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INDUCED GENERALIZED CONNECTIONS IN VECTOR SUBBUNDLES

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Abstract. Using special coordinate transformations we introduce subbundles and complementary subbundles of a vector bundle. The new results come from the fact that these bundles are considered together. In the former investigations as in Dragomir [**5**], Miron [**7**], Oproiu [**8**] the subbundle of the vector bundle was defined. In the relations between their tangent spaces the unit normal vectors of the subbundle were involved. Here, they are substituted by the tangent vectors of the complementary subbundle.

The coordinates in the vector bundle $\xi = (E, \pi, M)$ are (x^i, y^a) in the subbundle $\tilde{\xi}$ are (u^α, v^A) and in the complementary subbundle $\tilde{\tilde{\xi}}$ are $(\bar{u}^{\bar{\alpha}}, \bar{v}^{\bar{A}})$. We need six types of indices. With respect to the special coordinate transformations (given by (1.1), (2.1) and (2.5)) the non-linear connections $N_i^a(x,y)$, $N_\alpha^A(u,v)$ and $N_{\bar{\alpha}}^{\bar{A}}(\bar{u},\bar{v})$ are given. Using them, the adapted bases $B = \{\delta_i, \partial_a\}$ and $\widehat{B} = \{\delta_\alpha, \partial_A, \delta_{\bar{\alpha}}, \partial_{\bar{A}}\}$ of $T(E)$ are constructed. The generalized connection $\nabla : T(E) \otimes T(E) \to T(E)$ in the basis B has 2^3 types and in the basis $\widehat{B}$ 4^3 types of connection coefficients. The relations between these coefficients are given. These formulae are very general and have nice special cases.

When the second fundamental forms of the subbundle and complementary subbundle are equal to zero, i.e. when the so called induced nonlinear connections N_α^A and $N_{\bar{\alpha}}^{\bar{A}}$ are used, then these relations are simpler $((3.4)'–(3.7)')$. In this case we obtain that Miron's d-connection defined in $T(E)$ induces also d-connection in the tangent space of the subbundle, in $T(\widetilde{E})$.

1. The Geometry of Vector Bundles. Let $\xi = (E, \pi, M)$ be a C^∞ vector with $\dim M = n$, $\dim E = n + m$. In some local chart the point $u \in E$ has the coordinates

$$(x^1, \dots, x^n, y^1, \dots, y^m) = (x^i, y^a) = (x, y)$$
$$i, j, k, l, m = 1, \dots, n \qquad a, b, c, d, e, f = 1, \dots, m.$$

The allowable coordinate transformations $(x, y) \to (x', y')$ are given by

$$\begin{aligned} x^{i'} &= x^{i'}(x^1, \dots, x^n) \qquad && \operatorname{rank}\left[\partial x^{i'}/\partial x^i\right] = n \\ y^{a'} &= M_a^{a'}(x) y^a && \operatorname{rank}\left[M_a^{a'}\right] = m, \end{aligned} \tag{1.1}$$

AMS Subject Classification (1980): Primary 53 B 25, 53 B 40 so the inverse transformation $(x', y') \to (x, y)$ exists and is determined by

$$\begin{aligned} x^i &= x^i(x^{1'}, \dots, x^{n'}) \qquad & M^{a'}_a M^a_{b'} &= \delta^{a'}_{b'} \\ y^a &= M^a_{a'}(x') y^{a'} & M^{a'}_b M^a_{a'} &= \delta^a_b. \end{aligned} \tag{1.2}$$

The tangent space $T(E)$ is spanned by $\{\partial_i, \partial_a\}$, where $\partial_i = \partial/\partial x^i$ $\partial_a = \partial/\partial y^a$. They have the following law of transformation:

$$\begin{aligned} &\partial_a = M^{a'}_a(x)\partial_{a'} \qquad \partial_{a'} = M^a_{a'}(x')\partial_a \\ &\partial_i = (\partial_i x^{i'})\partial_{i'} + (\partial_i M^{a'}_b(x))y^b \partial_{a'}. \end{aligned} \tag{1.3}$$

If M is paracompact, then there exists a family of functions $N^a_i(x, y)$, obeying the following law of transformation:

$$N^a_i(x,y) = N^{a'}_{i'}(x', y')(\partial_i x^{i'}) M^a_{a'}(x') - (\partial_{i'} M^a_{b'}(x'))y^{b'}(\partial_i x^{i'}). \tag{1.4}$$

These functions are called the coefficients of nonlinear connection. By using them we can transform the basis $\{\partial_i, \partial_a\}$, whose vectors, under the coordinate transformation (1.1) or (1.2) do not transform as vectors, into the basis $B = \{\delta_i, \partial_a\}$ whose vectors have this property. δ_i is defined by

$$\delta_i = \partial_i - N^a_i \partial_a. \tag{1.5}$$

It is easy to prove, form (1.3) and (1.4), that $\delta_{i'} = \delta_i(\partial_{i'} x^i)$. Any vector field X in $T(E)$ can be represented in the basis B in the following form

$$X = X^i \delta_i + X^a \partial_a, \qquad X^i = X^{i'}(\partial_{i'} x^i), \qquad X^a = X^{a'} M^a_{a'}.$$

We call $X^a\partial_a$ the vertical vector field and $X^i\delta_i$ the horizontal vector field of $T(E)$, respectively. The subspace of $T(E)$ spanned by $\{\delta_i\}$ we shall denote by $T_H(E)$ and the subspace spanned by $\{\partial_a\}$ by $T_V(E)$. So we have

$$T(E) = T_H(E) \oplus T_V(E), \qquad \dim T_H(E) = n, \qquad \dim T_V(E) = m.$$

Let us consider $T^*(E)$, the dual tangent space of E. The natural basis in $T^*(E)$ is $\{dx^1, \dots, dx^n, dy^1, \dots, dy^m\} = \{dx^i, dy^a\}$. From (1.1) we have

$$(a) \quad dx^{i'} = (\partial_i x^{i'})dx^i \qquad (b) \quad dy^{a'} = (\partial_i M^{a'}_a)y^a dx^i + M^{a'}_a dy^a. \tag{1.6}$$

It is obvious that dy^a $a = 1, ..., m$ do not transform as tensors, so we introduce a new basis

$$B^* = \{dx^i, \delta y^a\} \text{ of } T^*(E), \quad \text{where } \delta y^a = dy^a + N^a_i dx^i.$$

According to the coordinate transformation (1.1) and (1.2) the bases B^* and $B^{*\prime} = \{dx^{i'}, \delta y^{a'}\}$ are related by (1.6a) and

$$\delta y^a = M^a_{a'}(x')\delta y^{a'}, \qquad \delta y^{a'} = M^{a'}_a(x)\delta y^a. \tag{1.7}$$

If B^* and $B^{*\prime}$ are two bases of $T^*(E)$ related by (1.6a) and (1.7), then any 1-form $w \in T^*(E)$ satisfies the relations

$$w = w_i dx^i + w_a \delta y^a = w_{i'} dx^{i'} + w_{a'} \delta y^{a'},$$

where

$$w_{i'} = w_i(\partial_{i'} x^i), \qquad w_{a'} = w_a M^a_{a'}.$$

Let us consider $T^*(E) \otimes T^*(E)$. In this space the metric tensor G with respect to the basis B^* is given by

$$G = g_{ij} dx^i \otimes dx^j + g_{ib} dx^i \otimes \delta y^b + g_{aj} \delta y^a \otimes dx^j + g_{ab} \delta y^a \otimes \delta y^b. \tag{1.8}$$

$T_H(E)$ is orthogonal to $T_V(E)$ with respect to the metric G iff $g_{ib} = 0$ and $g_{aj} = 0$ for all $a, b = 1, \ldots, m$.

With respect to the coordinate transformations (1.1) and (1.2) the coordinates of the metric tensor G transform in the following way

$$g_{i'j'} = g_{ij}(\partial_{i'} x^i)(\partial_{j'} x^j), \qquad g_{i'b'} = g_{ib}(\partial_{i'} x^i) M^b_{b'}$$
$$g_{a'j'} = g_{aj} M^a_{a'}(\partial_{j'} x^j), \qquad g_{a'b'} = g_{ab} M^a_{a'} M^b_{b'}.$$

We shall define the covariant coordinates of the vector $X = X^i \delta_i + X^a \partial_a$ by

$$X_i = g_{ij} X^j + g_{ib} X^b, \qquad X_a = g_{aj} X^j + g_{ab} X^b.$$

Definition 1.1. The generalized connection $\nabla: T(E) \otimes T(E) \to T(E)$ $(\nabla: (X, Y) \to \nabla_X Y)$ or equivalently $\nabla_X: T(E) \to T(E)$ $(\nabla_X: Y \to \nabla_X Y)$ is a linear connection defined by

$$\begin{aligned} \nabla_{\delta_i}\delta_j &= F_j{}^k{}_i \delta_k + F_j{}^c{}_i \partial_c, \qquad & \nabla_{\delta_j}\partial_a &= F_a{}^k{}_j \delta_k + F_a{}^c{}_j \partial_c \\ \nabla_{\partial_a}\delta_j &= C_j{}^k{}_a \delta_k + C_j{}^c{}_a \partial_c, \qquad & \nabla_{\partial_a}\partial_b &= C_b{}^k{}_a \delta_k + C_b{}^c{}_a \partial_c. \end{aligned} \tag{1.10}$$

Proposition 1.1. *If (x, y) and (x', y') are two coordinate systems connected by the transformation laws* (1.1) *and* (1.2), *then $\nabla_{X'} Y' = \nabla_X Y$ if and only if*

$$\begin{aligned} F_k{}^j{}_i &= F_{k'}{}^{j'}{}_{i'}(\partial_k x^{k'})(\partial_{j'} x^j)(\partial_i x^{i'}) + (\partial_k \partial_i x^{j'})(\partial_{j'} x^j) \\ F_c{}^b{}_i &= F_{c'}{}^{b'}{}_{i'}(\partial_i x^{i'}) M^b_{b'} M^{c'}_c + (\partial_i M^{b'}_c) M^b_{b'} \\ F_b{}^j{}_i &= F_{b'}{}^{j'}{}_{i'} M^{b'}_b (\partial_{j'} x^j)(\partial_i x^{i'}) \\ F_k{}^b{}_i &= F_{k'}{}^{b'}{}_{i'}(\partial_k x^{k'})(\partial_i x^{i'}) M^b_{b'} \\ C_k{}^j{}_a &= C_{k'}{}^{j'}{}_{a'}(\partial_{j'} x^j)(\partial_k x^{k'}) M^{a'}_a \\ C_b{}^j{}_a &= C_{b'}{}^{j'}{}_{a'}(\partial_{j'} x^j) M^{b'}_b M^{a'}_a \\ C_k{}^b{}_a &= C_{k'}{}^{b'}{}_{a'}(\partial_k x^{k'}) M^{b'}_b M^{a'}_a \\ C_c{}^b{}_a &= C_{c'}{}^{b'}{}_{a'} M^b_{b'} M^{c'}_c M^{a'}_a. \end{aligned}$$

Proposition 1.2. *The torsion tensor* $T(X,Y)=\nabla_X Y-\nabla_Y X-[XY]$ *for the connection* ∇ *has the form*

$$T(X,Y)=(T_j{}^k{}_iY^jX^i+T_b{}^k{}_iY^bX^i+T_j{}^k{}_aY^jX^a+T_b{}^k{}_aY^bX^a)\delta_k+ \\ (T_j{}^c{}_iY^jX^i+T_b{}^c{}_iY^bX^i+T_j{}^c{}_aY^jX^a+T_b{}^c{}_aY^bX^a)\partial_c, \tag{1.11}$$

where

$$\begin{aligned} T_j{}^k{}_i&=F_j{}^k{}_i-F_i{}^k{}_j, & T_j{}^c{}_i&=F_j{}^c{}_i-F_i{}^c{}_j+\delta_iN_j^c-\delta_jN_i^c\\ T_b{}^k{}_i&=F_b{}^k{}_i-C_i{}^k{}_b, & T_b{}^c{}_i&=F_b{}^c{}_i-C_b{}^c{}_i-\partial_bN_i^c\\ T_j{}^k{}_a&=C_j{}^k{}_a-F_a{}^k{}_j, & T_j{}^c{}_a&=C_j{}^c{}_a-F_a{}^c{}_j+\partial_aN_j^c\\ T_b{}^k{}_a&=C_b{}^k{}_a-C_a{}^k{}_b, & T_b{}^c{}_a&=C_b{}^c{}_a-C_a{}^c{}_b. \end{aligned} \tag{1.12}$$

2. Decomposition of $T(E)$. Let us consider a special transformation of (1.1) and (1.2), which has the form:

$$\begin{gathered} x^i=x^i(u^1,\dots,u^{\tilde n})+\hat x^i(\bar u^{\tilde n+1},\dots,\bar u^n)=x^i(u^\alpha)+\hat x^i(\bar u^{\bar\alpha})\\ (\alpha,\beta,\gamma,\dots=1,\dots,\tilde n\quad \bar\alpha,\bar\beta,\bar\gamma,\dots=\tilde n+1,\dots,n)\\ y^a=B^a_A(u)v^A+B^a_{\bar A}(\bar u)\bar v^{\bar A}\\ (A,B,C,\dots=1,\dots,\tilde m\quad \bar A,\bar B,\bar C,\dots=\tilde m+1,\dots,m), \end{gathered} \tag{2.1}$$

where

$$\begin{aligned} &\operatorname{rank}\left[\partial_\alpha x^i\right]=\operatorname{rank}\left[B^i_\alpha\right]=\tilde n\\ &\operatorname{rank}\left[\partial_{\bar\alpha} x^i\right]=\operatorname{rank}\left[B^i_{\bar\alpha}\right]=n-\tilde n,\qquad \partial_{\bar\alpha}=\partial/\partial\bar u^{\bar\alpha},\quad \operatorname{rank}\begin{bmatrix}B^i_\alpha\\ B^i_{\bar\alpha}\end{bmatrix}=n,\\ &\operatorname{rank}\left[\partial_A y^a\right]=\operatorname{rank}\left[B^a_A\right]=\tilde m,\\ &\operatorname{rank}\left[\partial_{\bar A} y^a\right]=\operatorname{rank}\left[B^a_{\bar A}\right]=m-\tilde m,\qquad \partial_{\bar A}=\partial/\partial\bar v^{\bar A},\quad \operatorname{rank}\begin{bmatrix}B^a_A\\ B^a_{\bar A}\end{bmatrix}=m. \end{aligned} \tag{2.2}$$

Let $\tilde\xi=(\tilde E,\tilde\pi,\tilde M)$ be a vector subbundle of a vector bundle $\xi=(E,\pi,M)$, having $\dim\tilde M=\tilde n$, $\dim\tilde E=\tilde n+\tilde m$, $1\le\tilde n\le n$, $1\le\tilde m\le m$. The function $\tilde\xi\to\xi$ is an embedding if: there are embeddings $f\colon\tilde E\to E$, $\tilde f\colon\tilde M\to M$, such that the following diagram is comutative:

$$\begin{array}{ccc} \tilde E & \xrightarrow{f} & E\\ \downarrow{\scriptstyle\tilde\pi} & & \downarrow{\scriptstyle\pi}\\ \tilde M & \xrightarrow{\tilde f} & M \end{array}$$

and $f_{(\tilde{E}_{\tilde{x}})}\colon \tilde{E}_{\tilde{x}} \to E_{\tilde{f}(\tilde{x})}$, $\tilde{x} \in \tilde{M}$ is a monomorphism. Therefore if (u^α, V^A) $(\alpha, \beta, \dots = 1, \dots, \tilde{n}, A, B, \dots = 1, \dots, \tilde{m})$ are the local coordinates on $\tilde{E}$, then the embeding $f\colon \tilde{\xi} \to \xi$ may be given by

$$x^i = x^i(u^1, \dots, u^{\tilde{n}}) + C^i, \qquad \operatorname{rank}\left[\partial x^i/\partial u^a\right] = \operatorname{rank}\left[B^i_\alpha\right] = \tilde{n}$$
$$y^a = B^a_A(u)v^A + C^a, \qquad \operatorname{rank}\left[B^a_A\right] = \tilde{m},$$

where $B^a_A(u)$ is a tensor field on $\tilde{E}$. The $\tilde{n}$ vectors $B^i_\alpha(u)$ and m vectors $B^a_A(u)$ are tangent vectors to the coordinate curves in $\tilde{E}$. They are, for fixed α and A, vectors in $T(E)$ and are defined only on $\tilde{E}$.

For $\bar{u}^{\bar{\alpha}} = C^{\bar{\alpha}}$ $\bar{v}^{\bar{A}} = C^{\bar{A}}$ $(C^{\bar{\alpha}}, C^{\bar{A}} \in R)$ the relations (2.1) and (2.2) give the equations of subbundle $\tilde{\xi}$ of the form (2.3). For $u^\alpha = C^\alpha$, $v^A = C^A$ (2.1) and (2.2) give the equations of an another subbundle denoted by $\tilde{\tilde{\xi}}$.

B^i_α, B^a_A are the tangent vectors of the coordinate curves in $\tilde{E}$ and they form a basis $B_1 = \{B^i_\alpha, B^a_A\}$ of $T(\tilde{E})$. In the same way $B^i_{\bar{\alpha}}$, $B^a_{\bar{A}}$ are the tangent vectors of the coordinate curves in $\tilde{\tilde{E}}$ and they form a basis $B_2 = \{B^i_{\bar{\alpha}}, B^a_{\bar{A}}\}$ of $T(\tilde{\tilde{E}})$. It is obvious that under the conditions (2.2) we have $T(E) = T(\tilde{E}) \oplus T(\tilde{\tilde{E}})$. From (2.1) we obtain

$$\text{(2.4)} \qquad \begin{aligned} &(a) \quad \partial_\alpha = B^i_\alpha(u)\partial_i + (\partial_\alpha B^a_A(u))v^A\partial_a, \\ &(b) \quad \partial_{\bar{\alpha}} = B^i_{\bar{\alpha}}(\bar{u})\partial_i + (\partial_{\bar{\alpha}} B^a_{\bar{A}}(\bar{u}))\bar{v}^{\bar{A}}\partial_a, \\ &(c) \quad \partial_A = B^a_A(u)\partial_a, \qquad \partial_{\bar{A}} = B^a_{\bar{A}}(\bar{u})\partial_a \end{aligned}$$

and

$$\begin{aligned} dx^i &= B^i_\alpha(u)du^\alpha + B^i_{\bar{\alpha}}(\bar{u})d\bar{u}^{\bar{\alpha}} \\ dy^a &= (\partial_\alpha B^a_A)v^A du^\alpha + (\partial_{\bar{\alpha}} B^a_{\bar{A}})\bar{v}^{\bar{A}} d\bar{u}^{\bar{\alpha}} + B^a_A(u)dv^A + B^a_{\bar{A}}(\bar{u})d\bar{v}^{\bar{A}}. \end{aligned}$$

We shall introduce the following coordinate transformations:

$$\text{(2.5)} \quad \begin{aligned} &(a) \quad u^{\alpha'} = u^{\alpha'}(u^1, \dots, u^{\tilde{n}}) && \Leftrightarrow \quad u^\alpha = u^\alpha(u^{1'}, \dots, u^{\tilde{n}'}) \\ &(b) \quad v^{A'} = M^{A'}_A(u)v^A && \Leftrightarrow \quad v^A = M^A_{A'}(u')v^{A'} \\ &(c) \quad \bar{u}^{\bar{\alpha}'} = \bar{u}^{\bar{\alpha}'}(\bar{u}^{\tilde{n}+1}, \dots, \bar{u}^n) = \bar{u}^{\bar{\alpha}'}(\bar{u}^{\bar{\alpha}}) && \Leftrightarrow \quad \bar{u}^{\bar{\alpha}} = \bar{u}^{\bar{\alpha}}(\bar{u}^{\bar{\alpha}'}) \\ &(d) \quad \bar{v}^{\bar{A}'} = M^{\bar{A}'}_{\bar{A}}(\bar{u})\bar{v}^{\bar{A}} && \Leftrightarrow \quad \bar{v}^{\bar{A}} = M^{\bar{A}}_{\bar{A}'}(\bar{u}')\bar{v}^{\bar{A}'}. \end{aligned}$$

From (2.5) it follows

$$\begin{aligned} \partial_{\alpha'} &= (\partial_{\alpha'} u^\alpha)\partial_\alpha + (\partial_{\alpha'} M^A_{A'}(u'))v^{A'}\partial_A, && \partial_{A'} = M^A_{A'}(u')\partial_A, \\ \partial_{\bar{\alpha}'} &= \partial_{\bar{\alpha}'}(\bar{u}^{\bar{\alpha}})\partial_{\bar{\alpha}} + (\partial_{\bar{\alpha}'} M^{\bar{A}}_{\bar{A}'}(\bar{u}'))\bar{v}^{\bar{A}'}\partial_{\bar{A}}, && \partial_{\bar{A}'} = M^{\bar{A}}_{\bar{A}'}(\bar{u}')\partial_{\bar{A}}. \end{aligned}$$

The following relations are valid:

$$M^A_{A'} M^{A'}_B = \delta^A_B, \qquad M^A_{B'} M^{A'}_A = \delta^{A'}_{B'}$$
$$(\partial_\alpha u^{\alpha'})(\partial_{\alpha'} u^\beta) = \delta^\beta_\alpha, \qquad (\partial_\alpha u^{\beta'})(\partial_{\alpha'} u^\alpha) = \delta^{\beta'}_{\alpha'}$$
$$M^{\bar A}_{\bar A'} M^{\bar A'}_{\bar B} = \delta^{\bar A}_{\bar B}, \qquad (\partial_{\bar\alpha} \bar u^{\bar\alpha'})(\partial_{\bar\alpha'} \bar u^{\bar\beta}) = B^{\bar\alpha'}_{\bar\alpha} B^{\bar\beta}_{\bar\alpha'} = \delta^{\bar\beta}_{\bar\alpha}.$$

As $B^a_A(u)$ is a tensor field on $\tilde E$ it transforms in the following way: $B^{a'}_{A'}(u') = B^a_A(u) M^{a'}_a(u) M^A_{A'}(u')$. The tangent space $T(\tilde E)$ is spanned by $\{\partial_\alpha, \partial_A\}$, but from (2.4a) it is obvious that ∂_α do not transform as tensor. We shall introduce in $T(\tilde E)$ a nonlinear connection with coefficients $N^A_\alpha(u,v)$ as a family of functions, which under coordinate transformations (2.5) transform in the following way

$$N^A_\alpha(u,v) = N^{A'}_{\alpha'}(u',v') M^A_{A'}(u') \partial_\alpha u^{\alpha'} - (\partial_{\alpha'} M^A_{A'}(u')) v^{A'} \partial_\alpha u^{\alpha'} \tag{2.6}$$

Substituting $\partial_i = \delta_i + N^a_i \partial_a$ from (1.5) into (2.4a) and adding on the both sides of this equation $-N^A_\alpha \partial_A$ we obtain

$$\delta_\alpha = B^i_\alpha \delta_i + H^a_\alpha \partial_a, \tag{2.7}$$

where

$$\begin{aligned} \delta_a &:= \partial_\alpha - N^A_\alpha \partial_A, \\ H^a_\alpha &:= (\partial_\alpha B^a_A) v^A + B^i_\alpha N^a_i - N^A_\alpha B^a_A. \end{aligned} \tag{2.8}$$

The intrinsic adapted basis in $T(\tilde E)$ is $\tilde B = \{\delta_\alpha, \partial_A\}$. It can be shown that δ_α, ∂_A and H^a_α are tensors with respect to the coordinate transformation (2.5) on $\tilde E$, i.e.

$$\delta_{\alpha'} = (\partial_{\alpha'} u^\alpha) \delta_\alpha, \quad \partial_{A'} = M^A_{A'} \partial_A, \quad H^a_\alpha = H^{a'}_{\alpha'} M^a_{a'} (\partial_\alpha u^{\alpha'}).$$

Introducing the nonlinear connection $N^{\bar A}_{\bar\alpha}(\bar u, \bar v)$ in $T(\tilde{\bar E})$, which has the transformation law of the form (2.6), we can form an adapted basis $\tilde{\bar B} = \{\delta_{\bar\alpha}, \partial_{\bar A}\}$ of $T(\tilde{\bar E})$, where

$$\begin{aligned} \delta_{\bar\alpha} &:= \partial_{\bar\alpha} - N^{\bar A}_{\bar\alpha} \partial_{\bar A}, \\ H^a_{\bar\alpha}(\bar u, \bar v) &:= (\partial_{\bar\alpha} B^a_{\bar A}) \bar v^{\bar A} + B^i_{\bar\alpha} N^a_i - B^a_{\bar A} N^{\bar A}_{\bar\alpha}, \end{aligned} \tag{2.9}$$

and

$$\delta_{\bar\alpha} = B^i_{\bar\alpha} \delta_i + H^a_{\bar\alpha} \partial_a. \tag{2.10}$$

It can be shown that $\delta_{\bar\alpha}$, $\partial_{\bar A}$ and $H^a_{\bar\alpha}$ are tensors with respect to the coordinate transformations (2.5) on $\tilde{\bar E}$, i.e.

$$\delta_{\bar\alpha'} = (\partial_{\bar\alpha'} \bar u^{\bar\alpha}) \delta_{\bar\alpha}, \quad \partial_{\bar A} = M^{\bar A}_{\bar A'} \partial_{\bar A}, \quad H^a_{\bar\alpha} = H^{a'}_{\bar\alpha'} M^a_{a'} (\partial_{\bar\alpha} \bar u^{\bar\alpha'}).$$

PROPOSITION 2.1. *The bases* $B = \{\delta_i, \partial_a\}$ *of* $T(E)$, $\bar{B} = \{\delta_\alpha, \partial_A\}$ *of* $T(E)$ *and* $\tilde{\tilde{B}} = \{\delta_{\bar\alpha}, \partial_{\bar A}\}$ *of* $T(\tilde{\tilde{E}})$ *are connected by*

$$\begin{aligned} \delta_\alpha &= B^i_\alpha \delta_i + H^a_\alpha \partial_a, & \delta_{\bar\alpha} &= B^i_{\bar\alpha}\delta_i + H^a_{\bar\alpha}\partial_a, \\ \partial_A &= B^a_A(u)\partial_a, & \partial_{\bar A} &= B^a_{\bar A}(\bar u)\partial_a. \end{aligned} \tag{2.11}$$

Proof. (2.11) follows from (2.7), (2.10) and (2.4).

The intrinsic adapted basis in $T^*(\tilde{E})$ is $\tilde{B}^* = \{du^\alpha, \delta v^A\}$, where $\delta v^A := dv^A + N^A_\alpha du^\alpha$. The adapted basis in $T^*(\tilde{\tilde{E}})$ is $\tilde{\tilde{B}}^* = \{d\bar u^{\bar\alpha}, \delta \bar v^{\bar A}\}$, where $\delta\bar v^{\bar A} := d\bar v^{\bar A} + N^{\bar A}_{\bar\alpha} d\bar u^{\bar\alpha}$.

PROPOSITION 2.2. *The adapted bases* $\tilde{B}$ *and* $\tilde{B}^*$ *of* $T(\tilde{E})$ *and* $T^*(\tilde{E})$ *respectively are dual to each other, i.e.*

$$\begin{aligned} \langle du^\alpha, \delta_\beta\rangle &= \delta^\alpha_\beta, & \langle du^\alpha, \partial_B\rangle &= 0, \\ \langle \delta v^A, \delta_\beta\rangle &= 0, & \langle \delta v^A, \partial_B\rangle &= \delta^A_B. \end{aligned} \tag{2.12}$$

The adapted bases $\tilde{\tilde{B}} = \{\delta_{\bar\alpha}, \partial_{\bar A}\}$ *and* $\tilde{\tilde{B}}^* = \{d\bar u^{\bar\alpha}, \delta\bar v^{\bar A}\}$ *are dual to each other, i e.*

$$\begin{aligned} \langle d\bar u^{\bar\alpha}, \delta_{\bar\beta}\rangle &= \delta^{\bar\alpha}_{\bar\beta}, & \langle d\bar u^{\bar\alpha}, \partial_{\bar B}\rangle &= 0, \\ \langle \delta\bar v^{\bar A}, \delta_{\bar\beta}\rangle &= 0, & \langle \delta\bar v^{\bar A}, \partial_{\bar B}\rangle &= \delta^{\bar A}_{\bar B}. \end{aligned} \tag{2.12}$$

PROPOSITION 2.3. *The bases* $B^* = \{dx^i, \delta y^a\}$ *of* $T^*(E)$ *and* $\tilde{B}^* = \{du^\alpha, \delta v^A\}$, $\tilde{\tilde{B}}^* = \{d\bar u^{\bar\alpha}, \delta\bar v^{\bar A}\}$ *of* $T^*(\tilde{E})$ *and* $T^*(\tilde{\tilde{E}})$ *respectively, are connected by the formulae*

$$dx^i = B^i_\alpha du^\alpha + B^i_{\bar\alpha} d\bar u^{\bar\alpha}, \qquad \delta y^a = H^a_\alpha du^\alpha + H^a_{\bar\alpha} d\bar u^\alpha + B^a_A \delta v^A + B^a_{\bar A}\delta\bar v^{\bar A}. \tag{2.13}$$

3. The Linear Connection ∇ Expressed in the Basis $\tilde{B}$. The point $u \in E$ in the coordinate system (1.1) and (2.5) (which are connected by (2.1)) has coordinates $(x^i, y^a) = (u^\alpha, \bar u^\alpha, v^A, \bar v^{\bar A})$. The adapted bases of $T(E)$ for these two coordinate systems are $B = \{\delta_i, \partial_a\}$ and $\hat{B} = \{\delta_\alpha, \delta_{\bar\alpha}, \partial_A, \partial_{\bar A}\}$ respecitvely. The linear connection $\nabla_X: T(E) \to T(E)$ in the basis $\hat{B}$ can be expressed by

PROPOSITION 3.1. *The linear connection* $\nabla_X: T(E) \to T(E)$, *introduced by Definition* 1.1, *in the basis* $\hat{B} = \{\delta_\alpha, \delta_{\bar\alpha}, \partial_A, \partial_{\bar A}\}$ *is given by*

$$\begin{aligned} (3.1) \qquad (a) \qquad & \nabla_{\delta_x}\delta_y = F_y{}^\gamma{}_x \delta_\gamma + F_y{}^{\bar\gamma}{}_x \delta_{\bar\gamma} + F_y{}^A{}_x \partial_A + F_y{}^{\bar A}{}_x \partial_{\bar A} \\ (b) \qquad & \nabla_{\delta_x}\partial_u = F_u{}^\gamma{}_x \delta_\gamma + F_u{}^{\bar\gamma}{}_x \delta_{\bar\gamma} + F_u{}^A{}_x \partial_A + F_u{}^{\bar A}{}_x \partial_{\bar A} \\ (c) \qquad & \nabla_{\partial_v}\delta_y = F_y{}^\gamma{}_v \delta_\gamma + F_y{}^{\bar\gamma}{}_v \delta_{\bar\gamma} + F_y{}^A{}_v \partial_A + F_y{}^{\bar A}{}_v \partial_{\bar A} \\ (d) \qquad & \nabla_{\partial_v}\partial_u = F_u{}^\gamma{}_v \delta_\gamma + F_u{}^{\bar\gamma}{}_v \delta_{\bar\gamma} + F_u{}^A{}_v \partial_A + F_u{}^{\bar A}{}_v \partial_{\bar A} \\ & x \in \{\alpha, \bar\alpha\}, \qquad y \in \{\beta, \bar\beta\}, \qquad u \in \{B, \bar B\}, \qquad v \in \{D, \bar D\}. \end{aligned}$$

In (3.1) there are 64 types of cooefficients. In [**3**] the relations between the different kinds of covariant derivatives of vector fields, expressed in bases B and $\hat{B}$, were given. It was proved, that the covarint derivatives of vector fields in the basis $\hat{B}$, with respect to the coordinate transformations of type (2.5) transform as tensors; iff $F_{\gamma}{}^{\alpha}{}_{\beta}$, $F_{\gamma'}{}^{\alpha'}{}_{\beta'}$ transform "as connection coefficients" (for instance $F_{\gamma'}{}^{\alpha'}{}_{\beta'}B^{\gamma'}_{\gamma} = B^{\alpha'}_{\alpha}B^{\beta}_{\beta'}F_{\gamma}{}^{\alpha}{}_{\beta} - B^{\beta}_{\beta'}\partial_{\beta}B^{\alpha'}_{\gamma}$) and the other 60 types of coefficients (from (3.1)) as tensors (for example $F_{C'}{}^{\alpha'}{}_{\beta'}M^{C'}_{C} = B^{\alpha'}_{\alpha}B^{\beta}_{\beta'}F_{C}{}^{\alpha}{}_{\beta}$). Here we want to give the relations between the connection coefficients from (1.10) and (3.1). To obtain the desired formulae, the relations (2.11) and the linearity of connection ∇ will be used.

From (3.1a) and (2.18) we have

$$\begin{aligned}\nabla_{\delta_\beta}\delta_\alpha &= F_{\alpha}{}^{\gamma}{}_{\beta}\delta_\gamma + F_{\alpha}{}^{\bar\gamma}{}_{\beta}\delta_{\bar\gamma} + F_{\alpha}{}^{C}{}_{\beta}\partial_C + F_{\alpha}{}^{\bar C}{}_{\beta}\partial_{\bar C}\\ &= F_{\alpha}{}^{\gamma}{}_{\beta}(B^k_\gamma\delta_k + H^c_\gamma\partial_c) + F_{\alpha}{}^{\bar\gamma}{}_{\beta}(B^k_{\bar\gamma}\delta_k + H^c_{\bar\gamma}\partial_c)\\ &\quad + F_{\alpha}{}^{C}{}_{\beta}B^c_C\partial_c + F_{\alpha}{}^{\bar C}{}_{\beta}B^c_{\bar C}\partial_c.\end{aligned} \tag{3.2}$$

On the other hand from the linearity of ∇ and (2.18) it follows

$$\begin{aligned}\nabla_{\delta_\beta}\delta_\alpha &= \nabla_{\left(B^j_\beta\delta_j + H^b_\beta\partial_b\right)}\left(B^i_\alpha\delta_i + H^a_\alpha\partial_a\right)\\ &= \left(\delta_\beta B^i_\alpha\right)\delta_i + \left(\delta_\beta H^a_\alpha\right)\partial_a + B^j_\beta B^i_\alpha\nabla_{\delta_j}\delta_i + H^b_\beta B^i_\alpha\nabla_{\partial_b}\delta_i\\ &\quad + B^j_\beta H^a_\alpha\nabla_{\delta_j}\partial_a + H^b_\beta H^a_\alpha\nabla_{\partial_b}\partial_a.\end{aligned} \tag{3.3}$$

Substituting (1.17) into (3.3) and equating the corresponding components of δ_k and ∂_c in (3.2) and (3.3) we obtain:

THEOREM 3.1. *The connection coefficients of linear connection ∇ in the bases B and $\hat{B}$ are related by*

$$\begin{aligned}&F_{\alpha}{}^{\gamma}{}_{\beta} + F_{\alpha}{}^{\bar\gamma}{}_{\beta}B^k_{\bar\gamma}\\ &\qquad = \delta_\beta B^k_\alpha + F_i{}^k{}_j B^j_\beta B^i_\alpha + F_a{}^k{}_j B^j_\beta H^a_\alpha + C_i{}^k{}_b H^b_\beta B^i_\alpha + C_a{}^k{}_b H^b_\beta H^a_\alpha,\\ &F_{\alpha}{}^{\gamma}{}_{\beta}H^c_\gamma + F_{\alpha}{}^{\bar\gamma}{}_{\beta}H^c_\gamma + F_{\alpha}{}^{C}{}_{\beta}B^c_C + F_{\alpha}{}^{\bar C}{}_{\beta}B^c_{\bar C}\\ &\qquad = \delta_\beta H^c_\alpha + F_i{}^c{}_j B^j_\beta B^i_\alpha + F_a{}^c{}_j B^j_\beta H^a_\alpha + C_i{}^c{}_b H^b_\beta B^i_\alpha + C_a{}^c{}_b H^b_\beta H^a_\alpha.\end{aligned} \tag{3.4}$$

In a similar way we obtain

$$\begin{aligned}&F_A{}^{\gamma}{}_{\beta}B^k_\gamma + F_A{}^{\bar\gamma}{}_{\beta}B^k_{\bar\gamma} = F_a{}^k{}_j B^j_\beta B^a_A + C_a{}^k{}_b H^b_\beta B^a_A,\\ &F_A{}^{\gamma}{}_{\beta}H^c_\gamma + F_A{}^{\bar\gamma}{}_{\beta}H^c_{\bar\gamma} + F_A{}^{C}{}_{\beta}B^c_C + F_A{}^{\bar C}{}_{\beta}B^c_{\bar C}\\ &\qquad = \delta_\beta B^c_A + F_a{}^c{}_j B^j_\beta B^a_A + C_a{}^c{}_b H^b_\beta B^a_A,\end{aligned} \tag{3.5}$$

$$\begin{aligned}&C^{\gamma}_{AB}B^k_\gamma + C^{\bar\gamma}_{AB}B^k_{\bar\gamma} = C^k_{i\,b}B^b_B B^i_\alpha + C^k_{a\,b}B^b_B H^a_\alpha,\\ &C^{\gamma}_{\alpha B}H^c_\gamma + C^{\bar\gamma}_{\alpha B}H^c_{\bar\gamma} + C^{C}_{\alpha\;B}B^c_C + C^{\bar C}_{\alpha\;B}B^c_{\bar C}\\ &\qquad = \partial_B H^c_\alpha + C^c_{i\,b}B^b_B B^i_\alpha + C^c_{a\,b}B^b_B H^a_\alpha,\end{aligned} \tag{3.6}$$

$$C_A{}^{\gamma}{}_B B^k_{\gamma} + C_A{}^{\bar\gamma}{}_B B^k_{\bar\gamma} = C_a{}^k{}_b B^b_B B^a_A, \tag{3.7}$$
$$C_A{}^{\gamma}{}_B H^c_{\gamma} + C_A{}^{\bar\gamma}{}_B H^c_{\bar\gamma} + C_A{}^C{}_B H^c_C + C_A{}^{\bar C}{}_B H^c_{\bar C} = C_a{}^c{}_b B^b_B B^a_A.$$

Remark 1. Formulae (3.4)–(3.7) are valid if one or two indices from the set $\{\alpha, \beta, A, B\}$ everywhere appear overlined. (It is known that $\delta_{\bar\beta} B^k_{\alpha} = \delta_{\beta} B^k_{\bar\alpha} = \delta_{\bar\beta} B^a_A = \delta_{\beta} B^a_{\bar A} = 0$.)

Theorem 3.1. has many consequences. For example

PROPOSITION 3.2. *If the nonlinear connections $N^A_{\alpha}(u,v)$ in $T(\tilde E)$ and $N^{\bar A}_{\bar\alpha}(\bar u, \bar v)$ in $T(\tilde{\tilde E})$ are chosen in such a way that*

$$H^a_{\alpha} = 0, \qquad H^a_{\bar\alpha} = 0 \tag{3.8}$$

(*see* (2.12) *and* (2.15)), *then the coeficients of the linear connection ∇ defined by the Definition* 1.1 *in the bases B and $\hat B$ satisfy the relations*

$$F_{\alpha}{}^{\gamma}{}_{\beta} B^k_{\gamma} + F_{\alpha}{}^{\bar\gamma}{}_{\beta} B^k_{\bar\gamma} = \delta_{\beta} B^k_{\alpha} + F_i{}^k{}_j B^j_{\beta} B^i_{\alpha}, \tag{3.4'}$$
$$F_{\alpha}{}^C{}_{\beta} B^c_C + F_{\alpha}{}^{\bar C}{}_{\beta} B^c_{\bar C} = F_i{}^c{}_j B^j_{\beta} B^i_{\alpha},$$

$$F_A{}^{\gamma}{}_{\beta} B^k_{\gamma} + F_A{}^{\bar\gamma}{}_{\beta} B^k_{\bar\gamma} = F_a{}^k{}_j B^j_{\beta} B^a_A \tag{3.5'}$$
$$F_A{}^C{}_{\beta} B^c_C + F_A{}^{\bar C}{}_{\beta} B^c_{\bar C} = \delta_{\beta} B^c_A + F_a{}^c{}_j B^j_{\beta} B^a_A,$$

$$C_{\alpha}{}^{\gamma}{}_B B^k_{\gamma} + C_{\alpha}{}^{\bar\gamma}{}_B B^k_{\bar\gamma} = C_i{}^k{}_b B^b_B B^i_{\alpha}, \tag{3.6'}$$
$$C_{\alpha}{}^C{}_B B^c_C + C_{\alpha}{}^{\bar C}{}_B B^c_{\bar C} = C_i{}^c{}_b B^b_B B^i_{\alpha},$$

$$C_A{}^{\gamma}{}_B B^k_{\gamma} + C_A{}^{\bar\gamma}{}_B B^k_{\bar\gamma} = C_a{}^k{}_b B^b_B B^a_A, \tag{3.7'}$$
$$C_A{}^C{}_B B^c_C + C_A{}^{\bar C}{}_B B^c_{\bar C} = C_a{}^c{}_b B^b_B B^a_A.$$

Remark 1 remains valid for (3.4)'–(3.7)'.

The relations (3.8) have very important geometric meaning. From (2.11) and (2.13) it follows, that, under the conditions (3.8), we have

$$T_H(E) = T_H(\tilde E) \oplus T_H(\tilde{\tilde E}), \qquad T_V(E) = T_V(\tilde E) \oplus T_V(\tilde{\tilde E}), \tag{3.9}$$
$$T^*_H(E) = T^*_H(\tilde E) \oplus T^*_H(\tilde{\tilde E}), \qquad T^*_V(E) = T^*_V(\tilde E) \oplus T^*_V(\tilde{\tilde E}).$$

In this case it is very easy to obtain the connection coeficients in the basis $\hat B$ of Miron's d-connection defined by

Definition 3.1. The linear Miron's d-connection is determined by

$$\bar\nabla_{\delta_i}\delta_j = F_j{}^k{}_i\delta_k, \qquad \bar\nabla_{\delta_j}\partial_a = F_a{}^c{}_j\partial_c, \tag{3.10}$$
$$\bar\nabla_{\partial_a}\delta_j = C_j{}^k{}_a\delta_k, \qquad \bar\nabla_{\partial_a}\partial_b = C_b{}^c{}_a\partial_c.$$

From Definition 3.1 it follows that the d-connection $\bar\nabla$ has the property:

$$\bar\nabla_X: T_H(E) \to T_H(E) \qquad \bar\nabla_X: T_V(E) \to T_V(E) \quad (\text{for every } X \in T(E)).$$

From (3.8), (3.9) and (3.10) it follows

PROPOSITION 3.3. *The d-connection $\bar\nabla$, which in the basis B was defined by* (3.10) *in the basis $\hat B$ for which* (3.8) *hold, i.e.*

$$\delta_\alpha = B^i_\alpha\delta_i, \quad \delta_{\bar\alpha} = B^i_{\bar\alpha}\delta_i, \quad \partial_A = B^a_A\partial_a, \quad \partial_{\bar A} = B^a_{\bar A}\partial_a, \tag{3.11}$$

is given by

$$\begin{aligned}
\bar\nabla_{\delta_x}\delta_y &= \bar F_y{}^\gamma{}_x\delta_\gamma + \bar F_y{}^{\bar\gamma}{}_x\delta_{\bar\gamma} && x \in \{\alpha, \bar\alpha\}\\
\bar\nabla_{\delta_x}\partial_u &= \bar F_u{}^A{}_x\partial_A + \bar F_u{}^{\bar A}{}_x\partial_{\bar A} && y \in \{\beta, \bar\beta\}\\
\bar\nabla_{\partial_v}\delta_y &= \bar C_y{}^\gamma{}_v\delta_\gamma + \bar C_y{}^{\bar\gamma}{}_v\delta_{\bar\gamma} && u \in \{B, \bar B\}\\
\bar\nabla_{\partial_v}\partial_u &= \bar C_u{}^A{}_v\partial_A + \bar C_u{}^{\bar A}{}_v\partial_{\bar A} && v \in \{C, \bar C\}.
\end{aligned} \tag{3.12}$$

THEOREM 3.2. *The coefficients of the d-connection $\bar\nabla$ in the bases B and $\hat B$ in the case when* (3.8) *and consequently,* (3.11) *hold, are connected by*

$$\bar F_\alpha{}^\gamma{}_\beta B^k_\gamma + \bar F_\alpha{}^{\bar\gamma}{}_\beta B^k_{\bar\gamma} = \delta_\beta B^k_\alpha + F_i{}^k{}_j B^j_\beta B^i_\alpha \tag{3.4"}$$

$$\bar F_A{}^C{}_\beta B^c_C + \bar F_A{}^{\bar C}{}_\beta B^c_{\bar C} = \delta_\beta B^c_A + F_i{}^c{}_j B^j_\beta B^i_A \tag{3.5"}$$

$$\bar C_\alpha{}^\gamma{}_B B^k_\gamma + \bar C_\alpha{}^{\bar\gamma}{}_B B^k_{\bar\gamma} = C_i{}^k{}_b B^b_B B^i_\alpha \tag{3.6"}$$

$$\bar C_A{}^C{}_B B^c_C + \bar C_A{}^{\bar C}{}_B B^c_{\bar C} = C_a{}^c{}_b B^b_B B^a_A. \tag{3.7"}$$

The Remark 1 remains valid for (3.4)"–(3.7)".

4. The Subbundles of Vector Bundles. Let us consider the case when (2.1) reduces to (2.3) i.e. when the subbundle $\tilde\xi$ is defined. In this case the coordinate transformations are given by (2.5a) and (2.5b). Now we have $B^i_{\bar\alpha} = 0$, $B^a_{\bar A} = 0 \Rightarrow H^a_{\bar\alpha} = 0$.

THEOREM 4.1. *The bases $B = \{\delta_i, \partial_a\}$ of $T(E)$ and $\tilde B = \{\delta_\alpha, \partial_A\}$ of $T(\tilde E)$ are connected by*

$$\delta_\alpha = B^i_\alpha\delta_i + H^a_\alpha\partial_a \qquad \partial_A = B^a_A(u)\partial_a \tag{4.2}$$

and the dual bases $B^* = \{dx^i, \delta y^a\}$ of $T^*(E)$ and $\tilde B^* = \{du^2, \delta v^A\}$ are connected by $dx^i = B^i_\alpha du^\alpha$, $\delta y^a = B^a_A\delta v^A + H^a_\alpha du^\alpha$. Formulae (2.12) *are valid.*

Definition 4.1. The linear connection $\tilde{\nabla}_X : T(\tilde{E}) \to T(\tilde{E})$ $(X \in T(\tilde{E}))$ in the basis $\tilde{B}$ is defined by

$$\tag{4.2} \begin{aligned} \tilde{\nabla}_{\delta_\beta}\delta_\alpha &= \tilde{F}_\alpha{}^\gamma{}_\beta \delta_\gamma + \tilde{F}_\alpha{}^C{}_\beta \partial_C, & \tilde{\nabla}_{\delta_\beta}\partial_A &= \tilde{F}_A{}^\gamma{}_\beta \delta_\gamma + \tilde{F}_A{}^C{}_\beta \partial_C,\\ \tilde{\nabla}_{\partial_B}\delta_\alpha &= \tilde{C}_\alpha{}^\gamma{}_B \delta_\gamma + \tilde{C}_\alpha{}^C{}_B \partial_C, & \tilde{\nabla}_{\partial_B}\partial_A &= \tilde{C}_A{}^\gamma{}_B \delta_\gamma + \tilde{C}_A{}^C{}_B \partial_C, \end{aligned}$$

In this case overlined indices do not exist, because the complementary subbundle is not considered.

The linear connection $\tilde{\nabla}_X : T(\tilde{E}) \to T(\tilde{E})$, defined by (4.2), in the basis B can be expressed by (1.10), but in this case the basis vectors of B and $\tilde{B}$ are connected by (4.1).

THEOREM 4.2. *The coefficients of linear connection $\tilde{\nabla}_X : T(\tilde{E}) \to T(\tilde{E})$ expressed in the bases B and $\tilde{B}$ are connected by the following relations:*

$$\tag{4.3} \begin{aligned} \tilde{F}_\alpha{}^\gamma{}_\beta B^k_\gamma &= \delta_\beta B^k_\alpha + F_i{}^k{}_j B^j_\beta B^i_\alpha \\ &\quad + F_a{}^k{}_j B^j_\beta H^a_\alpha + C_i{}^k{}_\beta H^b_\beta B^i_\alpha + C_a{}^k{}_\beta H^b_\beta H^a_\alpha,\\ \tilde{F}_\alpha{}^\gamma{}_\beta H^c_\gamma + \tilde{F}_\alpha{}^C{}_\beta B^c_C &= \delta_\beta H^c_\alpha + F_i{}^c{}_j B^j_\beta B^i_\alpha + F_a{}^c{}_j B^j_\beta H^a_\alpha \\ &\quad + C_i{}^c{}_b H^b_\beta B^i_\alpha + C_a{}^c{}_b H^b_\beta H^a_\alpha,\\ \tilde{F}_A{}^\gamma{}_\beta B^k_\gamma &= F_\alpha{}^k{}_j B^j_\beta B^a_A + C_\alpha{}^k{}_\beta H^b_\beta B^a_A,\\ \tilde{F}_A{}^\gamma{}_\beta H^c_\gamma + \tilde{F}_A{}^C{}_\beta B^c_C &= \delta_\beta B^C_A + F_a{}^c{}_j B^j_\beta B^a_A + C_a{}^c{}_b H^b_\beta B^a_A,\\ \tilde{C}_\alpha{}^\gamma{}_B B^k_\gamma &= C_i{}^k{}_b B^b_B B^i_\alpha + C_a{}^k{}_b B^b_B H^a_\alpha,\\ \tilde{C}_\alpha{}^\gamma{}_B H^C_\gamma + \tilde{C}_\alpha{}^C{}_B B^c_C &= \partial_B H^c_\alpha + C_i{}^c{}_b B^b_B B^i_\alpha + C_a{}^c{}_b B^b_b B H^a_\alpha,\\ \tilde{C}_A{}^\gamma{}_B B^k_\gamma &= C_a{}^k{}_b B^b_B B^a_A,\\ \tilde{C}_A{}^\gamma{}_B H^c_\gamma + \tilde{C}_A{}^C{}_B B^c_C &= C_a{}^c{}_b B^b_B B^a_A. \end{aligned}$$

It can be proved that, with respect to the coordinate transformations of type (2.5a) and (2.5b), all the coefficients in (4.2) transform as tensors, except $\tilde{F}_\alpha{}^\gamma{}_\beta$ and $\tilde{F}_A{}^C{}_\beta$, which transform "as connection coefficients".

THEOREM 4.3. *If the nonlinear connection $N^A_\alpha(u,v)$ in $T(\tilde{E})$ satisfies the relation $H^a_\alpha = 0$ (see (2.8)), then (4.3) reduces to the form*

$$\tag{4.3'} \begin{aligned} \tilde{F}_\alpha{}^\gamma{}_\beta B^k_\gamma &= \delta_\beta B^k_\alpha + F_i{}^k{}_j B^j_\beta B^i_\alpha, & \tilde{F}_\alpha{}^C{}_\beta B^c_C &= F_i{}^c{}_j B^j_\beta B^i_\alpha,\\ \tilde{F}_A{}^\gamma{}_\beta B^k_\gamma &= F_\alpha{}^k{}_j B^j_\beta B^a_A, & \tilde{F}_A{}^C{}_\beta B^c_C &= \delta_\beta B^C_A + F_\alpha{}^c{}_j B^j_\beta B^a_A,\\ \tilde{C}_\alpha{}^\gamma{}_B B^k_\gamma &= C_i{}^k{}_b B^b_B B^i_\alpha, & \tilde{C}_\alpha{}^C{}_B B^c_C &= C_i{}^c{}_b B^b_B B^i_\alpha,\\ \tilde{C}_A{}^\gamma{}_B B^k_\gamma &= C_a{}^k{}_b B^b_B B^a_A, & \tilde{C}_A{}^C{}_B B^c_C &= C_a{}^c{}_b B^b_B B^a_A. \end{aligned}$$

THEOREM 4.4. *The distinguished connection*

$$\bar\nabla_X : T_H(E) \to T_H(E), \qquad \bar\nabla_X : T_V(E) \to T_V(E) \quad \textit{(for every } X \in T(E),)$$

given by (3.10), *reduced on* $T(\tilde E)$ *is distinguished connection:*

$$\begin{aligned} &\bar\nabla_{\delta_\beta}\delta_\alpha = \bar F_\alpha{}^\gamma{}_\beta \delta_\gamma, \qquad &&\bar\nabla_{\delta_\beta}\partial_A = \bar F_A{}^C{}_\beta \partial_C,\\ &\bar\nabla_{\partial_B}\delta_\alpha = \bar C_\alpha{}^\gamma{}_B \delta_\gamma, &&\bar\nabla_{\partial_B}\partial_A = \bar C_A{}^C{}_B \partial_C. \end{aligned} \tag{4.4}$$

iff $H^a_\alpha = 0$.

THEOREM 4.5. *The coefficients of d-connection* $\bar\nabla$ *in the basis* B (*defined by* (3.10)) *and* $\tilde B$ (*defined by* (4.4)) *for* $H^a_\alpha = 0$ *are connected by the relations*

$$\begin{aligned} &\bar F_\alpha{}^\gamma{}_\beta B^k_\gamma = \delta_\beta B^k_\alpha + F_i{}^k{}_j B^j_\beta B^i_\alpha, \qquad &&\bar F_A{}^C{}_\beta B^c_C = \delta_\beta B^c_A + F_a{}^c{}_j B^j_\beta B^a_A,\\ &\bar C_\alpha{}^\beta{}_B B^k_\beta = C_i{}^k{}_b B^b_B B^i_\alpha, &&\bar C_A{}^C{}_B B^c_C = C_a{}^c{}_b B^b_B B^a_A. \end{aligned} \tag{4.3"}$$

The other connection coefficients appearing in (4.3)*' for the d-connection are equal to zero.*

The coefficients (appeared in (1.10)) for the recurrent and metric connection are determined in Čomić [**3**] as functions of the metric tensor G ((1.8), (1.9)), the vector of recurrency λ and the torsion tensor T ((1.11), (1.12)). The curvature theory of vector bundles and subbundles is given in Čomić [**4**]. In Anastasiu [**1**], using the d-connection (which is generalized in Čomić [**2**]) the dual spaces are examined. In Miron [**6**] the theory of vector bundles in Finsler spaces is given.

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