathcal{A}$

Let $(\Omega, \mathcal{F}, P)$ be a probability space and P a complete measure. Throughout the paper we shall assume that $(\Omega, \mathcal{F}, P)$ is fixed.

Definition 3.1. A *generalized random process* is a mapping $\xi : \Omega \times \mathcal{A} \to \mathbf{C}$ such that: (i) $\forall \varphi \in \mathcal{A}$, $\xi(\,\cdot\,,\varphi)$ is a random variable; (ii) $\forall \omega \in \Omega$, $\xi(\omega,\,\cdot\,)$ is an element from $\mathcal{A}'$.

THEOREM 3.1. *Let ξ be a g.r.p. on $\Omega \times \mathcal{A}$. Then for every $\varepsilon \in (0,1)$ there exists a non-negative integer $k = k(\varepsilon)$, a set $B \in \mathcal{F}$ with $P(B) \geq 1-\varepsilon$, and a sequence $\{c_n, n \in \mathbf{N}_0\}$ of random variables, such that*

$$\xi(\omega,\varphi) = \sum_{n=0}^{\infty} c_n(\omega)(\psi_n,\varphi), \qquad \omega \in B,\ \varphi \in \mathcal{A}$$
$$\left(\sum_{n=0}^{\infty} |c_n(\omega)|^2 \widetilde{\lambda}_n^{-2k}\right)^{1/2} < k, \qquad \omega \in B.$$

Proof. The proof is similar to the proof of Lemma 4 and Theorem 1 of [**13**]. See also [**1**,**12**,**10**]. For every $\omega_0 \in \Omega$ we have that $\xi(\omega_0,\,\cdot\,) \in \mathcal{A}'$. So, there exist $C(\omega_0)$ and $k(\omega_0)$ such that

$$|\xi(\omega_0,\varphi)| \leq C(\omega_0)\|\varphi\|_{k(\omega_0)}, \qquad \varphi \in \mathcal{A}.$$

Put

$$B_N(\varphi) = \big\{\omega \in \Omega : |\xi(\omega,\varphi)| \leq N\|\varphi\|_N,\ N \in \mathbf{N}_0,\ \varphi \in \mathcal{A}\big\},$$
$$B_N = \bigcap_{\varphi \in \mathcal{A}} B_N(\varphi), \quad N \in \mathbf{N}_0.$$

We have that

$$B_N = \bigcap_{\varphi \in S_r} B_N(\varphi), \qquad N \in \mathbf{N}_0.$$

Since S_r is a denumerable dense set in $\mathcal{A}$ it follows that B_N is a measurable set. Furthermore, $B_N \subset B_{N+1}$, $N \in \mathbf{N}_0$ and $\Omega = \bigcup_{N=0}^{\infty} B_N$. Hence, for a given $\varepsilon > 0$ there exists $k \in \mathbf{N}_0$ such that $P(B_k) \geq 1-\varepsilon$. If we put $B = B_k$ we obtain that $|\xi(\omega,\varphi)| \leq k\|\varphi\|_k$, $\varphi \in \mathcal{A}$, $\omega \in B$. For $\varphi \in \mathcal{A}$, define

$$\xi_1(\omega,\varphi) = \begin{cases} \xi(\omega,\varphi), & \omega \in B \\ 0, & \omega \notin B. \end{cases} \tag{3.1}$$

Put, for $\omega \in \Omega$,

$$\begin{aligned} R(\omega) &= \sup\big\{|\xi_1(\omega,\varphi)|,\ \varphi \in \mathcal{A},\ \|\varphi\|_k \leq 1\big\} \\ &= \sup\big\{|\xi_1(\omega,\varphi)|,\ \varphi \in S_r,\ \|\varphi\|_k \leq 1\big\}. \end{aligned}$$

We have that $R(\cdot)$ is a measurable function, $R(\cdot) \leq k$, and

$$|\xi_1(\omega,\varphi)| \leq R(\omega)\|\varphi\|_k, \qquad \varphi \in \mathcal{A},\ \omega \in B.$$

According to the probabilistic Hahn-Banach theorem, [**4**], ξ_1 can be extended to $\mathcal{A}_k$. Denote this extension by $\widetilde{\xi}_1$. It follows that $|\widetilde{\xi}_1(\omega,\varphi)| \le R(\omega)\|\varphi\|_k$, $\varphi \in \mathcal{A}_k$, $\omega \in \Omega$. A mapping from $\Omega \times \mathcal{A}_k$ to $\Omega \times l^2$ defined by

$$i : (\omega,\varphi) \to (\omega, \{\widetilde{\lambda}_n^k a_n\}), \quad \varphi = \sum_{n=0}^{\infty} a_n \psi_n \in \mathcal{A},$$

is an isometry of the spaces $\Omega \times \mathcal{A}_k$ and $\Theta = i(\Omega \times \mathcal{A}_k) \subset \Omega \times l^2$. A g.r.p. ξ_2 on Θ is defined by

$$\xi_2(\omega, \{\widetilde{\lambda}_n^k a_n\}) = \widetilde{\xi}_1(\omega,\varphi), \qquad \omega \in \Omega, \ \varphi \in \mathcal{A}_k,$$

where $(\omega, \{\widetilde{\lambda}_n^k a_n\}) = i(\omega,\varphi)$ and

$$|\xi_2(\omega, \{\widetilde{\lambda}_n^k a_n\})| \le R(\omega)\left(\sum_{n=0}^{\infty} |a_n|^2 \widetilde{\lambda}_n^{2k}\right)^{1/2}, \qquad \omega \in \Omega.$$

According to the probabilistic Hahn-Banach theorem ξ_2 can be extended to $\Omega \times l^2$. Denote this extension by $\widetilde{\xi}_2$. We have that

$$\widetilde{\xi}_2(\omega, \{\widetilde{\lambda}_n^k a_n\}) = \xi_2(\omega, \{\widetilde{\lambda}_n^k a_n\}), \qquad (\omega, \{\widetilde{\lambda}_n^k a_n\}) \in \Theta,$$
$$|\widetilde{\xi}_2(\omega, \{b_n\})| \le R(\omega)\|\{b_n\}\|_{l^2}, \qquad \{b_n\} \in l^2, \quad \omega \in \Omega.$$

For every $\omega \in \Omega$, $\widetilde{\xi}_2(\omega,\cdot)$ is a continuous linear functional on l^2. Therefore, there exists a sequence $\{\widetilde{c}_n(\omega), n \in \mathbf{N}_0\}$ such that

$$\sum_{n=0}^{\infty} |\widetilde{c}_n(\omega)|^2 < \infty, \quad \text{and} \quad \widetilde{\xi}_2(\omega, \{b_n\}) = \sum_{n=0}^{\infty} \widetilde{c}_n(\omega)\overline{b}_n, \qquad \{b_n\} \in l^2, \quad \omega \in \Omega.$$

In an obivous way we define, and denote by the same letters,

$$\widetilde{\xi}_2 : \Omega \times L^2(\Omega) \to \mathbf{C}, \quad \widetilde{\xi}_2(\omega,\varphi) = \widetilde{\xi}_2(\omega, \{b_n\}), \quad \varphi = \sum_{n=0}^{\infty} b_n \psi_n, \quad \{b_n\} \in l^2.$$

Since $\widetilde{\xi}_2(\cdot,\varphi)$ is a random variable for every $\varphi \in L^2(I)$, it follows, putting $\varphi = \psi_n$, that $\widetilde{c}_n(\omega) = \widetilde{\xi}_2(\omega,\psi_n)$ are random variables. Moreover, for the dual norm we have

$$|\widetilde{\xi}_2(\omega,\cdot)|'_{L^2(I)} = \left(\sum_{n=0}^{\infty} |\widetilde{c}_n(\omega)|^2\right)^{1/2} = R(\omega), \qquad \omega \in \Omega.$$

We have for $\omega \in B$, $\varphi \in \mathcal{A}$,

$$\begin{aligned} \xi(\omega,\varphi) = \xi_1(\omega,\varphi) = \widetilde{\xi}_1(\omega,\varphi) &= \xi_2(\omega, \{\widetilde{\lambda}_n^k a_n\}) = \widetilde{\xi}_2(\omega, \{\widetilde{\lambda}_n^k a_n\}) \\ &= \sum_{n=0}^{\infty} \widetilde{c}_n(\omega)\widetilde{\lambda}_n^k \overline{a}_n = \sum_{n=0}^{\infty} \widetilde{c}_n(\omega)\widetilde{\lambda}_n^k (\psi_n,\varphi). \end{aligned}$$

Define $c_n(\omega) = \widetilde{c}_n(\omega)\widetilde{\lambda}_n^k$, $n \in \mathbf{N}_0$ and the assertion follows.

THEOREM 3.2. *Let ξ be a g.r.p. on $\Omega \times \mathcal{A}$. Suppose there exist a random variable r, a set $Z \in \mathcal{F}$ with $P(Z) = 0$, and a non-negative integer k, such that $|\xi(\omega,\varphi)| \le r(\omega)\|\varphi\|_k$, for $\omega \in \Omega \setminus Z$, $\varphi \in \mathcal{A}$. Then there exists a sequence $\{c_n, n \in \mathbf{N}_0\}$ of random variables, such that*

$$\xi(\omega,\varphi) = \sum_{n=0}^{\infty} c_n(\omega)(\psi_n,\varphi) \qquad \omega \in \Omega \setminus Z, \quad \varphi \in \mathcal{A},$$
$$\left(\sum_{n=0}^{\infty} |c_n(\omega)|^2 \widetilde{\lambda}_n^{-2k}\right)^{1/2} < r(\omega), \qquad \omega \in \Omega \setminus Z.$$

The proof is similar to the proof of Theorem 3.1, putting $\Omega \setminus Z$ instead of B in (3.1).

We define the differential operator $\widetilde{\mathcal{R}}^k$, $k \in \mathbf{N}_0$, on the set of g.r.p.'s by

$$\widetilde{\mathcal{R}}_k \xi(\omega,\varphi) = \xi(\omega, \mathcal{R}^k\varphi), \quad \omega \in \Omega,\ \varphi \in \mathcal{A},$$
$$\widetilde{\mathcal{R}}^{k+1} = \widetilde{\mathcal{R}}(\widetilde{\mathcal{R}}^k), \quad k \in \mathbf{N}_0, \qquad \widetilde{\mathcal{R}}^0 = \Im.$$

We shall denote $\widetilde{\mathcal{R}}$ by $\mathcal{R}$.

Next, we shall give representation theorems of a g.r.p. that are analogous to the Theorem 9.6.2 from [**14**, Ch. 9.6]. Put $\Lambda = \{n \in \mathbf{N}_0 : \lambda_n = 0\}$, $\Lambda^c = \mathbf{N}_0 \setminus \Lambda$.

THEOREM 3.3. *Let ξ be a g.r.p. on $\Omega \times \mathcal{A}$. For every $\varepsilon \in (0,1)$ there exist $B \in \mathcal{F}$ with $P(B) \ge 1 - \varepsilon$, a non-negative integer $k_0 = k_0(\varepsilon)$, a g.r.p. ξ_0 on $\Omega \times L^2(I)$, and random variables c_n, $n \in \Lambda$, such that*

$$\xi(\omega,\varphi) = \mathcal{R}^{k_0}\xi_0(\omega,\varphi) + \sum_{n\in\Lambda} c_n(\omega)(\psi_n,\varphi), \qquad \omega \in B, \quad \varphi \in \mathcal{A}.$$

Proof. From Theorem 3.1 it follows that there exist $B \subset \Omega$ with $P(B) \ge 1-\varepsilon$, and $k_0 = k_0(\varepsilon)$ such that

$$\xi(\omega,\varphi) = \sum_{n=0}^{\infty} c_n(\omega)(\psi_n,\varphi), \qquad \omega \in B, \quad \varphi \in \mathcal{A},$$

where $c_n(\omega)$ are random variables with

$$\left(\sum_{n=0}^{\infty} |c_n(\omega)|^2 \widetilde{\lambda}_n^{-2k_0}\right)^{1/2} < k_0, \qquad \omega \in B.$$

Put

$$b_n(\omega) = \begin{cases} c_n(\omega)/\widetilde{\lambda}_n^{k_0}, & \omega \in B \\ 0, & \omega \notin B. \end{cases}$$

We have that
$$\xi_0(\omega,\varphi)=\sum_{n=0}^{\infty}b_n(\omega)(\psi_n,\varphi)$$
is a g.r.p. on $\Omega\times L^2(I)$. Namely ξ_0 is determined by the function $X_0(\omega,t)=\sum_{n=0}^{\infty}b_n(\omega)\psi_n(t)$, on $\Omega\times I$, where, for fixed $\omega\in\Omega$, X_0 is in $L^2(I)$. We have that, for $\omega\in\Omega$, $\varphi\in\mathcal{A}$,
$$\mathcal{R}^{k_0}\xi_0(\omega,\varphi)=\xi_0(\omega,\mathcal{R}^{k_0}\varphi)=\sum_{n=0}^{\infty}b_n(\omega)\lambda_n^{k_0}(\psi_n,\varphi)=\sum_{n\in\Lambda^c}c_n(\omega)(\psi_n,\varphi).$$
So,
$$\xi(\omega,\varphi)=\mathcal{R}^{k_0}\xi_0(\omega,\varphi)+\sum_{n\in\Lambda}c_n(\omega)(\psi_n,\varphi).\ \square$$

Remark 3.1. From the proof of Theorem 3.3 we can conclude that instead of k_0 and ξ_0 Theorem 3.3 holds for every $k\geq k_0$ and the corresponding g.r.p. on $\Omega\times L^2(I)$, ξ_k.

Remark 3.2. The representation of $\xi(\omega,\varphi)$ in Theeorem 3.3 means that
$$\xi(\omega,\varphi)=\int_I X_0(\omega,t)\mathcal{R}^{k_0}\varphi(t)\,dt+\sum_{n\in\Lambda}c_n(\omega)(\psi_n,\varphi),\qquad \omega\in B,\ \varphi\in\mathcal{A}.$$

In the same way as in Theorems 3.2 and 3.3 we can prove the following

THEOREM 3.4. *Let ξ be a g.r.p. on $\Omega\times\mathcal{A}$. Suppose there exist a random variable r such that $E(r)<\infty$, a set $Z\in\mathcal{F}$ such that $P(Z)=0$, a non-negative integer k_0 such that $|\xi(\omega,\varphi)|\leq r(\omega)\|\varphi\|_{k_0}$, $\omega\in\Omega\setminus Z$, $\varphi\in\mathcal{A}$. Then for every $k\geq k_0$ there exist a g.r.p. on $\Omega\times L^2(I)$, $\xi_k(\omega,\varphi)$, and random variables c_n, $n\in\Lambda$, idependent of k, such that*
$$\begin{aligned}\xi(\omega,\varphi)&=\mathcal{R}^k\xi_k(\omega,\varphi)+\sum_{n\in\Lambda}c_n(\omega)(\psi_n,\varphi)\\&=\int_I X_k(\omega,t)\mathcal{R}^k\varphi(t)\,dt+\sum_{n\in\Lambda}c_n(\omega)(\psi_n,\varphi),\quad \omega\in\Omega\setminus Z,\ \varphi\in\mathcal{A},\end{aligned}$$
where X_k is the function on $\Omega\times I$ which determines ξ_k.

Next we shall give a similar representation where X_k is a continuous random process. By continuous stohastic process on $\Omega\times I$ we shall mean the process which for almost every $\omega\in\Omega$ is a continuous function on I. In our next theorem we shall suppose that sequences $\{\psi_n,n\in\mathbf{N}_0\}$ and $\{\lambda_n,n\in\mathbf{N}_0\}$ satisfy the following conditions:

(∗) there exist $s_0\in\mathbf{N}_0$ and a constant K, such that $\sup\{|\psi_n(t)/\widetilde{\lambda}_n^{s}|:n\in\mathbf{N}_0,\ t\in I\}<K$,

$(**)$ there exists $p_0 \in \mathbf{N}_0$ such that for $p \geq p_0$, $\sum_{n \in \Lambda^c} \lambda_n^{-2p} < \infty$.

Conditions $(*)$ and $(**)$ are not too restrictive. For example, Hermite, Fourier and Laguerre complete orthonormal suystems satisfy these conditions. For other o.n.s. which satisfy these conditions we refer to [**14**, ch. 9.8] and [**2**, ch. 10.18].

THEOREM 3.5. *Let ξ be a g.r.p. on $\Omega \times \mathcal{A}$. Suppose that there exist a random variable r such that $E(r) < \infty$, a set $Z \in \mathcal{F}$, such that $P(Z) = 0$, a non-negative integer k_0, such that $|\xi(\omega, \varphi)| \leq r(\omega)\|\varphi\|_{k_0}$, for $\omega \in \Omega \setminus Z$, $\varphi \in \mathcal{A}$. Then, for $k \geq k_0$, there exist a continuous random process $X_k(\omega, t)$ on $\Omega \times I$, and random variables c_n, $n \in \Lambda$, such that*

$$\xi(\omega, \varphi) = \int_I X_k(\omega, t)\mathcal{R}^{k+p+s}\varphi(t)\, dt + \sum_{n \in \Lambda} c_n(\omega)(\psi_n, \varphi), \quad \forall \omega \in \Omega \setminus Z,\ \varphi \in \mathcal{A},$$

where $s \geq s_0$, s_0 is from $(*)$, and $p \geq p_0$, p_0 is from $(**)$.

Proof. From Theorem 3.2 it follows that there exists a sequence of random variables $\{c_n, n \in \mathbf{N}_0\}$ such that

$$\xi(\omega, \varphi) = \sum_{n=0}^{\infty} c_n(\omega)(\psi_n, \varphi), \quad \omega \in \Omega \setminus Z,\ \varphi \in \mathcal{A},$$

$$\left(\sum_{n=0}^{\infty} |c_n(\omega)|^2 \widetilde{\lambda}_n^{-2k}\right)^{1/2} \leq r(\omega), \quad \omega \in \Omega \setminus Z.$$

Let $k \geq k_0$. Define

$$X_k(\omega, t) = \sum_{n \in \Lambda^c} c_n(\omega)\lambda_n^{-(k+p+s)}\psi_n(t), \quad \omega \in \Omega,\ t \in I.$$

We have that for every $\omega \in \Omega \setminus Z$, $t \in I$,

$$\sum_{n \in \Lambda^c} |c_n(\omega)\lambda_n^{-(k+p+s)}\psi_n(t)| \leq K \sum_{n \in \Lambda^c} |(c_n(\omega)\lambda_n^{-k})(\lambda_n^{-p})|$$
$$\leq K\left(\sum_{n \in \Lambda^c} |c_n(\omega)|^2 \lambda_n^{-2k}\right)^{1/2} \left(\sum_{n \in \Lambda^c} \lambda_n^{-2p}\right)^{1/2} < \infty.$$

It follows that for every $\omega \in \Omega \setminus Z$, $X_k(\omega, \cdot)$ is a continuous function. Since $X_k(\cdot, t)$, $t \in I$, is measurable it follows that X_k is jointly measurable on $\Omega \times I$. For $\varphi = \sum_{n=0}^{\infty} a_n \psi_n \in \mathcal{A}$ we have that

$$\sum_{n=0}^{\infty} |a_n|^2 \widetilde{\lambda}_n^{2(k+p+s)} = C < \infty.$$

Also,

$$\left(\sum_{n \in \Lambda^c} |c_n(\omega)|^2 \lambda_n^{-2(k+p+s)}\right)^{1/2} \leq r(\omega), \quad \omega \in \Omega.$$

Hence,

$$\int_{\Omega\setminus Z}\int_I |X_k(\omega,t)\mathcal{R}^{k+p+s}\varphi(t)|\,dtdP(\omega)$$
$$\le \int_{\Omega\setminus Z}\left\{\left(\int_I |X_k(\omega,t)|^2\,dt\right)^{1/2}\left(\int_I |\mathcal{R}^{k+p+s}\varphi(t)|^2\,dt\right)^{1/2}\right\}dP(\omega)$$
$$\le \int_{\Omega\setminus Z}\left\{\left(\sum_{n\in\Lambda^c}|c_n(\omega)|^2\lambda_n^{-2(k+p+s)}\right)^{1/2}\left(\sum_{n\in\Lambda^c}|a_n|^2\lambda_n^{2(k+p+s)}\right)^{1/2}\right\}dP(\omega)$$
$$\le C\int_{\Omega\setminus Z}|r(\omega)|\,dP(\omega)<\infty.$$

It follows from Fubini's theorem that $X_k(\cdot,\cdot)\mathcal{R}^{k+p+s}\varphi(\cdot)\in L^1((\Omega\setminus Z)\times I)$ and, again, from the Fubini theorem that

$$\xi_k(\cdot,\varphi)=\int_I X_k(\cdot,t)\mathcal{R}^{k+p+s}\varphi(t)\,dt$$

is a random variable for every $\varphi\in\mathcal{A}$ and hence a g.r.p. It is obvious that for every $\omega\in\Omega\setminus Z$ and every $\varphi\in\mathcal{A}$,

$$\xi_k(\omega,\varphi)=\sum_{n\in\Lambda^c}c_n(\omega)\lambda_n^{-(k+p+s)}(\psi_n,\mathcal{R}^{k+p+s}\varphi)=\sum_{n\in\Lambda^c}c_n(\omega)(\psi_n,\varphi).$$

So, finally we have

$$\xi(\omega,\varphi)=\xi_k(\omega,\varphi)+\sum_{n\in\Lambda}c_n(\omega)(\psi_n,\varphi)$$
$$=\int_I X_k(\omega,t)\mathcal{R}^{k+p+s}\varphi(t)\,dt+\sum_{n\in\Lambda}c_n(\omega)(\psi_n,\varphi),\quad \omega\in\Omega\setminus Z,\ \varphi\in\mathcal{A}.$$

4. Almost sure convergence of a sequence of g.r.p. on $\mathcal{A}$

Definition 4.1. A sequence $\{\xi_n,n\in\mathbf{N}\}$ of g.r.p. on $\mathcal{A}'$ converges to a g.r.p. ξ almost surely $(\mathcal{A}')$ if there exists a set $Z\in\mathcal{F}$ such that $P(Z)=0$ and for $\omega\in\Omega\setminus Z$, $\xi_n(\omega,\cdot)$ converges to $\xi(\omega,\cdot)$ in $\mathcal{A}'$.

Since $\xi_n\to\xi$ iff $\xi_n-\xi\to 0$ we shall consider the case $\xi_n\to 0$.

THEOREM 4.1. *Let $\{\xi_n,n\in\mathbf{N}\}$ be a sequence of g.r.p. on $\mathcal{A}$. The following conditions are equivalent*:

A. *The sequence $\{\xi_n\}$ converges to zero almost surely $(\mathcal{A}')$.*

B. (i) *For every $\varphi\in\mathcal{A}$, $\xi_n(\cdot,\varphi)\to 0$ almost surely, $n\to\infty$;*
   (ii) *There exist a set $Z\in\mathcal{F}$ such that $P(Z)=0$, and for every $\omega\in\Omega\setminus Z$, $\{\xi_n(\omega,\cdot),n\in\mathbf{N}\}$ is bounded in $\mathcal{A}'$.*

C. (i) *For every* $\varphi \in \mathcal{A}$, $\xi_n(\cdot, \varphi) \to 0$ *almost surely,* $n \to \infty$;

(ii) *For every* $\varepsilon \in (0,1)$ *there exist a set* $B \in \mathcal{F}$, *a non-negative integer* k, *both independent of* n, *such that* $P(B) \geq 1 - \varepsilon$, *and for every* $\omega \in B$, $\varphi \in \mathcal{A}$, $|\xi_n(\omega, \varphi)| \leq k\|\varphi\|_k$.

Proof. The proof is similar to the proof of Theorem 2.2 [**7**]. We shall prove $A \implies C \implies B \implies A$.

$A \implies C$. Assume that $\xi_n \to 0$ almost surely $(\mathcal{A}')$, then C (i) follows immediately. We have that for each $\omega \in \Omega \setminus Z$ the sequence $\xi_n(\omega, \cdot)$ is bounded, and since $\mathcal{A}' = \bigcup_{k=0}^{\infty} \mathcal{A}'_k$, it follows that there exists a non-negative integer $k = k(\omega)$, independent of n, such that $|\xi_n(\omega, \varphi)| \leq k(\omega)\|\varphi\|_{k(\omega)}$, $\varphi \in \mathcal{A}$. Now, as in Theorem 3.1, put

$$A_n(\varphi) = \{\omega \in \Omega : |\xi_n(\omega, \varphi)| \leq N\|\varphi\|_N\}, \quad \varphi \in \mathcal{A},\ N \in \mathbf{N},$$
$$A_N = \bigcap_{\varphi \in S_r} A_n(\varphi).$$

We have that $A_N \in \mathcal{F}$, and that $\Omega \setminus Z \subset \bigcup_{N=1}^{\infty} A_N$, $A_N \subset A_{N+1}$, $N \in \mathbf{N}$. Thus, we obtain that for a given $\varepsilon \in (0,1)$ there exist $k \in \mathbf{N}_0$ such that $P(A_k) \geq 1 - \varepsilon$. Put $B = A_k$ and C (ii) follows.

$C \implies B$. The conditions C (i) and B (i) are the same, and to show that C (ii) $\implies$ B (ii), choose $\varepsilon = 1/p$, $p \in \mathbf{N}$. Then there exist B_p and k_p with $P(B_p) \geq 1 - 1/p$ such that for $\omega \in B_p$, $|\xi_n(\omega, \varphi)| \leq k_p\|\varphi\|_{k_p}$, $\varphi \in \mathcal{A}$. Let $Z = \Omega \setminus \bigcup_{p=1}^{\infty} B_p$. Then $P(Z) = 0$ and B (ii) follows.

$B \implies A$. From B (i) it follows that for each $\varphi \in S_r$ there exists a set $Z_\varphi \in \mathcal{F}$ such that $P(Z_\varphi) = 0$ and for each $\omega \in \Omega \setminus Z_\varphi$, $\xi_n(\omega, \varphi) \to 0$. Let $Z' = Z \cup \left(\bigcup_{\varphi \in S_r} Z_\varphi\right)$, (where Z is from B (ii)). Then $P(Z') = 0$. For each $\varphi \in S_r$ and $\omega \in \Omega \setminus Z'$, $\xi_n(\omega, \varphi) \to 0$. Since from B (ii) we have that for each $\omega \in \Omega \setminus Z'$ $\{\xi_n(\omega, \cdot)\}$ is bounded on $\mathcal{A}$, it follows by the Banach-Steinhaus theorem that for every $\omega \in \Omega \setminus Z'$, $\xi_n(\omega, \cdot) \to 0$, that is, ξ_n converges almost surely $(\mathcal{A}')$. $\square$

THEOREM 4.2. *Let* $\{\xi_n\}$ *be a sequence of g.r.p.'s on* $\mathcal{A}$. *Then* $\xi_n \to 0$ *almost surely* $(\mathcal{A}')$ *iff for every* $\varepsilon \in (0,1)$ *there exist a set* $A \in \mathcal{F}$, *with* $P(A) \geq 1 - \varepsilon$, *and a non-negative integer* k_0 *such that for* $k > k_0$ *and* $\omega \in A$

$$\sup_{\|\varphi\|_k \leq 1} |\xi_n(\omega, \varphi)| \to 0, \quad n \to \infty. \tag{4.1}$$

Proof. From Theorem 4.1, part C (ii) it follows that for a given $\varepsilon \in (0,1)$ there exist $B \in \mathcal{F}$, $P(B) \geq 1 - \varepsilon$, and a non-negative integer k_0 such that, for $\omega \in B$, $\varphi \in \mathcal{A}$, $n \in \mathbf{N}$, $|\xi_n(\omega, \varphi)| \leq k_0\|\varphi\|_{k_0}$. Then, from Theorem 3.1 it follows that for every $n \in \mathbf{N}$ there exists a sequence of random variables $\{c_{m,n} : m \in \mathbf{N}_0\}$ such that for every $\omega \in B$, $\varphi \in \mathcal{A}$,

$$\xi_n(\omega, \varphi) = \sum_{m=0}^{\infty} c_{m,n}(\omega)(\psi_m, \varphi), \tag{4.2}$$

and, for every $n \in \mathbf{N}$,

$$\left(\sum_{m=0}^{\infty} |c_{m,n}(\omega)|^2 \widetilde{\lambda}_m^{-2k} \right)^{1/2} < C, \quad k \geq k_0,\ \omega \in \Omega. \tag{4.3}$$

Since $\xi_n \to 0$ almost surely $(\mathcal{A}')$, there exists $Z \in \mathcal{F}$, $P(Z) = 0$, such that for $\omega \in \Omega \setminus Z$, $\xi_n(\omega, \varphi) \to 0$, for every $\varphi \in \mathcal{A}$. Put $A = B \setminus Z$. Then $P(A) \geq 1 - \varepsilon$ since $P(\,\cdot\,)$ is a complete measure. Putting $\varphi = \psi_m$, $m \in \mathbf{N}_0$, in (4.2) we obtain for $\omega \in \Omega$

$$\xi_n(\omega, \psi_m) = c_{m,n}(\omega) \to 0, \quad n \to \infty,\ m \in \mathbf{N}_0. \tag{4.4}$$

Let $k > k_0$ be fixed. We have $\left(\varphi = \sum_{m=0}^{\infty} a_m \psi_m \in \mathcal{A}\right)$

$$|\xi_n(\omega, \varphi)| = \left| \sum_{m=0}^{\infty} c_{m,n}(\omega)(\psi_m, \varphi) \right| \tag{4.5}$$

$$= \left| \sum_{m=0}^{\infty} c_{m,n}(\omega) |\widetilde{\lambda}_m|^{-k} |\widetilde{\lambda}_m|^k a_m \right| \leq \left(\sum_{m=0}^{\infty} |c_{m,n}(\omega)|^2 \widetilde{\lambda}_m^{-2k} \right)^{1/2} \|\varphi\|_k.$$

Further,

$$\sum_{m=0}^{\infty} |c_{m,n}(\omega)|^2 \widetilde{\lambda}_m^{-2k} = \sum_{m=0}^{m_0} |c_{m,n}(\omega)|^2 \widetilde{\lambda}_m^{-2k} + \widetilde{\lambda}_{m_0+1}^{-2} \cdot \sum_{m=m_0+1}^{\infty} |c_{m,n}(\omega)|^2 \widetilde{\lambda}_m^{-2(k-1)},$$

where m_0 is chosen so that for C from (4.3)

$$\widetilde{\lambda}_{m_0+1}^{-2} \leq \varepsilon^2 / 2C^2.$$

From (4.4) it follows that there exists $n_0 = n_0(\varepsilon)$ such that

$$\sum_{m=0}^{m_0} |c_{m,n}(\omega)|^2 \widetilde{\lambda}_m^{-2k} < \varepsilon^2/2, \quad n \geq n_0.$$

Thus, we have that, $\omega \in A$, $\varphi \in \mathcal{A}$, $k > k_0$,

$$|\xi_n(\omega, \varphi)| \leq \varepsilon \|\varphi\|_k;$$

so (4.1) follows.

Conversely, C (ii) follows immediately. For each integer p choose $\varepsilon = 1/p$. There exist A_p and k_p with $P(A_p) \geq 1 - 1/p$, such that

$$\sup_{\|\varphi\|_{k_p} \leq 1} |\xi_n(\omega, \varphi)| \to 0, \qquad \omega \in A_p.$$

Let $Z = \Omega \setminus \bigcup_{p=1}^{\infty} A_p$, then $P(Z) = 0$ and for $\omega \in \Omega \setminus Z$ there exists $p(\omega)$ such that $\omega \in A_{p(\omega)}$ and there exists $k_{p(\omega)}$ such that

$$\sup_{\|\varphi\|_{k_{p(\omega)}} \leq 1} |\xi_n(\omega, \varphi)| \to 0$$

Thus, for given $\varphi \in \mathcal{A}$, and for $\omega \in \Omega \setminus Z$

$$|\xi_n(\omega,\varphi)| \leq \sup_{\|\varphi\|_{k_{p(\omega)}} \leq 1} |\xi_n(\omega,\varphi)| \, \|\varphi\|_{k_{p(\omega)}} \to 0.$$

So C (i) follows.

THEOREM 4.3. *Let $\{\xi_n\}$ be a sequence of g.r.p. on $\mathcal{A}$. Then $\xi_n \to 0$ almost surely $(\mathcal{A}')$ iff for every $\varepsilon \in (0,1)$ there exist a set $B \in \mathcal{F}$ with $P(B) \geq 1-\varepsilon$, an integer $k_0 \in \mathbf{N}_0$, (where B and k_0 are independent of n), for each $m \in \Lambda$ a sequence of random variables $\{c_{m,n}, n \in \mathbf{N}\}$, and for every $k > k_0$ a sequence of functions $X_{k,n}$ on $\Omega \times I$, $n \in \mathbf{N}$, such that, for $n \in \mathbf{N}$*

(4.6)
$$\xi_n(\omega,\varphi) = \int_I X_{k,n}(\omega,t)\mathcal{R}^k\varphi(t)\,dt + \sum_{m\in\Lambda} c_{m,n}(\omega)(\psi_m,\varphi), \quad \omega \in B,\ \varphi \in A;$$

$$\|X_{k,n}(\omega,\cdot)\|_{L^2} < k, \qquad \omega \in \Omega; \tag{4.7}$$

$$\|X_{k,n}(\omega,\cdot)\|_{L^2} \to 0, \qquad n\to\infty, \quad \omega \in \Omega\setminus Z; \tag{4.8}$$

$$\sum_{m\in\Lambda} c_{m,n}(\cdot) \to 0, \qquad n\to\infty, \quad \omega \in B\setminus Z, \tag{4.9}$$

where B is the set from Definition 4.1.

Proof. Assume that $\xi_n \to 0$ almost surely $(\mathcal{A}')$ and let $\varepsilon \in (0,1)$ be given. From C (ii) of Theorem 4.1 it follows that there exist a set $B \in \mathcal{F}$, with $P(B) \geq 1-\varepsilon$, a positive integer k_0, both independent of n, such that $|\xi_n(\omega,\varphi)| \leq k_0\|\varphi\|_{k_0}$, $\omega \in B$, $\varphi \in \mathcal{A}$. Then, from Theorem 3.2 it follows that for every $n \in \mathbf{N}$ and for $k \geq k_0$ there exist a function $X_{k,n}$ on $\Omega \times I$, and random variables $c_{m,n}$, $m \in \Lambda$, such that (4.6) and (4.7) hold.

Now, since

$$\|X_{k,n}(\omega,\cdot)\|_{L^2} = \begin{cases} \sup_{\|\varphi\|_k \leq 1} |\xi_n(\omega,\varphi)|, & \omega \in B \\ 0, & \omega \notin B, \end{cases}$$

from Theorem 4.2, (4.8) follows. □

As in Theorem 4.2 we obtain that $c_{m,n}(\cdot) \to 0$, $\omega \in B\setminus Z$, $m \in \mathbf{N}_0$. Since Λ is finite, (4.9) follows. □

Suppose that the conditions $(*)$ and $(**)$ are satisfied.

THEOREM 4.4. *Let $\{\xi_n, n \in \mathbf{N}\}$ be a sequence of g.r.p. on $\mathcal{A}$. If $\xi_n \to 0$ almost surely $(\mathcal{A}')$, then for every $\varepsilon \in (0,1)$ there exist a set $B \in \mathcal{F}$, with $P(B) \geq 1-\varepsilon$, an integer $k_0 \in \mathbf{N}_0$, (where B and n are independent of n), for each $m \in \Lambda$ a sequence of random variables $\{c_{m,n}, n \in \mathbf{N}\}$, and for every $k > k_0$, a sequence of continuous random processes $X_{k,n}$, on $\Omega \times I$, such that for $n \in \mathbf{N}$*

$$\xi_n(\omega,\varphi) = \int_I X_{k,n}(\omega,t)\mathcal{R}^{k+p+s}\varphi(t)\,dt + \sum_{m\in\Lambda} c_{m,n}(\omega)(\psi_m,\varphi), \tag{4.10}$$

$$\omega\in B,\ \varphi\in\mathcal{A},\ \textit{where } s\geq s_0,\ p\geq p_0;$$

$$\|X_{k,n}(\omega,\cdot)\|_{L^2}\leq k,\quad \omega\in\Omega; \tag{4.11}$$

$$\textit{for each } \omega\in\Omega\setminus Z,\ X_{k,n}(\omega,\cdot)\to 0 \textit{ on } I,\quad n\to\infty; \tag{4.12}$$

$$\{X_{k,n}(\omega,\cdot)\} \textit{ is equicontinuous on } I,\quad \omega\in\Omega\setminus Z; \tag{4.13}$$

$$\textit{for each } t\in I,\ X_{k,n}(\cdot,t)\to 0 \textit{ on } \Omega\setminus Z,\quad n\to\infty; \tag{4.14}$$

$$\sum_{m\in\Lambda} c_{m,n}(\cdot)\to 0,\quad n\to\infty,\ \omega\in B\setminus Z, \tag{4.15}$$

where Z is the set from Definition 4.1.

Proof. Theorem 3.3 and C (ii) of Theorem 4.1 imply (4.10) and (4.11) where for $n\in\mathbf{N}_0$ and $k\geq k_0$

$$X_{k,n}(\omega,t)=\begin{cases}\sum_{m=0}^{\infty} c_{m,n}(\omega)\widetilde{\lambda}_m^{-(k+p+s)}\psi_m(t), & \omega\in B,\ t\in I\\ 0, & \omega\notin B,\ t\in I.\end{cases}$$

Now let $t\in I$. For $\omega\notin B$, $X_{k,n}(\omega,t)=0$ and for $\omega\in B\setminus Z$,

$$\begin{aligned}|X_{k,n}(\omega,t)| &= \left|\sum_{m=0}^{\infty} c_{m,n}(\omega)\widetilde{\lambda}_m^{-(k+p+s)}\psi_m(t)\right|\\ &\leq K\sum_{m=0}^{\infty}|c_{m,n}(\omega)\widetilde{\lambda}_m^{-(k+p)}|<\varepsilon,\quad n\geq n_0(\varepsilon),\end{aligned}$$

in the same way as in Theorem 4.2, since $k+p>k_0$, hence (4.12) follows. (4.15) follows in the same way as in Theorem 4.3.

To establish (4.13) observe that from the condition $(**)$ it follows that for $p>p_0$, $\sum_{m=0}^{\infty}\widetilde{\lambda}_m^{-2p}=A<\infty$. We can choose l_0 such that

$$\left(\min_{m\geq l_0}\widetilde{\lambda}_m^2\right)^{-1}<\varepsilon^2/(4AK^2k^2).$$

$\{\widetilde{\lambda}_n, n\in\mathbf{N}_0\}$ is monotone sequence, hence $\widetilde{\lambda}_{l_0}^2=\min_{m\geq l_0}\widetilde{\lambda}_m^2$. Since $\psi_m(t)$, $m\in\mathbf{N}_0$, are continuous functions, for every $t,t'\in I$ and every $\varepsilon>0$ there exists $\delta(\varepsilon,t)$ such that

$$\sum_{m=0}^{l_0-1}|\psi_m(t)-\psi_m(t')|^2\widetilde{\lambda}_m^{-2(s+p)}<\frac{\varepsilon^2}{2k^2},$$

if $|t-t'|<\delta(\varepsilon,t)$. Now we have for $t,t'\in I$, $|t-t'|<\delta(\varepsilon,t)$, $\omega\in B$

$$|X_{k,n}(\omega,t)-X_{k,n}(\omega,t')|\leq\sum_{m=0}^{\infty}|c_{m,n}(\omega)|\,|\widetilde{\lambda}_m|^{-(k+p+s)}|\psi_m(t)-\psi_m(t')|$$

$$\leq \left(\sum_{m=0}^{\infty} |c_{m,n}(\omega)|^2 |\widetilde{\lambda}_m|^{-2k}\right)^{1/2} \left(\sum_{m=0}^{\infty} |\psi_m(t) - \psi_m(t')|^2 \widetilde{\lambda}_m^{-2(p+s)}\right)^{1/2}$$

$$\leq k \sum_{m=0}^{l_0-1} \left(|\psi_m(t) - \psi_m(t')|^2 \widetilde{\lambda}_m^{-2(p+s)} + 2K^2 \sum_{m=l_0}^{\infty} \widetilde{\lambda}_m^{-2p}\right)^{1/2}$$

$$\leq k\left(\frac{\varepsilon^2}{2k^2} + \frac{2K^2}{\widetilde{\lambda}_{l_0}^2} \sum_{m=l_0}^{\infty} \widetilde{\lambda}_m^{-2(p-1)}\right)^{1/2} \leq k\left(\frac{\varepsilon^2}{2k^2} + \frac{\varepsilon^2}{2k^2}\right)^{1/2} = \varepsilon.$$

So (4.13) follows. Further on, for $t_0 \in I$

$$\begin{aligned} |X_{k,n}(\cdot, t_0)| &= |X_{k,n}(\cdot, t_0) - X_{k,n}(\cdot, t) + X_{k,n}(\cdot, t)| \\ &\leq |X_{k,n}(\cdot, t_0) - X_{k,n}(\cdot, t)| + |X_{k,n}(\cdot, t)|. \end{aligned}$$

We have from (4.13) that $|X_{k,n}(\cdot, t_0) - X_{k,n}(\cdot, t)| \leq \varepsilon/2$ when $|t - t_0| < \delta(\varepsilon, t_0)$, and, from (4.12) $|X_{k,n}(\cdot, t)| \leq \varepsilon/2$, for $n \geq n_0(\varepsilon)$; so (4.14) follows.

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