PUBLICATIONS DE L'INSTITUT MATHÉMATIQUE
Nouvelle série, tome 56 (70), 1994, 7–17

A CONSTRUCTIVE PROOF OF EQUIVALENCE OF FORMALISM OF DCG'S WITH THE FORMALISM OF TYPE 0 PHRASE–STRUCTURE GRAMMARS

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Communicated by Žarko Mijajlović

Abstract. We present a proof that definite clause grammars (DCG's) are equivalent in their generative power to type 0 phrase-structure grammars. The proof is constructive and it actually describes an algorithm for transferring from a language description by type 0 grammar to DCG characterization. The proof has been inspired by the proof given in [MA93] but our approach is considerably simpler and the constructed DCG grammar is much more efficient. The paper also suggests how computer implementation of the algorithm can be developed.

1. Description of definite clause grammar G_P

Let $G = (N, T, S, P)$ be a type 0 grammar in Chomsky's hierarchy. In [MA93] it has been proved that there exists a definite clause grammar, which we denote by G_M, such that $L(G) = L(G_M)$. In this part of the paper we construct a new but simpler definite clause grammar G_P for which the equality $L(G) = L(G_P)$ holds.

The components of the grammar G_P are specified as follows: *Terminals* are the same as the terminals for the grammar G (set T); *Non-terminals* are all and only symbols whose main (outermost) functor is the list constructor denoted with "·"; *Axiom* (starting symbol) is $[[\,]|[S]]$; *Set of rewriting rules* consists of the following rules:

(j) $[[\,]|[S]] \to [[\,]|\underline{\alpha}]$ if $S \to \alpha$ was in P and $\underline{\alpha}$ is the list of all symbols in α. [The elements of the list $\underline{\alpha}$ are symbols of the word α taken in the order they occur from left to right.]

(jj) $[X|[a_1, a_2, \dots, a_m|Y]] \to [X|[b_1, b_2, \dots, b_n|Y]]$ if $a_1a_2 \dots a_m \to b_1b_2 \dots b_n$ was in P

We recall that $[x_1, x_2, \dots, x_n|Y]$ is defined as the following list $(x_1 \cdot (x_2 \cdot (\dots (x_n \cdot Y) \dots)))$, where atoms are denoted by italics and lists by capitals – symbolism which is used throughout the paper.

AMS Subject Classification (1991): Primary 03 B 65

(jjj) $[[y|X]|Z] \to [X|[y|Z]]$ (vj) $[\,] \to e$, where e is empty word
(jv) $[X|[y|Z]] \to [[y|X]|Z]$ (vjj) $[x|Y] \to x, Y$
(v) $[[\,]|Y] \to Y$

Thus instead of the atom $sym\,(X, [y_1, y_2, \dots, y_n], Z)$, which plays the key role in the grammar G_M of [MA93], we now have the atom $[X|[y_1, y_2, \dots, y_n|Z]]$ which represents the symbols from the current word in the derivation in grammar G. This derivation is to be simulated in grammar G_P. The first parameter X contains the list of symbols to the left of the symbol currently being expanded by a production from P. The symbol being expanded in G is $y_1 y_2 \dots y_n$ and it is represented by the head of the list $[y_1, y_2, \dots, y_n|Z]$. The third parameter Z contains the list of symbols to the right of the symbol currently being expanded in G. The list for the first parameter X is given in right-to-left with respect to how its elements appear in the current word and the second and third parameters, $y_1, y_2, \dots, y_n$ and Z, are given in standard left-to-right order. Between the rules (j) through (vjj) and the rules (i) through (xii) of [MA93] there is the following correspondence: the rule (j) corresponds to the rules (i), the rule (jj) corresponds to the rule (ii), the rule (jjj) corresponds to (iv), the rule (jv) corresponds to (vi), the rule (v) corresponds to (vii), the rules (vj), (vjj) are the same as the rules (xi), (xii) respectively.

The rule (jjj) gives the possibility for taking letters from left part of the current word and merging them to the middle part and the rule (jv) gives the possibility of taking letters from the left of the middle part and merging them to the left part of the current word. This is possible at any time of a derivation for arbitrary numbers of letters.

In the grammar G_P there are no rules corresponding to the rules (iii), (v), (viii), (ix), (x) which in [MA93] serve for taking letters from right part of the current word and merging them to the middle part and for taking letters from the right of the middle part and merging them to the right part of the current word as well as for elimination of predicate *sym* and *merge-right*. The main reasons for absence of the rules of this kind are the the following:

In the grammar G_P there are no other non-terminals except those built up from the list constructor "$\cdot$".

In order to apply a rule of the form (jj) corresponding to the rule $\alpha \to \beta$ from P it suffices to clean the word α only from the l e f t side.

2. Comparing efficiency of grammars G_M, G_P

In this part of the paper we compare the efficiency of the grammars G_M, G_P on the basis of the number of steps in the corresponding proofs. We start with the following example.

Example. Consider the language $\{a^n b^n | n > 2\}$ given in [MA93] which can obviously be generated by the grammar $G = (N, T, S, P)$, where $N = \{S, A\}$, $T = \{a, b\}$, $P = \{S \to aAb, A \to aAb, A \to ab\}$ For the word $aaabbb$ a derivation in G may look like (using the meaning of bold case letters and the operator $==$ in the way explained in [MA93]):

$$S \to \mathbf{aAb} == a\mathbf{A}b \to a\mathbf{aAb}b == aa\mathbf{A}bb \to aa\mathbf{ab}bb == aaabbb$$

The derivation in G_P which simulates these steps is as follows:

$$
\begin{array}{rll}
[[\,]|[\mathbf{S}]] & \longrightarrow [[\,]|[\mathbf{a},\mathbf{A},\mathbf{b}]] & \text{[Application of the rule (j)]} \\
& == [[\,]|[\mathbf{a}|[A,b]]] & \\
& \longrightarrow [[\mathbf{a}]|[A,b]] & \text{[Application of the rule (jv)]} \\
& == [[a]|[\mathbf{A}|[b]]] & \\
& \longrightarrow [[a]|[\mathbf{a},\mathbf{A},\mathbf{b}|[b]]] & \text{[Application of the rule (jj)]} \\
& == [[a]|[\mathbf{a}|[A,b,b]]] & \\
& \longrightarrow [[a,\mathbf{a}]|[A,b,b]] & \text{[Application of the rule (jv)]} \\
& == [[a,a]|[\mathbf{A}|[b,b]] & \\
& \longrightarrow [[a,a]|[\mathbf{a},\mathbf{b}|[b,b]] & \text{[Application of the rule (jj)]} \\
& == [[a,a]|[a,b,b,b]] & \\
& == [[\mathbf{a}|[a]|[a,b,b,b]] & \\
& \longrightarrow [[a]|[\mathbf{a}|[a,b,b,b]]] & \text{[Application of the rule (jjj)]} \\
& == [[\mathbf{a}|[\,]]|[a,a,b,b,b]] & \\
& \longrightarrow [[\,]|[\mathbf{a}|[a,a,b,b,b]]] & \text{[Application of the rule (jjj)]} \\
& == [[\,]|[a,a,a,b,b,b]] & \\
& \longrightarrow [\,],[a,a,a,b,b,b] & \text{[Application of the rule (v)]} \\
& \longrightarrow e,[a,a,a,b,b,b] & \text{[Application of the rule (vj)]} \\
& \longrightarrow a,[a,a,b,b,b] & \text{[Application of the rule (vjj)]} \\
& \longrightarrow a,a,[a,b,b,b] & \text{[Application of the rule (vjj)]} \\
& \longrightarrow a,a,a,[b,b,b] & \text{[Application of the rule (vjj)]} \\
& \longrightarrow a,a,a,b,[b,b] & \text{[Application of the rule (vjj)]} \\
& \longrightarrow a,a,a,b,b,[b] & \text{[Application of the rule (vjj)]} \\
& \longrightarrow a,a,a,b,b,b,[\,] & \text{[Application of the rule (vjj)]} \\
& \longrightarrow a,a,a,b,b,b & \text{[Application of the rule (vj)]}
\end{array}
$$

All together 16 steps of the proof which is considerably smaller number comparing with the 34 steps of the corresponding proof in the grammar G_M of [MA93]. The similar situation is in the general case which we shall prove in the sequel.

We start with the following four lemmae for the grammars G_M, G_P.

Lemma A. (i) *The proof in the grammar G_M (where Y, Z, U are lists):*

$$sym\,([x_1,x_2,\dots,x_n|Y],Z,U) \longrightarrow^* sym\,(Y,[x_n,x_{n-1},\dots,x_1|Z],U)$$

has n steps (applications of the rules of G_M).

(ii) *The corresponding proof in the grammar G_P (where Y, Z are lists):*

$$[[x_1, x_2, \dots, x_n|Y]|Z] \longrightarrow^* [Y|[x_n, x_{n-1}, \dots, x_1|Z]]$$

has also n steps (applications of the rules of G_P).

Proof. Starting from x_1 we move $x_1, x_2, \dots, x_n$ from the first to the second place moving one atom at a time and using in each step the rule (iv) of the grammar G_M i.e. the rule (jjj) of the grammar G_P. □

LEMMA B. (i) *The proof in the grammar G_M (where Y, Z, U are lists):*

$$sym\,(Y, [x_n, x_{n-1}, \dots, x_1|Z], U) \longrightarrow^* sym\,([x_1, x_2, \dots, x_n|Y], Z, U)$$

has n steps (applications of the rules of G_M).

(ii) *The corresponding proof in the grammar G_P (where Y, Z are lists):*

$$[Y|[x_n, x_{n-1}, \dots, x_1|Z]] \longrightarrow^* [[x_1, x_2, \dots, x_n|Y]|Z]$$

has also n steps (applications of the rules of G_P).

Proof. Similarly to the previous proof starting from x_1 we move $x_1, x_2, \dots, x_n$ from the second to the first place moving one atom at a time but now using in each step the rule (vi) of the grammar G_M i.e. the rule (jv) of the grammar G_P. □

To simplify the deductions in the grammar G_M it is convenient to redefine the predicate *merge-right* in the following way:

$$\textit{merge-right}\ (X, A, B) \text{ iff } X = A\underline{B}$$

Using this the rules (iii) and (v) of [MA93] can be written in the form:
(iii) $sym\,(X, Y, [Z|U]) \rightarrow sym\,(X, Y\underline{Z}, U])$
(v) $sym\,(X, Y\underline{Z}, U]) \rightarrow sym\,(X, Y, [Z|U])$

As in these two rules the predicate *merge-right*, i.e. the operation concatenation of lists is present, both of which are defined recursively, the real length of the rules (iii), (v) depends essentially on the length of the list Y.

LEMMA C. *The number of steps in the proof*

$$sym\,(X, Y, [z_1, z_2, \dots, z_n|U]) \longrightarrow^* sym\,(X, Y[z_1, z_2, \dots, z_n], U)$$

of grammar G_M equals to $n|Y| + n(n+3)/2$.

Proof. (by induction on n). If $n = 1$ the proof reduces to:

$$sym\,(X, Y, [z_1|U]) \longrightarrow sym\,(X, Y[z_1], U)$$

in fact to the rule (iii). As the definition of the expression $Y[z_1]$ is by induction on Y, the steps which are necessary to deduce $sym\,(X, Y[z_1], U)$ are the following:

one application of the rule (iii), $|Y|$ applications of the rule (ix), one application of the rule (viii)

All together: $|Y|+2$ steps. Using this in the case $n>1$ we have step by step:

$$\begin{aligned} \mathit{sym}\,(X,Y,[z_1,z_2,\dots,z_n|U]) &\longrightarrow^* \mathit{sym}\,(X,Y[z_1],[z_2,\dots,z_n|U]) \\ &\longrightarrow^* \mathit{sym}\,(X,Y[z_1,z_2],[z_3,\dots,z_n|U]) \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ &\longrightarrow^* \mathit{sym}\,(X,Y[z_1,z_2,\dots,z_n],U) \end{aligned}$$

wherefrom it follows that the total number of the steps in the proof equals to:

$$(|Y|+0+2)+(|Y|+1+2)+\cdots+(|Y|+(n-1)+2)$$

which yields: $n|Y|+n(n+3)/2$. $\square$

LEMMA D. *The number of steps in the proof*

$$\mathit{sym}\,(X,Y[z_1,z_2,\dots,z_n],U) \longrightarrow^* \mathit{sym}\,(X,Y,[z_1,z_2,\dots,z_n|U])$$

of grammar G_M equals to $n|Y|+n(n+3)/2$.

Proof. The proof is similar to the proof of the previous lemma but in the case $n=1$ instead of the rules (iii), (ix), (viii) we have:

one applications of the rule (v), $|Y|$ application of the rule (ix), one application of the rule (viii)

which all together is again: $|Y|+2$. Using this step by step it can easily be proved that the total number of steps in the case $n>1$ is again $n|Y|+n(n+3)/2$. $\square$

Let now $\pi(S)$ be a proof in the Chomsky grammar G:

$$S \longrightarrow^* \omega$$

(ω is a terminal word in T, i.e. the word built up from terminal symbols from T) and let $\pi(\mathit{sym}\,([\,],\underline{S},[\,]))$ be the corresponding proof in the logic grammar G_M:

$$\mathit{sym}\,([\,],\underline{S},[\,]) \longrightarrow^* \omega$$

which is shortly denoted by $\pi_M(\underline{S})$.

Let further $\pi([[\,]|\underline{S}])$ be the corresponding proof in the logic grammar G_P:

$$[[\,]|\underline{S}] \longrightarrow^* \omega$$

which is shortly denoted by $\pi_P(\underline{S})$. The proofs $\pi_M(\underline{S})$, $\pi_P(\underline{S})$ can in a natural way be splitted into two parts:

I part: Obtained by application of sequences of rewriting rules of the grammars G_M, G_P having the forms $\mathit{sym}\,(X,\underline{\alpha},Y) \to \mathit{sym}\,(X,\underline{\beta},Y)$, $[X|[\alpha|Y]] \to [X|[\beta|Y]]$ respectively which correspond to the rules $\alpha\to\beta$ of the grammar G.

$$\pi_M(S)_I: \qquad \mathit{sym}\,([\,],\underline{S},[\,]) \longrightarrow^* \mathit{sym}\,(\underline{\rho^\sim},\underline{\theta},\underline{\sigma})$$

$$\pi_P(S)_I: \qquad [[\,]|\underline{S}] \longrightarrow^* [\underline{\rho}^\sim|[\theta|\underline{\sigma}]]$$

where $\rho^\sim$ is the mirror image of the word ρ.

II part: Obtained by rules which transform the formulae $sym\,(\underline{\rho}^{\sim}, \underline{\theta}, \underline{\sigma})$, $[\underline{\rho}|[\theta|\underline{\rho}]]$ into the word $\omega = \rho\theta\sigma$.

$$\pi_M(S)_{II}: \qquad sym\,(\underline{\rho}^{\sim}, \underline{\theta}, \underline{\sigma}) \longrightarrow^* \omega$$

$$\pi_P(S)_{II}: \qquad [\underline{\rho}^{\sim}|[\theta|\underline{\sigma}]] \longrightarrow^* \omega$$

We consider each of these parts in all detail.

(I) The proofs $\pi_M(S)_I, \pi_P(S)_I$ are composed of the fragments related to the neighbour applications of some rules of the form $\alpha \to \beta$, i.e. the corresponding rules

$$sym\,(X, \underline{\alpha}, Y) \longrightarrow sym\,(X, \underline{\beta}, Y), \quad [X|[\alpha|Y]] \longrightarrow [X|[\beta|Y]]$$

respectively. We confine ourself first of all to the grammar G_M.

Thus suppose that the mentioned neighbour rules of G are the following two:

$$\alpha \longrightarrow \beta, \;\; \gamma \longrightarrow \delta$$

and that after application of the first rule the following formula $sym\,(\underline{\xi}, \underline{\beta}, \underline{\zeta})$ has been obtained, then for the considered fragment of proof the five cases are possible:

$1°\gamma$ is a subword of β, $2°\gamma$ is a subword of ξ, $3°\gamma$ is a subword of ζ

$4°\gamma$ is splitted between ξ and β, $5°\gamma$ is splitted between β and ζ

1° Let $\beta = \beta_0\gamma\boldsymbol{\beta}$. The corresponding fragment of proof in the grammar G_M reeds:

$$\begin{aligned} sym\,(\underline{\xi}, \underline{\alpha}, \underline{\zeta}) &\longrightarrow sym\,(\underline{\xi}, \underline{\beta_0\gamma\boldsymbol{\beta}}, \underline{\zeta}) \\ &\longrightarrow^* sym\,(\underline{\beta_0^{\sim}\xi}, \underline{\gamma\boldsymbol{\beta}}, \underline{\zeta}) \\ &\longrightarrow^* sym\,(\underline{\beta_0^{\sim}\xi}, \underline{\gamma}, \underline{\boldsymbol{\beta}\zeta}) \end{aligned}$$

[Firstly the rule of G_M corresponding to $\alpha \to \beta$ has been employed, then the list $\underline{\gamma}$ has been cleaned from the left side using the rule (vi) of [MA93], and at last the list $\underline{\gamma}$ has been cleaned from the right side by means of the rules (v), (ix), (viii) of [MA93].]

LEMMA M1. *Let $k = |\boldsymbol{\beta}|$, then for number of steps in the above fragment of proof the following equality:* NUMBER OF STEPS $= [1 + |\beta_0|] + [k|\gamma| + k(k+3)/2]$ *holds. In the case $k = 0$ this equality reduces to:* NUMBER OF STEPS $= |\beta| + |\beta_0|$

Proof. For the above equality we have the following deduction:

NUMBER OF STEPS	$= 1$	[By the rule of G_M corresponding to $\alpha \longrightarrow \beta$]
	$+ \lvert\beta_0\rvert$	[By Lemma B]
	$+ k\lvert\gamma\rvert + k(k+3)/2$	[By Lemma D]
	$= [1 + \lvert\beta_0\rvert] + [k\lvert\gamma\rvert + k(k+3)/2]$	

It is easy to check that for $k = 0$ this equality reduces to: NUMBER OF STEPS $= 1 + |\beta_0|$. $\square$

2° Let $\xi = \xi_0(\gamma)^{\sim}\xi$. Then the considered fragment of proof reads:

$$\begin{aligned} sym\,(\underline{\xi_0\gamma\xi}, \underline{\alpha}, \underline{\zeta}) &\longrightarrow sym\,(\underline{\xi_0\gamma^{\sim}\xi}, \underline{\beta}, \underline{\zeta}) \\ &\longrightarrow^* sym\,(\underline{\xi_0\gamma^{\sim}\xi}, [\,], \underline{\beta\zeta}) \\ &\longrightarrow^* sym\,(\underline{\gamma^{\sim}\xi}, \underline{\xi_0^{\sim}}, \underline{\beta\zeta}) \\ &\longrightarrow^* sym\,(\underline{\gamma^{\sim}\xi}, [\,], \underline{\xi_0^{\sim}\beta\zeta}) \\ &\longrightarrow^* sym\,(\underline{\xi}, \underline{\gamma}, \underline{\xi_0^{\sim}\beta\zeta}) \end{aligned}$$

[Firstly the rule of G_M corresponding to $\alpha \to \beta$ has been used, then the list $\underline{\beta}$ has been moved to the third place, after that the list $\underline{\xi_0}$ has been moved from the first to the second and then to the third place, and at last the list $\underline{\gamma}$ has been moved to the second place.]

LEMMA M2. *Let $k_1 = |\beta|$, $k_2 = |\xi_0|$. It is obvious that $k \geq 1$. Then the number of steps in the above fragment of proof satisfies the following equality:*

$$\text{NUMBER OF STEPS} = [1 + |\gamma| + k_2] + [k_1(k_1 + 3)/2 + k_2(k_2 + 3)/2]$$

Proof. For this equality we have the following deduction:

NUMBER OF STEPS	$= 1$	[By the rule of G_M corresponding to $\alpha \longrightarrow \beta$]
	$+ k_1(k_1 + 3)/2$	[By Lemma D]
	$+ k_2$	[By Lemma A]
	$+ k_2(k_2 + 3)/2$	[By Lemma D]
	$+ \|\gamma\|$	[By Lemma A]
	$= [1 + \|\gamma\| + k_2] + [k_1(k_1 + 3)/2 + k_2(k_2 + 3)/2]$. □	

3° Let $\zeta = \zeta_0\gamma\zeta$. Then the considered fragment of proof reads:

$$\begin{aligned} sym\,(\underline{\xi}, \underline{\alpha}, \underline{\zeta}) &\longrightarrow sym\,(\underline{\xi}, \underline{\beta}, \underline{\zeta_0\gamma\zeta}) \\ &\longrightarrow^* sym\,(\underline{\beta^{\sim}\xi}, [\,], \underline{\zeta_0\gamma\zeta}) \\ &\longrightarrow^* sym\,(\underline{\beta^{\sim}\xi}, \underline{\zeta_0\gamma}, \underline{\zeta}) \\ &\longrightarrow^* sym\,(\underline{\zeta_0^{\sim}\beta^{\sim}\xi}, \underline{\gamma}, \underline{\zeta}) \end{aligned}$$

[Firstly the rule of G_M corresponding to $\alpha \to \beta$ has been used, then the list $\underline{\beta}$ has been moved to the first place, after that the list $\underline{\zeta_0\gamma}$ has been moved from the third to the second place, and at last the list $\underline{\zeta_0}$ has been moved to the first place.]

LEMMA M3. *Let $k = |\zeta_0\gamma|$. Then for the number of steps in the above fragment of proof we have the following equality:*

$$\text{NUMBER OF STEPS} = [1 + |\beta| + |\zeta_0|] + [k(k + 3)/2]$$

Proof. For the above equality we have the following deduction:

NUMBER OF STEPS $= 1$	[By the rule of G_M corresponding to $\alpha \longrightarrow \beta$]
$+ \lvert\beta\rvert$	[By Lemma B]
$+ k(k+3)/2$	[By Lemma C]
$+ \lvert\zeta_0\rvert$	[By Lemma B]
$= [1 + \lvert\beta\rvert + \lvert\zeta_0\rvert] + [k(k+3)/2]$. $\square$	

4° Let $\xi = \gamma_1^{\sim}\xi$, $\beta = \gamma_2\boldsymbol{\beta}$, $\gamma = \gamma_1\gamma_2$, $|\gamma_2| > 0$. The possibility $|\gamma_2| = 0$ has been included in case 2°. Then the considered fragment of proof is based on the Lemma D and Lemma A respectively. In the case $\boldsymbol{\beta}$ is non-empty word this fragment reads:

$$\begin{aligned} sym\,(\underline{\xi}, \underline{\alpha}, \underline{\zeta}) &\longrightarrow\; sym\,(\gamma_1^{\sim}\underline{\xi}, \underline{\gamma}_2\underline{\boldsymbol{\beta}}, \underline{\zeta}) \\ &\longrightarrow^*\; sym\,(\underline{\gamma}_1^{\sim}\underline{\xi}, \underline{\gamma}_2, \underline{\boldsymbol{\beta}}\zeta) \\ &\longrightarrow^*\; sym\,(\underline{\xi}, \underline{\gamma}_1\underline{\gamma}_2, \underline{\boldsymbol{\beta}}\zeta) \end{aligned}$$

LEMMA M4. *Let $k = |\boldsymbol{\beta}|$. Then for the number of steps in the above fragment of proof in the case $k > 0$ the following equality:*

$$\text{NUMBER OF STEPS} \;= [1 + |\gamma_1|] + [|\gamma_2|k + k(k+3)/2]$$

holds. In the case $k = 0$ instead of this we have: NUMBER OF STEPS $= 1 + |\gamma_1|$

Proof. For the equality in the case $k > 0$ we have the following deduction:

NUMBER OF STEPS $= 1$	[By the rule of G_M corresponding to $\alpha \longrightarrow \beta$]
$+ \lvert\gamma_2\rvert k + k(k+3)/2$	[By Lemma D]
$+ \lvert\gamma_1\rvert$	[By Lemma A]
$= [1 + \lvert\gamma_1\rvert] + [\lvert\gamma_2\rvert k + k(k+3)/2]$	

In the case $k = 0$ only the first and last steps of the proof remain and the equality reduces to NUMBER OF STEPS $= 1 + |\gamma_1|$. $\square$

5° Let $\beta = \beta_0\gamma_1$, $\zeta = \gamma_2\zeta$, $\gamma = \gamma_1\gamma_2$, $|\gamma_2| > 0$. The possibility $|\gamma_2| = 0$ has been included in case 1°. Then the considered fragment of proof is based on the Lemma B and Lemma C respectively:

$$\begin{aligned} sym\,(\underline{\xi}, \underline{\alpha}, \underline{\zeta}) &\longrightarrow\; sym\,(\underline{\xi}, \underline{\beta}_0\underline{\gamma}_1, \underline{\gamma}_2\underline{\zeta}) \\ &\longrightarrow^*\; sym\,(\underline{\beta}_0^{\sim}\underline{\xi}, \underline{\gamma}_1, \underline{\gamma}_2\underline{\zeta}) \\ &\longrightarrow^*\; sym\,(\underline{\beta}_0^{\sim}\underline{\xi}, \underline{\gamma}_1\underline{\gamma}_2, \underline{\zeta}) \end{aligned}$$

The number of steps is determined in the next lemma.

LEMMA M5. *Let $k = |\gamma_2|$. Then for the number of steps in the above fragment of proof we gave the following equality:*

$$\text{NUMBER OF STEPS } = [1 + |\beta_0|] + [k|\gamma_1| + k(k+3)/2]$$

Proof. For this equality we have the following deduction:

$$\begin{aligned} \text{NUMBER OF STEPS } &= 1 && \text{[By the rule of } G_M \text{ corresponding to } \alpha \longrightarrow \beta] \\ &\quad + |\beta_0| && \text{[By Lemma B]} \\ &\quad + k|\gamma_1| + k(k+3)/2 && \text{[By Lemma C]} \\ &= [1 + |\beta_0|] + [k|\gamma_1| + k(k+3)/2]. \quad \square \end{aligned}$$

The fragments of proofs in the grammar G_P related to the naighbour applications of the rules corresponding to $\alpha \to \beta$, $\gamma \to \delta$ are the following:

1° In the case $\beta = \beta_0\gamma\boldsymbol{\beta}$:

$$[\underline{\xi}|[\alpha|\underline{\zeta}]] \longrightarrow [\underline{\xi}|[\beta_0\gamma\boldsymbol{\beta}|\underline{\zeta}]] \longrightarrow^* [\underline{\beta}_0^{\sim}\underline{\xi}|[\gamma|\underline{\boldsymbol{\beta}}\zeta]]$$

[One application of the rule corresponding to $\alpha \to \beta$ and $|\beta_0|$ applications of the rule (jv)]

2° In the case $\xi = \xi_0\gamma^{\sim}\boldsymbol{\xi}$:

$$[\underline{\xi}|[\alpha|\underline{\zeta}]] \longrightarrow [\xi_0\gamma^{\sim}\boldsymbol{\xi}|[\beta|\underline{\zeta}]] \longrightarrow^* [\underline{\xi}|[\gamma|\underline{\xi}_0^{\sim}\underline{\boldsymbol{\beta}}\zeta]]$$

[One application of the rule corresponding to $\alpha \to \beta$ and $|\gamma\xi_0|$ applications of the rule (jjj)]

3° In the case $\zeta = \zeta_0\gamma\boldsymbol{\zeta}$:

$$[\underline{\xi}|[\alpha|\underline{\zeta}_0\underline{\gamma}\boldsymbol{\zeta}]] \longrightarrow [\underline{\xi}|[\beta|\underline{\zeta}_0\underline{\gamma}\boldsymbol{\zeta}]] \longrightarrow^* [(\underline{\beta}\zeta_0)^{\sim}\underline{\xi}|[\gamma|\underline{\boldsymbol{\zeta}}]]|\underline{\beta}\zeta_0]$$

[One application of the rule corresponding to $\alpha \to \beta$ and $|\beta\zeta_0|$ applications of the rule (jv)]

4° In the case $\xi = \gamma_1^{\sim}\boldsymbol{\xi}$, $\beta = \gamma_2\boldsymbol{\beta}$, $\gamma = \gamma_1\gamma_2$:

$$[\underline{\gamma}_1^{\sim}\underline{\boldsymbol{\xi}}|[\alpha|\underline{\zeta}]] \longrightarrow [\underline{\gamma}_1^{\sim}\underline{\boldsymbol{\xi}}|[\gamma_2\underline{\boldsymbol{\beta}}|\underline{\boldsymbol{\zeta}}]] \longrightarrow^* [\underline{\boldsymbol{\xi}}|[\underline{\gamma}_1\gamma_2\underline{\boldsymbol{\beta}}|\underline{\zeta}]]$$

[One application of the rule corresponding to $\alpha \to \beta$ and $|\gamma_1|$ applications of the rule (jjj)]

5° In the case $\beta = \beta_0\gamma_1$, $\zeta = \gamma_2\boldsymbol{\zeta}$, $\gamma = \gamma_1\gamma_2$:

$$[\underline{\xi}|[\alpha|\underline{\gamma}_2\underline{\boldsymbol{\zeta}}]] \longrightarrow [\underline{\xi}|[\beta_0\gamma_1|\underline{\gamma}_2\underline{\boldsymbol{\zeta}}]] \longrightarrow^* [\beta_0^{\sim}\underline{\xi}|[\underline{\gamma}_1\gamma_2|\underline{\boldsymbol{\zeta}}]]$$

[One application of the rule corresponding to $\alpha \to \beta$ and $|\beta_0|$ applications of the rule (jv)]

The related numbers of steps are given in the following lemmae:

LEMMA P1. *In the case 1°:* NUMBER OF STEPS $= 1 + |\beta_0|$. $\square$

LEMMA P2. *In the case 2°:* NUMBER OF STEPS $= 1 + |\gamma| + |\xi_0|$. $\square$

LEMMA P3. *In the case* 3°*:* NUMBER OF STEPS $= 1 + |\beta| + |\zeta_0|$. □

LEMMA P4. *In the case* 4°*:* NUMBER OF STEPS $= 1 + |\gamma_1|$. □

LEMMA P5. *In the case* 5°*:* NUMBER OF STEPS $= 1 + |\beta_0|$. □

The above numbers are in fact the numbers occurring in the first brackets in the Lemmae M1 through M5 respectively, wherefrom it follows immediately:

$$\text{NUMBER OF STEPS IN } \pi_M(S)_I > \text{NUMBER OF STEPS IN } \pi_P(S)_I$$

We turn now to the second parts $\pi_M(S)_{II}$, $\pi_P(S)_{II}$ of the considered proof. As the last formula in the proof $\pi_M(S)_I$ was $sym\,(\underline{\rho}, \underline{\theta}, \underline{\sigma})$, we further deduce:

$sym\,(\underline{\rho}^{\sim}, \underline{\theta}, \underline{\sigma}) \longrightarrow^* sym\,(\underline{\rho}^{\sim}, \underline{\theta\sigma}, [\,])$

[Moving $\underline{\sigma}$ from the third to the second place. By Lemma C: $|\theta||\sigma|+|\sigma|(|\sigma|+3)/2$ steps.]

$\longrightarrow^* sym\,([\,], \underline{\rho\theta\sigma}, [\,])$

[Moving $\underline{\rho}$ from the first to the second place. By Lemma A: $|\rho|$ steps.]

$\longrightarrow^* \underline{\rho\theta\sigma}$

[Application of the rule (vii) of [MA93]: 1 step.]

$\longrightarrow^* \rho\theta\sigma, [\,]$

[Application of the rule (xii) of [MA93]: $|\rho\theta\sigma|$ step.]

$\longrightarrow^* \omega$, where $\omega = \rho\theta\sigma$

[Application of the rule (xi) of [MA93]: 1 step.]

Thus for the length of the proof we have:

$$\text{NUMBER OF STEPS IN } \pi_M(S)_{II} = [|\theta||\sigma| + |\sigma|(|\sigma|+3)/2] + [2|\rho| + |\theta| + |\sigma| + 2] \quad (1)$$

Similarly for the the proof $\pi_p(S)_{II}$ in the grammar G_P we have the deduction:

$$\begin{aligned}
[\underline{\rho}^{\sim}|[\theta|\underline{\sigma}]] &\longrightarrow^* [[\,]|[\rho\theta|\underline{\sigma}]] && \text{[Applications of the rule (jjj): } |\rho| \text{ step.]}\\
&\longrightarrow [\rho\theta|\underline{\sigma}] && \text{[Applications of the rule (v): 1 step.]}\\
&\longrightarrow^* \rho\theta\sigma, [\,] && \text{[Applications of the rule (vjj): } |\rho\theta\sigma| \text{ step.]}\\
&\longrightarrow \omega, \text{ where } \omega = \rho\theta\sigma && \text{[Applications of the rule (vj): 1 step.]}
\end{aligned}$$

Thus all together for the length of this proof it holds the following equality:

$$\text{NUMBER OF STEP IN } \pi_P(S)_{II} = 2|\rho| + |\theta| + |\sigma| + 2 \quad (2)$$

The obtained number of steps is in fact the number occurring in the second bracket of the equality (1) wherefrom it follows immediately the inequality:

$$\text{NUMBER OF STEP IN } \pi_M(S)_{II} > \text{ NUMBER OF STEP IN } \pi_P(S)_{II}$$

Thus we have just completed the proof of our main result:

THEOREM. *Let $\pi_M(S)$, $\pi_P(S)$ be corresponding proofs in the grammars G_M, G_P respectively. Then the lengths of these proofs satisfy the following inequality:*

$$\text{NUMBER OF STEP IN } \pi_M(S) > \text{ NUMBER OF STEP IN } \pi_P(S). \quad \square$$

It is easy to see that generally the length of the proof $\pi_M(S)$ considerably exceeds the length of the proof $\pi_P(S)$. For the first part of the proof, for example, the differences in length between two applications of the rules are the numbers occurring in the second brackets in the equalities obtained in Lemae M1 trough M5.

Acknowledgments. We would like to thank Goran Nenadić from the Faculty of mathematics, University of Belgrade for valuable comments and remarks.

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(Received 06 04 1994)