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THE STRONGLY ASYMMETRIC GRAPHS OF ORDER 6 AND 7

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Abstract. Let G be an arbitrary connected simple graph of order n. G is called strongly asymmetric graph if all induced overgraphs of G of order $n+1$ are nonisomorphic. We give the list of all strongly asymmetric graphs of order 6 and 7. Also we prove that there exist exactly 16 asymmetric graphs of order 7 which are not strongly asymmetric.

We consider only finite connected graphs having no loops or multiple edges. The vertex set of a graph G is denoted by $V(G)$, and its order (number of vertices) by $|G|$.

Let S be any (possibly empty) subset of the vertex set $V(G)$. Denote by G_S the graph obtained of the graph G by adding a new vertex $x(x \in V(G))$, which is adjacent exactly to vertices from S. Graph G is obviously an induced subgraph of the graph G_S, and G_S is an induced overgraph of G. Varying the set $S \subseteq V(G)$ we get $2^{|G|}$ such graphs G_S. The family of all such graphs is denoted by $\mathcal{G}(G)$. We shall call $\mathcal{G}(G)$ the *overset* of G. We say that $\mathcal{G}(G)$ is connected if every graph $H \in \mathcal{G}(G)$ except $G_\emptyset$ so does. It is easily seen that this happens if and only if the graph G is connected.

In [**4**] we introduced the following definition.

Definition 1. Graph G is called strongly asymmetric graph if for any two sets $S_1, S_2 \subseteq V(G)$ $(S_1 \neq S_2)$, the graphs G_{S_1}, G_{S_2} are nonisomorphic.

Let $\Gamma(G)$ be the automorphism group of a graph G. As is well known, if $\Gamma(G)$ is trivial, then G is called asymmetric.

PROPOSITION 1. [**4**] *Every strongly asymmetric graph is also asymmetric.*

In [**4**] was proved that the class of all strongly asymmetric graphs is nonempty. In the same paper, many other properties of the strongly asymmetric graphs are also proved. We suppose that it can be also of some interest to describe all strongly

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asymmetric graphs with small number of vertices. In [**4**] was proved that there are no strongly asymmetric graphs of order $3, 4, 5$. In the following theorem we describe all strongly asymmetric graphs of order 6 and 7. We shall use here the list of all connected graphs with 6 vertices (published in [**1**]), and the corresponding list of all connected graphs with 7 vertices (published in [**2**]).

THEOREM 1. *There exist exactly* 8 *strongly asymmetric graphs of order* 6. *Their ordering numbers (according to* [**1**]*) are* 46, 59, 60, 67, 77, 85, 87, 98.

There exist exactly 128 *strongly asymmetric graphs of order* 7. *Their ordering numbers (according to* [**2**]*) are* 3, 21, 39, 52, 64, 67, 68, 74, 75, 77, 98, 100, 126, 128, 130, 133, 136, 149, 150, 158, 159, 160, 161, 165, 167, 171, 177, 189, 194, 196, 201, 205, 226, 232, 234, 236, 243, 249, 255, 267, 268, 270, 272, 280, 281, 283, 284, 286, 288, 289, 291, 295, 296, 306, 307, 309, 314, 315, 322, 348, 367, 368, 370, 373, 376, 377, 380, 389, 394, 395, 399, 400, 404, 406, 407, 414, 416, 418, 419, 428, 442, 443, 448, 455, 458, 470, 473, 477, 502, 506, 517, 519, 520, 522, 527, 532, 533, 535, 544, 547, 552, 554, 559, 560, 578, 581, 584, 591, 606, 627, 628, 641, 645, 646, 650, 652, 655, 671, 674, 676, 692, 727, 745, 748, 757, 761, 785, 793.

The proof is obtained by a straightforward use of computer.

We also note that there are asymmetric graphs which are not strongly asymmetric. Moreover, in [**4**] was proved that there are infinitely many such graphs.

It is not difficult to see that every asymmetric graph of order 6 is also strongly asymmetric. But, this is not so for graphs of order 7 (and for higher orders).

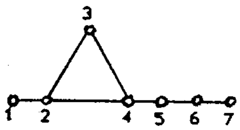
The following proposition describes all asymmetric graphs of order 7, which are not strongly asymmetric. It is also checked by the computer methods.

PROSITION 2. *There exist exactly* 16 *asymmetric graphs of order* 7, *which are not strongly asymmetric. They are represented in the figure.*

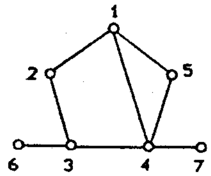
Proof. It is easy to check that all mentioned graphs are asymmetric. In order to prove that a graph G is not strongly asymmetric, it is sufficient to find two subset $R, S \subseteq V(G)$ $(R \neq S)$ such that the corresponding overgraphs $G_R, G_S \in \mathcal{G}(G)$ are isomorphic.

Indeed, for any graph G_i $(i = 1, 2, \dots, 16)$ from the figure we define the subsets R_i, S_i in the following way:

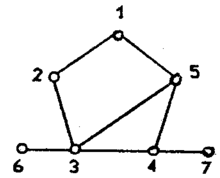
$$\begin{array}{llll}
R_1 = \{3,6,7\}, & S_1 = \{1,5,6\}; & R_2 = \{3,5,6\}, & S_2 = \{2,3,7\}; \\
R_3 = \{5,7\}, & S_3 = \{4,6\}; & R_4 = \{2,3,6\}, & S_4 = \{3,4,7\}; \\
R_5 = \{1,6,7\}, & S_5 = \{2,4,7\}; & R_6 = \{1,3,6,7\}, & S_6 = \{1,2,4,7\}; \\
R_7 = \{2,3,5\}, & S_7 = \{3,6,7\}; & R_8 = \{2,4,7\}, & S_8 = \{2,3,6\}; \\
R_9 = \{1,2,4\}, & S_9 = \{2,3,7\}; & R_{10} = \{4,5\}, & S_{10} = \{6,7\}; \\
R_{11} = \{1,3,4\}, & S_{11} = \{2,3,5\}; & R_{12} = \{3,4,7\}, & S_{12} = \{2,4,6\}; \\
R_{13} = \{2,5,6\}, & S_{13} = \{1,5,7\}; & R_{14} = \{1,5,7\}, & S_{14} = \{3,6,7\}; \\
R_{15} = \{4,5,6,7\}, & S_{15} = \{1,2,6,7\}; & R_{16} = \{1,5,7\}, & S_{16} = \{2,5,6\}.
\end{array}$$



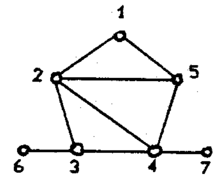
G_1



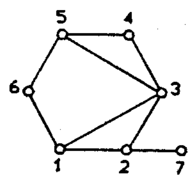
G_2



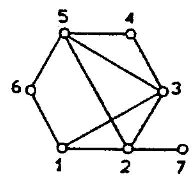
G_3



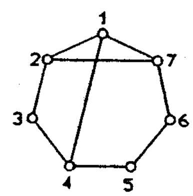
G_4



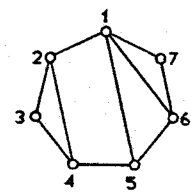
G_5



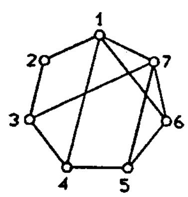
G_6



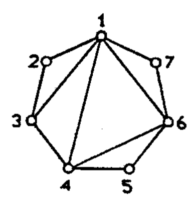
G_7



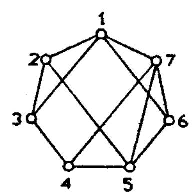
G_8



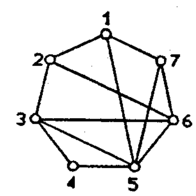
G_9



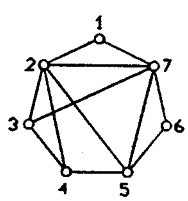
G_{10}



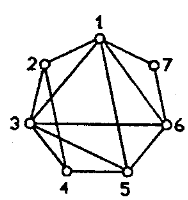
G_{11}



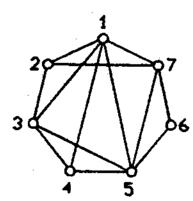
G_{12}



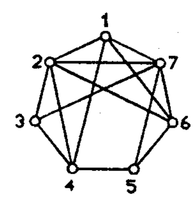
G_{13}



G_{14}



G_{15}



G_{16}

For any two graphs $G_{R_i}, G_{S_i} \in \mathcal{G}(G_i)$ $(i = 1, 2, \dots, 16)$ the new vertex x added to the graph G_i is denoted by 8. The corresponding vertex sets of the graphs G_{R_i}, G_{S_i} are denoted by $V(G_R^i)$ and $V(G_S^i)$, respectively. Now, we define the permutations $f_i\colon V(G_R^i) \to V(G_S^i)$ in the following way:

$$
\begin{aligned}
f_1 &= (7,6,5,8,1,2,3,4), & f_2 &= (8,7,4,3,2,5,6,1),\\
f_3 &= (6,8,4,5,3,7,1,2), & f_4 &= (7,4,2,3,8,1,6,5),\\
f_5 &= (3,1,2,7,8,4,6,5), & f_6 &= (3,1,2,7,8,4,6,5),\\
f_7 &= (8,7,2,3,4,5,6,1), & f_8 &= (2,1,7,6,8,3,4,5),\\
f_9 &= (7,5,4,1,2,8,3,6), & f_{10} &= (6,8,7,1,3,4,5,2),\\
f_{11} &= (5,4,7,1,3,8,2,6), & f_{12} &= (8,4,5,7,6,3,2,1),\\
f_{13} &= (6,7,8,1,2,4,5,3), & f_{14} &= (3,8,6,7,1,5,4,2),\\
f_{15} &= (7,6,8,2,1,4,5,3), & f_{16} &= (7,6,8,5,4,1,2,3).
\end{aligned}
$$

By these permutations, we immediately get that graph G_{R_i} is isomorphic to the graph G_{S_i} $(i = 1, 2, \dots, 16)$, and consequently, the graphs from the figure are not strongly asymmetric. $\square$

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